

# TARGET

## CLASS-X BOARD 2019

### MATHEMATICS

#### SOLUTION COPY SECTION-A

1. Without actually performing the long division, state whether  $\frac{77}{210}$  will have a terminating decimal expansion or a non-terminating repeating decimal expansion:

**Sol.** Let  $x = \frac{77}{210} = \frac{11}{30}$

Comparing equation with  $x = \frac{p}{q}$ , we get  $p = 11$  and  $q = 30 = 2^1 \times 3^1 \times 5^1$

Here  $q$  is not of the form  $2^n \times 5^m$

Thus,  $x = \frac{77}{210}$  have a non-terminating decimal expansion.

2. Determine  $k$  for the given equation  $x^2 + 2ax - k = 0$  for which  $x = -a$  is a solution of the equation.

**Sol.** Since  $x = -a$  is a root of the equation  $x^2 + 2ax - k = 0$

$$\therefore a^2 + 2a \times -a - k = 0 \Rightarrow k = -a^2.$$

3. If  $3x + k$ ,  $2x + 9$  and  $x + 13$  are three consecutive terms of an AP, find  $k$

**Sol.** Common difference =  $(2x + 9) - (3x + k) = (x + 13) - (2x + 9)$

$$-x + 9 - k = -x + 4$$

$$9 - k = 4$$

$$k = 5$$

4. Find the points on the  $y$ -axis, each of which is at a distance of 13 units from the point  $(-5, 7)$ .

**Sol.** Let  $A(-5, 7)$  be the given point and let  $P(0, y)$  be the required point on the  $y$ -axis.

$$\text{Then, } PA = 13 \text{ units} \Rightarrow PA^2 = 169$$

$$\Rightarrow (0 + 5)^2 + (y - 7)^2 = 169$$

$$\Rightarrow y^2 - 14y + 74 = 169$$

$$\Rightarrow y^2 - 14y - 95 = 0$$

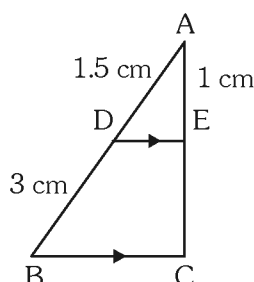
$$\Rightarrow (y - 19)(y + 5) = 0$$

$$\Rightarrow y - 19 = 0 \text{ or } y + 5 = 0$$

$$\Rightarrow y = 19 \text{ or } y = -5$$

Hence, the required points are  $(0, 19)$  and  $(0, -5)$ .

5. In given figure,  $DE \parallel BC$ ,  $AD = 1.5$  cm,  $BD = 3$  cm,  $AE = 1$  cm, Find  $EC$ .



**Sol.** In  $\Delta ABC$ ,  
 $\because DE \parallel BC$

$$\therefore \frac{AD}{DB} = \frac{AE}{EC} \quad \text{| By Basic Proportionality Theorem}$$

$$\Rightarrow \frac{1.5}{3} = \frac{1}{EC}$$

$$\Rightarrow EC = \frac{3}{1.5}$$

$$\Rightarrow EC = 2 \text{ cm}$$

**6.** If  $\sin \alpha = \frac{4}{5}$  and  $\cos \beta = \frac{12}{13}$ , find the value of  $\sin \alpha \cos \beta + \cos \alpha \sin \beta$ .

**Sol.** Given :  $\sin \alpha = \frac{4}{5} \Rightarrow \cos \alpha = \frac{3}{5}$   
 $\cos \beta = \frac{12}{13} \Rightarrow \sin \beta = \frac{5}{13}$   
 $\sin \alpha \cdot \cos \beta + \cos \alpha \cdot \sin \beta$   
 $= \frac{4}{5} \times \frac{12}{13} + \frac{3}{5} \times \frac{5}{13}$   
 $= \frac{48 + 15}{5 \times 13} = \frac{63}{65}$

**SECTION-B**

**7.** Find the H.C.F of 81 and 237 using euclid division algorithm.

**Sol.** Given integers are 81 and 237 such that  $81 < 237$ .

Applying division lemma to 81 and 237, we get

$$237 = 81 \times 2 + 75 \quad \text{.....(i)}$$

Since, the remainder  $75 \neq 0$ . So, consider the divisor 81 and the remainder 75 and apply division lemma to get

$$81 = 75 \times 1 + 6 \quad \text{.....(ii)}$$

We consider the new divisor 75 and the new remainder 6 and apply division lemma to get

$$75 = 6 \times 12 + 3 \quad \text{.....(iii)}$$

We consider the new divisor 6 and the new remainder 3 and apply division lemma to get

$$6 = 3 \times 2 + 0 \quad \text{.....(iv)}$$

The remainder at this stage is zero. So, the divisor at this stage or the remainder at the earlier stage i.e., 3 is the H.C.F of 81 and 237.

**8.** Check whether - 150 is a term of the AP : 11, 8, 5, 2, .....

**Sol.** The given list of numbers is

$$11, 8, 5, 2, \dots$$

So, first term  $a = 11$  and the common difference  $d = 8 - 11 = -3$ .

Let - 150 be the nth term of the given AP.

Then,  $a_n = - 150$

$$\Rightarrow a + (n - 1)d = - 150$$

$$\begin{aligned} \Rightarrow 11 + (n - 1)(-3) &= -150 \\ \Rightarrow -3(n - 1) &= -150 - 11 \\ \Rightarrow -3(n - 1) &= -161 \\ \Rightarrow 3(n - 1) &= 161 \\ \Rightarrow n - 1 &= \frac{161}{3} \\ \Rightarrow n &= \frac{161}{3} + 1 \\ \Rightarrow n &= \frac{164}{3} \end{aligned}$$

But  $n$  must be a positive integer. So  $-150$  is not a term of  $11, 8, 5, 2, \dots$

9. For what value of  $m$  and  $n$ , the following system of linear equations has an infinite number of solutions?  
 $3x + 4y = 12$ ,  $(m + n)x + 2(m - n)y = 5m - 1$ .

**Sol.**  $a_1 = 3, b_1 = 4, c_1 = -12$   
 $a_2 = m + n, b_2 = 2(m - n), c_2 = -(5m - 1)$

Therefore,  $\frac{a_1}{a_2} = \frac{3}{m+n}, \frac{b_1}{b_2} = \frac{4}{2(m-n)} = \frac{2}{m-n}$

$$\frac{c_1}{c_2} = \frac{-12}{-(5m-1)} = \frac{12}{5m-1}$$

For infinite number of solutions, we have

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

$$\begin{aligned} \Rightarrow \frac{3}{m+n} &= \frac{2}{m-n} = \frac{12}{5m-1} \\ \Rightarrow 3(m-n) &= 2(m+n) \text{ and } 2(5m-1) = 12(m-n) \\ \Rightarrow 3m - 3n &= 2m + 2n \text{ and } 10m - 2 = 12m - 12n \\ \Rightarrow m &= 5n \text{ and } m - 6n = -1 \end{aligned}$$

Putting  $m = 5n$  in  $m - 6n = -1$ , we get

$$\begin{aligned} 5n - 6n &= -1 \\ \Rightarrow -n &= -1 \\ \Rightarrow n &= 1 \\ \therefore m &= 5n = 5(1) = 5 \end{aligned}$$

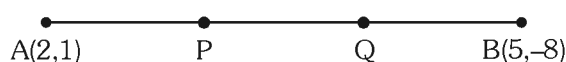
Hence,  $m = 5, n = 1$

10. The line segment joining the points  $A(2, 1)$  and  $B(5, -8)$  is trisected at the points  $P$  and  $Q$  such that  $P$  is nearer to  $A$ . If  $P$  also lies on the line given by  $2x - y + k = 0$ , find the value of  $k$ .

**Sol.** Since the line segment  $AB$  is trisected at point  $P$  and  $Q$ .

So,  $AP = PQ = QB$

$\Rightarrow AP : PB = 1 : 2$



$$\text{x coordinate of P is } \frac{1 \times 5 + 2 \times 2}{1 + 2} = 3$$

$$\text{y coordinate of P is } \frac{1 \times (-8) + 2 \times 1}{1 + 2} = -2$$

$\therefore$  Coordinates of P are (3, -2)

Since, P lies on the line  $2x - y + k = 0$

$$\Rightarrow 2 \times 3 + 2 + k = 0 \Rightarrow k = -8.$$

**11.** A bag contain 5 red balls and some blue balls. If the probability of drawing a blue ball is double that of a red ball, determine the number of blue balls in the bag.

**Sol.** Let the number of blue balls = x

So, total number of balls = 5 + x

Probability of drawing a blue ball

$$= \frac{\text{Number of favourable outcomes}}{\text{Total number of ball}}$$

$$= \frac{x}{5 + x} \quad \dots \text{ (i)}$$

Number of red balls = 5

And, total number of balls = 5 + x

Probability of drawing a red ball

$$= \frac{\text{Number of favourable outcomes}}{\text{Total number of ball}}$$

$$= \frac{5}{5 + x} \quad \dots \text{ (ii)}$$

As per condition,

$$\frac{x}{5 + x} = 2 \left( \frac{5}{5 + x} \right)$$

$$\Rightarrow \frac{x}{5 + x} = \frac{10}{5 + x}$$

$$\Rightarrow x = 10$$

Hence, the number of blue balls in the bag is 10.

**12.** A card is drawn at random from a well-shuffled deck of playing cards. Find the probability that the card drawn is :

(i) a King or a Jack

(ii) A non-Ace

(iii) a red card

(iv) neither King nor a Queen.

**Sol. (i)**  $P(\text{a King or a Jack}) = \frac{8}{52} = \frac{2}{13}$ .

**(ii)**  $P(\text{a non-Ace}) = 1 - P(\text{an Ace})$   
 $= 1 - \frac{4}{52} = 1 - \frac{1}{13} = \frac{12}{13}$ .

(iii) Number of red cards = 26

$$P(\text{a red card}) = \frac{26}{52} = \frac{1}{2}.$$

(iv) Number of Kings or Queens = 4 + 4 = 8

$$\begin{aligned} P(\text{neither a King nor a Queen}) &= 1 - P(\text{King or Queen}) \\ &= 1 - \frac{8}{52} = 1 - \frac{2}{13} = \frac{11}{13} \end{aligned}$$

### SECTION-C

13. Show that  $n^2 - 1$  is divisible by 8, if  $n$  is an odd positive integer.

**Sol.** Let  $n$  is in the form of  $a = bq + r$  where  $b = 4$ , then number  $n$  is in the form of  $4q, 4q + 1, 4q + 2$  and  $4q + 3$ . Here  $4q$  and  $4q + 2$  are even positive integer therefore  $n$  will be considered for  $4q + 1$  and  $4q + 3$  only for some integer  $q$ .

So, we have the following cases :

**Case I:** When  $n = 4q + 1$

In this case, we have

$$\begin{aligned} n^2 - 1 &= (4q + 1)^2 - 1 = 16q^2 + 8q + 1 - 1 \\ &= 16q^2 + 8q = 8q(2q + 1) \end{aligned}$$

$$\Rightarrow n^2 - 1 \text{ is divisible by } 8 \text{ [}\therefore 8_q (2q + 1) \text{ is divisible by } 8]$$

**Case II :** When  $n = 4q + 3$

In the case, we have

$$\begin{aligned} n^2 - 1 &= (4q + 3)^2 - 1 = 16q^2 + 24q + 9 - 1 \\ &= 16q^2 + 24q + 8 \end{aligned}$$

$$\Rightarrow n^2 - 1 \text{ is divisible by } 8$$

$$[\therefore 8(2q + 1)(q + 1) \text{ is divisible by } 8]$$

Hence,  $n^2 - 1$  is divisible by 8.

14. If two zeroes of the polynomial  $x^4 - 6x^3 - 26x^2 + 138x - 35$  are  $2 \pm \sqrt{3}$ , find other zeroes.

**Sol.** Let  $p(x) = x^4 - 6x^3 - 26x^2 + 138x - 35$

Since  $2 + \sqrt{3}$  and  $2 - \sqrt{3}$  are zeroes of  $p(x)$ , therefore  $(x - (2 + \sqrt{3}))$  and  $(x - (2 - \sqrt{3}))$  are factors of  $p(x)$ .

$$\begin{aligned} \therefore (x - (2 + \sqrt{3}))(x - (2 - \sqrt{3})) &= x^2 - (2 + \sqrt{3} + 2 - \sqrt{3})x + (2 + \sqrt{3})(2 - \sqrt{3}) \\ &= x^2 - 4x + 1 \text{ is also a factor of } p(x). \end{aligned}$$

Now we divide  $p(x)$  by  $x^2 - 4x + 1$ .

$$\begin{array}{r}
 x^2 - 2x - 35 \\
 \hline
 x^2 - 4x + 1 \left) \begin{array}{l} x^4 - 6x^3 - 26x^2 + 138x - 35 \\ x^4 - 4x^3 + x^2 \\ \hline -2x^3 - 27x^2 + 138x \\ -2x^3 + 8x^2 - 2x \\ \hline -35x^2 + 140x - 35 \\ -35x^2 + 140x - 35 \\ \hline 0 \end{array} \\
 \hline
 \end{array}
 \quad
 \begin{array}{l}
 \frac{x^4}{x^2} = x^2 \\
 \frac{-2x^3}{x^2} = -2x \\
 \frac{-35x^2}{x^2} = -35
 \end{array}$$

$$\begin{aligned}
 \therefore x^4 - 6x^3 - 26x^2 + 138x - 35 &= (x^2 - 4x + 1)(x^2 - 2x - 35) \\
 &= [x - (2 + \sqrt{3})][x - (2 - \sqrt{3})](x - 7)(x + 5)
 \end{aligned}$$

$\therefore$  Other zeroes of polynomial are 7 and -5.

15. A fraction becomes  $\frac{4}{5}$  when 3 is added to both numerator and denominator. If 3 is subtracted from both numerator and denominator it becomes  $\frac{1}{2}$ . Find the fraction. [3]

**Sol.** Let the required fraction be  $\frac{x}{y}$ .

$$\begin{aligned}
 \text{ATQ, } \frac{x+3}{y+3} &= \frac{4}{5} \\
 5x + 15 &= 4y + 12 \\
 5x - 4y &= -3 \quad \dots (1)
 \end{aligned}$$

$$\begin{aligned}
 \text{ATQ, } \frac{x-3}{y-3} &= \frac{1}{2} \\
 \Rightarrow 2x - 6 &= y - 3 \\
 \Rightarrow 2x - y &= 3 \quad \dots (2) \\
 \Rightarrow y &= 2x - 3
 \end{aligned}$$

Substituting  $y = 2x - 3$  in equation (1), we get

$$\begin{aligned}
 5x - 4(2x - 3) &= -3 \\
 \Rightarrow 5x - 8x + 12 &= -3 \\
 \Rightarrow -3x &= -15 \\
 \Rightarrow x &= 5
 \end{aligned}$$

Substituting  $x = 5$  in equation (2), we get

$$\begin{aligned}
 y &= 2 \times 5 - 3 \\
 \Rightarrow y &= 7
 \end{aligned}$$

Hence, the required fraction is  $\frac{5}{7}$ .

**16.** Show that the points  $(a, a)$ ,  $(-a, -a)$  and  $(-\sqrt{3}a, \sqrt{3}a)$  are the vertices of an equilateral triangle. Also find its area.

**OR**

Find the coordinates of the centre of a circle passing through the points  $A(2, 1)$ ,  $B(5, -8)$  and  $C(2, -9)$ . Also, find the radius of this circle.

**Sol.** Let  $A(a, a)$ ,  $B(-a, -a)$  and  $C(-\sqrt{3}a, \sqrt{3}a)$  be the given points. Then, we have

$$\begin{aligned} AB &= \sqrt{(-a-a)^2 + (-a-a)^2} \\ &= \sqrt{4a^2 + 4a^2} = 2\sqrt{2}a \end{aligned}$$

$$\begin{aligned} BC &= \sqrt{(-\sqrt{3}a+a)^2 + (\sqrt{3}a+a)^2} \\ &= a\sqrt{(1-\sqrt{3})^2 + (1+\sqrt{3})^2} \\ &= a\sqrt{1+3-2\sqrt{3}+1+3+2\sqrt{3}} \\ &= a\sqrt{8} = 2\sqrt{2}a \end{aligned}$$

and

$$\begin{aligned} AC &= \sqrt{(-\sqrt{3}a-a)^2 + (\sqrt{3}a-a)^2} \\ &= \sqrt{a^2(\sqrt{3}+1)^2 + a^2(\sqrt{3}-1)^2} \\ &= a\sqrt{(\sqrt{3}+1)^2 + (\sqrt{3}-1)^2} \\ &= a\sqrt{3+1+2\sqrt{3}+3+1-2\sqrt{3}} \\ &= a\sqrt{8} = 2\sqrt{2}a \end{aligned}$$

Clearly, we have  $AB = BC = AC$ .

Hence, the triangle  $ABC$  is an equilateral triangle.

**Proved.**

Now, area of triangle  $ABC = \frac{\sqrt{3}}{4}(\text{side})^2$  [∵ area of an equilateral triangle =  $\frac{\sqrt{3}}{4}(\text{side})^2$ ]

$$\begin{aligned} \Rightarrow \text{Area of triangle } ABC &= \frac{\sqrt{3}}{4} \times AB^2 \\ &= \frac{\sqrt{3}}{4} \times (2\sqrt{2}a)^2 = 2\sqrt{3}a^2 \text{ sq. units.} \end{aligned}$$

**OR**

**Sol.** Let  $P(x, y)$  be the centre of the circle passing through the points  $A(2, 1)$ ,  $B(5, -8)$  and  $C(2, -9)$ .

Then  $PA = PB = PC$

$$\Rightarrow PA^2 = PB^2 = PC^2$$

Now,  $PA^2 = PB^2$

$$\Rightarrow (x-2)^2 + (y-1)^2 = (x-5)^2 + (y+8)^2$$

$$\Rightarrow x^2 + y^2 - 4x - 2y + 5 = x^2 + y^2 - 10x + 16y + 89$$

$$\Rightarrow 6x - 18y - 84 = 0$$

$$\Rightarrow x - 3y = 14 \quad \dots (i)$$

And,  $PB^2 = PC^2$

$$\Rightarrow (x - 5)^2 + (y + 8)^2 = (x - 2)^2 + (y + 9)^2$$

$$\Rightarrow x^2 + y^2 - 10x + 16y + 89 = x^2 + y^2 - 4x + 18y + 85$$

$$\Rightarrow 6x + 2y - 4 = 0 \Leftrightarrow 3x + y = 2 \quad \dots (ii)$$

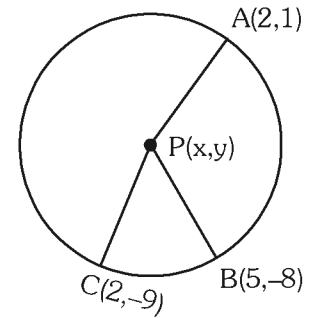
On solving (i) and (ii), we get  $x = 2$  and  $y = -4$

Hence, the coordinates of the centre of the circle are  $P(2, -4)$ .

Radius of the circle =  $PA$  ( $PA = PB = PC$ )

$$= \sqrt{(2 - 2)^2 + (1 + 4)^2}$$

$$= \sqrt{0^2 + 5^2} = \sqrt{25} = 5 \text{ units}$$

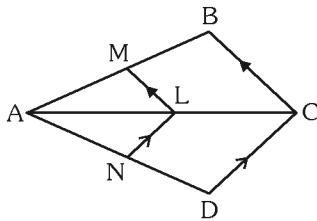


- 17.** ABC is an isosceles triangle right-angled at B. Similar triangles ACD and ABE are constructed on sides AC and AB. Find the ratio between the areas of  $\triangle ABE$  and  $\triangle ACD$ .

**OR**

In figure, if  $LM \parallel CB$  and  $LN \parallel CD$ , prove that

$$\frac{AM}{AB} = \frac{AN}{AD}$$



**Sol.** Let  $AB = BC = x$ .

It is given that  $\triangle ABC$  is right-angled at B.

$$\therefore AC^2 = AB^2 + BC^2$$

$$\Rightarrow AC^2 = x^2 + x^2$$

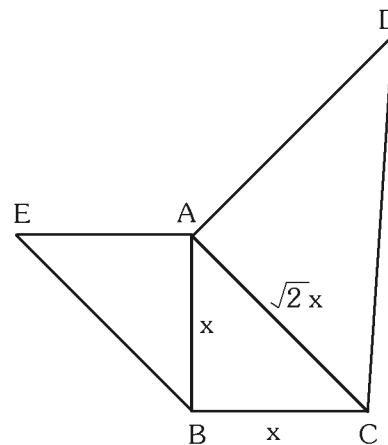
$$\Rightarrow AC = \sqrt{2} x$$

It is given that  $\triangle ABE \sim \triangle ACD$

$$\Rightarrow \frac{\text{Area}(\triangle ABE)}{\text{Area}(\triangle ACD)} = \frac{AB^2}{AC^2}$$

$$\Rightarrow \frac{\text{Area}(\triangle ABE)}{\text{Area}(\triangle ACD)} = \frac{x^2}{(\sqrt{2} x)^2}$$

$$\Rightarrow \frac{\text{Area}(\triangle ABE)}{\text{Area}(\triangle ACD)} = \frac{1}{2}$$



OR

**Sol.** In  $\triangle ACB$ ,  
 $\because LM \parallel CB$   
 Applying BPT

$$\therefore \frac{AM}{MB} = \frac{AL}{LC} \quad \dots (1)$$

In  $\triangle ACD$ ,  
 $\because LN \parallel CD$

$$\therefore \text{By BPT} \quad \frac{AL}{LC} = \frac{AN}{ND} \quad \dots (2)$$

From (1) and (2), we get

$$\frac{AM}{MB} = \frac{AN}{ND}$$

Now take the reciprocals on both sides  
 We have

$$\Rightarrow \frac{MB}{AM} = \frac{ND}{AN}$$

$$\Rightarrow \frac{MB}{AM} + 1 = \frac{ND}{AN} + 1 \quad | \text{ Adding 1 to both sides}$$

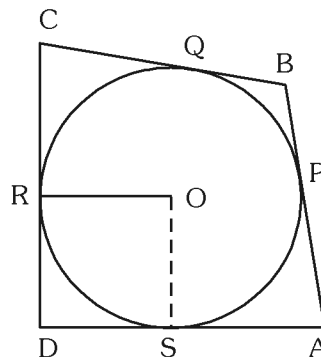
$$\Rightarrow \frac{MB + AM}{AM} = \frac{ND + AN}{AN}$$

$$\Rightarrow \frac{AB}{AM} = \frac{AD}{AN}$$

Now take the reciprocals on the both sides

$$\Rightarrow \frac{AM}{AB} = \frac{AN}{AD} \quad \text{Hence Proved}$$

**18.** In the figure,  $\angle ADC = 90^\circ$ ,  $BC = 38$  cm,  $CD = 28$  cm and  $BP = 25$  cm. Find the radius of the circle.



**Sol.** Since tangent to a circle is perpendicular to the radius through the point of contact.

$$\therefore \angle OSD = \angle ORD = 90^\circ, \text{ OR} = \text{OS}$$

$\Rightarrow$  DROS is a square.

Also  $BP = BQ$  [Tangents from an external point are equal]

$$\Rightarrow BQ = 25 \text{ cm} [BP = 25 \text{ cm}]$$

$$\begin{aligned} \Rightarrow BC - CQ &= 25 \\ \Rightarrow 38 - CQ &= 25 \text{ [BC = 38 cm]} \\ \Rightarrow CQ &= 38 - 25 = 13 \text{ cm} \\ \Rightarrow CR = CQ &= 13 \text{ [CQ = 13 cm]} \\ \Rightarrow CD - DR &= 13 \text{ [CR = CD - DR]} \\ \Rightarrow 28 - DR &= 13 \text{ [CD = 28 cm]} \\ \Rightarrow DR &= 28 - 13 = 15 \text{ cm} \end{aligned}$$

Since DROS is a square, so  $OR = DR = 15$  cm.

Hence, radius of the circle = 15 cm.

19. If  $\sin \theta = \frac{a^2 - b^2}{a^2 + b^2}$ , find the values of other five trigonometric ratios.

OR

If each of  $\alpha$ ,  $\beta$  and  $\gamma$  is a positive acute angle such that

$$\sin(\alpha + \beta - \gamma) = \frac{1}{2}, \cos(\beta + \gamma - \alpha) = \frac{1}{2} \text{ and } \tan(\gamma + \alpha - \beta) = 1$$

Find the value of  $\alpha$ ,  $\beta$  and  $\gamma$ .

Sol. We have

$$\sin \theta = \frac{\text{Perpendicular}}{\text{Hypotenuse}} = \frac{a^2 - b^2}{a^2 + b^2}$$

So, draw a right triangle right angled at B such that

$$\text{Perpendicular} = a^2 - b^2$$

$$\text{Hypotenuse} = a^2 + b^2 \text{ and } \angle BAC = \theta$$

By pythagoras theorem, we have

$$AC^2 = AB^2 + BC^2$$

$$\Rightarrow (a^2 + b^2)^2 = AB^2 + (a^2 - b^2)^2$$

$$\Rightarrow AB^2 = (a^2 + b^2)^2 - (a^2 - b^2)^2$$

$$\Rightarrow AB^2 = (a^4 + b^4 + 2a^2b^2) - (a^4 + b^4 - 2a^2b^2)$$

$$\Rightarrow AB^2 = 4a^2b^2 = (2ab)^2$$

$$\Rightarrow AB = 2ab$$

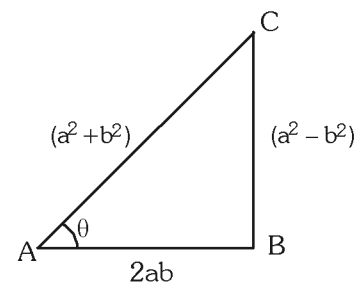
$$\text{Base} = AB = 2ab, \text{Perpendicular} = BC = a^2 - b^2 \text{ and, Hypotenuse} = AC = a^2 + b^2$$

$$\therefore \cos \theta = \frac{\text{Base}}{\text{Hypotenuse}} = \frac{2ab}{a^2 + b^2}$$

$$\Rightarrow \tan \theta = \frac{\text{Perpendicular}}{\text{Base}} = \frac{a^2 - b^2}{2ab}$$

$$\Rightarrow \operatorname{cosec} \theta = \frac{\text{Hypotenuse}}{\text{Perpendicular}} = \frac{a^2 + b^2}{a^2 - b^2}$$

$$\Rightarrow \sec \theta = \frac{\text{Hypotenuse}}{\text{Base}} = \frac{a^2 + b^2}{2ab}$$



$$\text{and, } \cot\theta = \frac{\text{Base}}{\text{Perpendicular}} = \frac{2ab}{a^2 - b^2}$$

OR

**Sol.** We have

$$\sin(\alpha + \beta - \gamma) = \frac{1}{2}, \cos(\beta + \gamma - \alpha) = \frac{1}{2} \text{ and } \tan(\gamma + \alpha - \beta) = 1$$

$$\Rightarrow \sin(\alpha + \beta - \gamma) = \sin 30^\circ, \cos(\beta + \gamma - \alpha) = \cos 60^\circ \text{ and } \tan(\gamma + \alpha - \beta) = \tan 45^\circ$$

$$\Rightarrow \alpha + \beta - \gamma = 30^\circ \quad \dots(\text{i})$$

$$\Rightarrow \beta + \gamma - \alpha = 60^\circ \quad \dots(\text{ii})$$

$$\Rightarrow \gamma + \alpha - \beta = 45^\circ \quad \dots(\text{iii})$$

Adding (i) to (ii) and (iii) respectively, we get

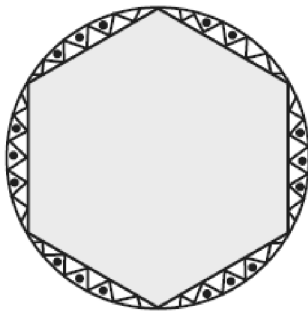
$$2\beta = 90^\circ \text{ and } 2\alpha = 75^\circ \Rightarrow \beta = 45^\circ \text{ and } \alpha = 37\frac{1}{2}^\circ$$

Putting the values of  $\alpha$  and  $\beta$  in (i), we get,

$$37\frac{1}{2}^\circ + 45^\circ - \gamma = 30^\circ \Rightarrow \gamma = 52\frac{1}{2}^\circ$$

$$\text{Hence } \alpha = 37\frac{1}{2}^\circ, \beta = 45^\circ \text{ and } \gamma = 52\frac{1}{2}^\circ$$

- 20.** A round table cover has six equal designs as shown in figure. If the radius of the cover is 28 cm, find the cost of making the designs at the rate of Rs. 0.35 per  $\text{cm}^2$ . (Use  $\sqrt{3} = 1.7$ )



**Sol.** Here,

$$r = 28 \text{ cm}$$

$$\theta = \frac{360^\circ}{6} = 60^\circ$$

Area of six designs = 6  $\times$  Area of one design

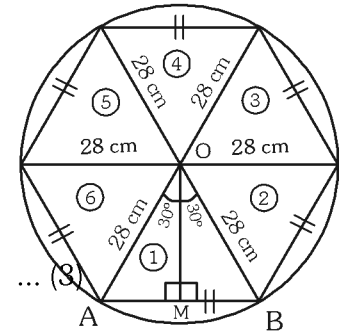
$$= 6(\text{Area of sector OAB} - \text{Area of } \Delta\text{OAB}) \quad \dots (1)$$

$$\text{Area of sector OAB} = \pi r^2 \cdot \frac{\theta}{360^\circ} = \frac{22}{7} (28)^2 \frac{60^\circ}{360^\circ} = \frac{1232}{3} \text{ cm}^2$$

$$= 410.67 \text{ cm}^2 \quad \dots (2)$$

$\therefore$  Area of  $\Delta\text{AOB}$

$$\begin{aligned}
 &= \frac{1}{2} \cdot AO \cdot OB \cdot \sin 60^\circ \\
 &= \frac{1}{2} \cdot 28 \cdot 28 \times \frac{\sqrt{3}}{2} \\
 &= 196\sqrt{3} = 196 \times 1.7 \\
 &= 333.2 \text{ cm}^2
 \end{aligned}$$



From (1), (2) and (3),

$$\begin{aligned}
 \text{Area of six designs} &= 6(410.67 - 333.2) \\
 &= 464.82 \text{ cm}^2
 \end{aligned}$$

$$\begin{aligned}
 \therefore \text{Cost of making the designs at the rate of Rs. } 0.35 \text{ per cm}^2 & \\
 &= 464.82 \times 0.35 \\
 &= \text{Rs. } 162.68
 \end{aligned}$$

- 21.** A 20 m deep well with diameter 7m is dug and the earth from digging is evenly spread out to form a platform 22m by 14m. Find the height of the platform.

**OR**

Metallic spheres of radii 6 cm, 8 cm and 10 cm, respectively, are melted to form a single solid sphere. Find the radius of the resulting sphere.

**Sol.** For well

$$\text{Diameter} = 7 \text{ m; Radius } (r) = \frac{7}{2} \text{ m; Depth } (h) = 20 \text{ m}$$

$$\begin{aligned}
 \therefore \text{Volume} &= \pi r^2 h \\
 &= \pi \left(\frac{7}{2}\right)^2 (20) = 245 \pi \text{ m}^3
 \end{aligned}$$

For Platform

$$\text{Length } (L) = 22 \text{ m}$$

$$\text{Breadth } (B) = 14 \text{ m}$$

Let the height of the platform be H m.

Then, Volume of the platform

$$= LBH = 22 \times 14 \times H = 308 H \text{ m}^3$$

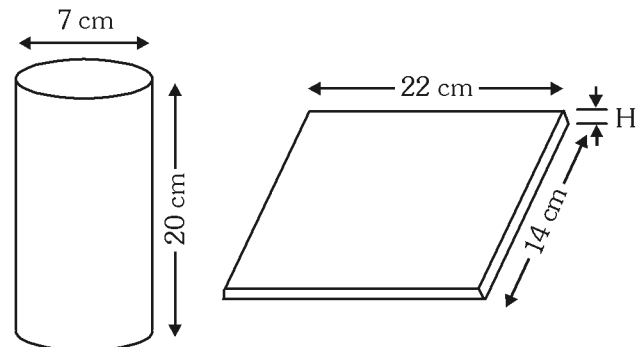
According to the question,

$$308 H = 245 \pi$$

$$H = \frac{245\pi}{308} = \frac{245 \times 22}{308 \times 7}$$

$$\Rightarrow H = 2.5$$

Hence, the height of the platform is 2.5 m.

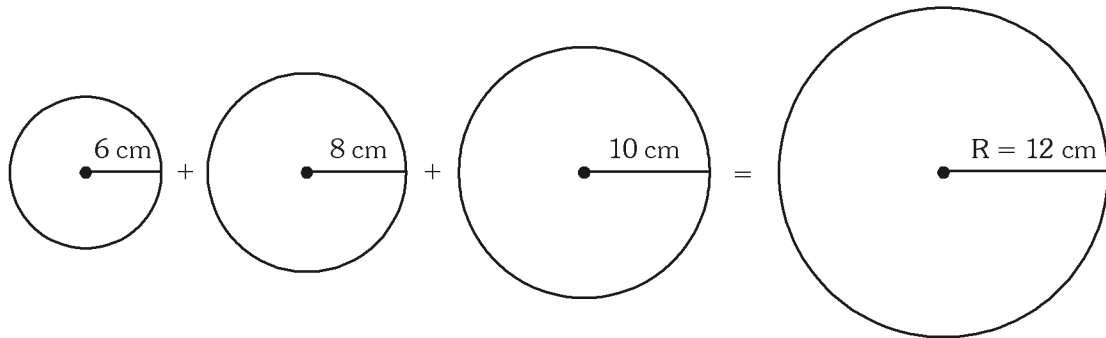


**OR**

$$\text{Sol. Volume of sphere of radius 6 cm} = \frac{4}{3} \pi (6)^3 \text{ cm}^3$$

$$\text{Volume of sphere of radius 8 cm} = \frac{4}{3} \pi (8)^3 \text{ cm}^3$$

$$\text{Volume of sphere of radius 10 cm} = \frac{4}{3} \pi (10)^3 \text{ cm}^3$$



Let the radius of the resulting sphere be R cm.

Then, volume of the resulting sphere =  $\frac{4}{3} \pi R^3 \text{ cm}^3$

According to the question,  $\frac{4}{3} \pi R^3 = \frac{4}{3} \pi (6)^3 + \frac{4}{3} \pi (8)^3 + \frac{4}{3} \pi (10)^3$

$$\Rightarrow R^3 = (6)^3 + (8)^3 + (10)^3$$

$$\Rightarrow R^3 = 216 + 512 + 1000$$

$$\Rightarrow R^3 = 1728$$

$$\Rightarrow R = (1728)^{1/3}$$

$$\Rightarrow R = 12$$

Hence, the radius of the resulting sphere is 12 cm.

22. Find the mean of the following distribution using step deviation method :

Class	0 - 10	10 - 20	20 - 30	30 - 40	40 - 50
Frequency	8	12	10	11	9

Sol.

Class Interval	Midvalue ( $x_i$ )	( $f_i$ )	$d_i = x_i - 25$	$u_i = (x_i - 25)/10$	$f_i u_i$
0 - 10	5	8	- 20	- 2	- 16
10 - 20	15	12	- 10	- 1	- 12
20 - 30	25	10	0	0	0
30 - 40	35	11	10	1	11
40 - 50	45	9	20	2	18
<b>Total</b>		<b>50</b>			<b>1</b>

$$A = 25, h = 10, N = 50 \text{ and } \sum f_i u_i = 1$$

$$\text{Mean} = A + \left( \frac{\sum f_i u_i}{N} \right) \times h$$

$$= 25 + \left( \frac{1}{50} \right) \times 10$$

$$= \frac{126}{5} = 25.20$$

**SECTION-D**

- 23.** While boarding an aeroplane, a passenger got hurt. The pilot showing promptness and concern, made arrangements to hospitalise the injured and so the plane started late by 30 minutes. To reach the destination, 1500 km away in time, the pilot increased the speed by 100 km/h. Find the original speed of the plane.

**OR**

A swimming pool is filled with three pipes with uniform flow. The first two pipes operating simultaneously, fill the pool in the same time during which the pool is filled by the third pipe alone. The second pipe fills the pool five hours faster than the first pipe and four hours slower than the third pipe. Find the time required by each pipe to fill the pool separately.

**Sol.** Let the original speed of the plane be  $x$  km/h

Destination = 1500 km

$$\text{So, usual time taken} = \frac{1500}{x} \text{ hrs}$$

Now, speed is increased by 100 km/h and plane started late by 30 minutes.

$$\text{So, } \left( \frac{1500}{x} - \frac{30}{60} \right) (x + 100) = 1500$$

$$\Rightarrow \left( \frac{1500}{x} - \frac{1}{2} \right) (x + 100) = 1500$$

$$\Rightarrow 1500 + \frac{150000}{x} - \frac{x}{2} - 50 = 1500$$

$$\Rightarrow 3000x + 300000 - x^2 - 100x = 3000x$$

$$\Rightarrow x^2 + 100x - 300000 = 0$$

$$\Rightarrow x = \frac{-100 \pm \sqrt{10000 + 4 \times 300000}}{2}$$

$$\Rightarrow x = \frac{-100 \pm 1100}{2}$$

$$\Rightarrow x = 500 \text{ as } x = -600 \text{ is not possible}$$

Hence, original speed of the plane = 500 km/h

**OR**

**Sol.** Let  $V$  be the volume of the pool and  $x$  the number of hours required by the second pipe alone to fill the pool. Then, the first pipe takes  $(x + 5)$  hours, while the third pipe takes  $(x - 4)$  hours to fill the pool. So, the parts of the pool filled by the first, second and third pipes in one hour are respectively.

$$\frac{V}{x+5}, \frac{V}{x} \text{ and } \frac{V}{x-4}$$

Let the time taken by the first and second pipes to fill the pool simultaneously be  $t$  hours. Then, the third pipe also takes the same time to fill the pool.

$$\therefore \left( \frac{V}{x+5} + \frac{V}{x} \right) t = \text{Volume of the pool.}$$

$$\text{Also, } \frac{V}{x-4} t = \text{Volume of the pool.}$$

$$\Rightarrow \left( \frac{V}{x+5} + \frac{V}{x} \right) t = \frac{V}{x-4} t$$

$$\Rightarrow \frac{1}{x+5} + \frac{1}{x} = \frac{1}{x-4}$$

$$\Rightarrow (2x+5)(x-4) = x^2 + 5x$$

$$\Rightarrow x^2 - 8x - 20 = 0$$

$$\Rightarrow x^2 - 10x + 2x - 20 = 0$$

$$\Rightarrow (x-10)(x+2) = 0 \Rightarrow x = 10 \text{ or } x = -2$$

But,  $x$  cannot be negative. So,  $x = 10$ .

Hence, the timings required by first, second and third pipes to fill the pool individually are 15 hours, 10 hours and 6 hours respectively.

**24.** If  $a, b, c$  are the  $p^{\text{th}}, q^{\text{th}}$  and  $r^{\text{th}}$  terms of an AP, then prove that

$$a(q-r) + b(r-p) + c(p-q) = 0.$$

**Sol.** Let the first term and the common difference of the AP be  $A$  and  $D$  respectively.

$$\begin{aligned} p^{\text{th}} \text{ term} &= a && | \text{ Given} \\ \Rightarrow A + (p-1)D &= a && \dots (1) \\ q^{\text{th}} \text{ term} &= b && \} \text{ Given} \\ \Rightarrow A + (q-1)D &= b && \dots (2) \\ r^{\text{th}} \text{ term} &= c && | \text{ Given} \\ \Rightarrow A + (r-1)D &= c && \dots (3) \end{aligned}$$

Multiplying equations (1), (2) and (3) by  $q-r, r-p$  and  $p-q$  respectively, we get

$$\begin{aligned} a(q-r) + b(r-p) + c(p-q) &= [A + (p-1)D](q-r) + [A + (q-1)D](r-p) + [A + (r-1)D](p-q) \\ &= A[q-r + r-p + p-q] + D[(p-1)(q-r) + (q-1)(r-p) + (r-1)(p-q)] \\ &= A(0) + D(0) \\ &= 0 \end{aligned}$$

**25.** State and prove basic proportionality Theorem (Thales theorem)

**OR**

If two sides and a median bisecting the third side of a triangle are respectively proportional to the corresponding sides and the median of another triangle, then the two triangles are similar.

**Sol. Statement :** If a line is drawn parallel to one side of a triangle intersecting the other two sides, then it divides the two sides in the same ratio.

**Given :** A triangle  $ABC$  in which  $DE \parallel BC$ , and intersects  $AB$  in  $D$  and  $AC$  in  $E$ .

**To prove :**  $\frac{AD}{DB} = \frac{AE}{EC}$

**Construction :** Join  $BE, CD$  and draw  $EF \perp BA$  and  $DG \perp CA$ .

**Proof :**  $\because EF \perp AB$ . Therefore, EF is the height of triangles ADE and DBE.

Now,  $\text{Area}(\triangle ADE) = \frac{1}{2}(\text{base} \times \text{height}) = \frac{1}{2}(AD \cdot EF)$

and,  $\text{Area}(\triangle DBE) = \frac{1}{2}(\text{base} \times \text{height}) = \frac{1}{2}(DB \cdot EF)$

$$\therefore \frac{\text{Area}(\triangle ADE)}{\text{Area}(\triangle DBE)} = \frac{1/2(AD \cdot EF)}{1/2(DB \cdot EF)} = \frac{AD}{DB} \quad \dots (1)$$

Similarly, we have

$$\frac{\text{Area}(\triangle ADE)}{\text{Area}(\triangle DEC)} = \frac{1/2(AE \cdot DG)}{1/2(EC \cdot DG)} = \frac{AE}{EC} \quad \dots (2)$$

But,  $\triangle DBE$  and  $\triangle DEC$  are on the same base DE and between the same parallels DE and BC.

$$\therefore \text{Area}(\triangle DBE) = \text{Area}(\triangle DEC)$$

$$\Rightarrow \frac{1}{\text{Area}(\triangle DBE)} = \frac{1}{\text{Area}(\triangle DEC)} \quad [\text{Taking reciprocals of both sides}]$$

$$\Rightarrow \frac{\text{Area}(\triangle ADE)}{\text{Area}(\triangle DBE)} = \frac{\text{Area}(\triangle ADE)}{\text{Area}(\triangle DEC)} \quad [\text{Multiplying both sides by Area}(\triangle ADE)]$$

$$\Rightarrow \frac{AD}{DB} = \frac{AE}{EC} \quad \text{Hence Proved}$$

**OR**

**Sol. Given :** Two triangles ABC and DEF in which AP and DQ are the medians such that  $\frac{AB}{DE} = \frac{AC}{DF} = \frac{AP}{DQ}$

**To Prove :**  $\triangle ABC \sim \triangle DEF$

**Construction :** Produce AP to G so that  $PG = AP$ . Join CG. Also, produce DQ to H so that  $QH = DQ$ . Join FH.

**Proof :** In  $\triangle APB$  and  $\triangle GPC$ , we have

$$BP = CP \quad [\because AP \text{ is the median}]$$

$$AP = GP \quad [\text{By construction}]$$

and,  $\angle APB = \angle CPG$  [Vertically opposite angles]

So, by SAS-criterion of congruence, we have

$$\triangle APB \cong \triangle GPC$$

$$\Rightarrow AB = GC \quad \dots (1)$$

Again, In  $\triangle DQE$  and  $\triangle HQF$ , we have

$$EQ = FQ \quad [\because DQ \text{ is the median}]$$

$$DQ = HQ \quad [\text{By construction}]$$

and,  $\angle DQE = \angle HQF$  [Vertically opposite angles]

So, by SAS-criterion of congruence, we have

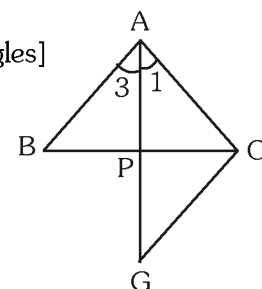
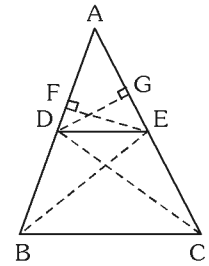
$$\triangle DQE \cong \triangle HQF$$

$$\Rightarrow DE = HF \quad \dots (2)$$

Now,  $\frac{AB}{DE} = \frac{AC}{DF} = \frac{AP}{DQ}$  [Given]

From equation (1) and (2), we have

$$\Rightarrow \frac{GC}{HF} = \frac{AC}{DF} = \frac{AP}{DQ}$$



$$\Rightarrow \frac{GC}{HF} = \frac{AC}{DF} = \frac{2AP}{2DQ}$$

$$\Rightarrow \frac{GC}{HF} = \frac{AC}{DF} = \frac{AG}{DH}$$

[∵ 2AP = AG and 2DQ = DH]

By SSS similarity, we have

$$\Rightarrow \triangle AGC \sim \triangle DHF$$

$$\Rightarrow \angle 1 = \angle 2$$

Similarly, we have

$$\angle 3 = \angle 4$$

$$\therefore \angle 1 + \angle 3 = \angle 2 + \angle 4$$

$$\Rightarrow \angle A = \angle D$$

... (3)

Thus, in  $\triangle ABC$  and  $\triangle DEF$ , we have

$$\angle A = \angle D$$

[From (3)]

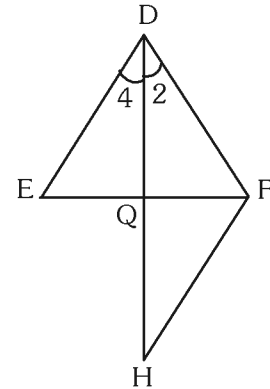
and, 
$$\frac{AB}{DE} = \frac{AC}{DF}$$

[Given]

So, by SAS-criterion of similarity, we have

$$\triangle ABC \sim \triangle DEF$$

Hence Proved



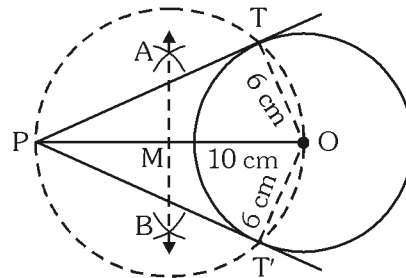
**26.** Draw a circle of radius 6 cm. From a point 10 cm away from its centre, construct a pair of tangents to the circle and measure their lengths. Write steps of constructions.

**Sol. Given :** A circle of radius 6 cm and a point P is 10 cm away from its centre.

**Required :** A pair of tangents.

**Steps of Construction :**

1. Draw a circle  $C(O, r)$  with centre O and radius 6 cm.
2. Take a point P, such that  $OP = 10$  cm.
3. Draw AB, the perpendicular bisector of OP and let it intersect OP in M.
4. With M as centre and PM or MO as radius, draw another circle intersecting the given circle in T and T'.
5. Join PT and PT'.



Thus, PT and PT' are the required tangents from point P to the circle  $C(O, r)$ .

**27.** Prove that :  $(\sin \theta + \sec \theta)^2 + (\cos \theta + \operatorname{cosec} \theta)^2 = (1 + \sec \theta \operatorname{cosec} \theta)^2$

**Sol.**  $(\sin \theta + \sec \theta)^2 + (\cos \theta + \operatorname{cosec} \theta)^2$

$$\left( \sin \theta + \frac{1}{\cos \theta} \right)^2 + \left( \cos \theta + \frac{1}{\sin \theta} \right)^2$$

$$\Rightarrow \sin^2 \theta + \frac{1}{\cos^2 \theta} + \frac{2 \sin \theta}{\cos \theta} + \cos^2 \theta + \frac{1}{\sin^2 \theta} + \frac{2 \cos \theta}{\sin \theta}$$

$$\Rightarrow (\sin^2 \theta + \cos^2 \theta) + \left( \frac{1}{\cos^2 \theta} + \frac{1}{\sin^2 \theta} \right) + 2 \left( \frac{\sin \theta}{\cos \theta} + \frac{\cos \theta}{\sin \theta} \right)$$

$$\Rightarrow (\sin^2 \theta + \cos^2 \theta) + \left( \frac{\sin^2 \theta + \cos^2 \theta}{\sin^2 \theta \cos^2 \theta} \right) + \frac{2(\sin^2 \theta + \cos^2 \theta)}{\sin \theta \cos \theta}$$

$$\Rightarrow 1 + \frac{1}{\sin^2 \theta \cos^2 \theta} + \frac{2}{\sin \theta \cos \theta}$$

$$\Rightarrow (1 + \sec \theta \operatorname{cosec} \theta)^2 = \text{RHS}$$

- 28.** A 1.5 m tall boy is standing at some distance from a 30 m tall building. The angle of elevation from his eyes to the top of the building increases from  $30^\circ$  to  $60^\circ$  as he walks towards the building. Find the distance he walked towards the building.

**Sol.** Let DG be the building of height 30 m. AE is the height of the boy i.e., 1.5 m.

Let 'A' and 'B' be the two positions of the eyes.

$$\therefore \quad \quad \quad AE = BF = CG = 1.5 \text{ m}$$

As  $\quad \quad \quad DG = 30 \text{ m}$

$$\therefore \quad \quad \quad DC = DG - CG = 30 - 1.5 = 28.5 \text{ m}$$

As  $\quad \quad \quad \angle DAB = 30^\circ \Rightarrow \angle DAC = 30^\circ \text{ and } \angle DBC = 60^\circ$

In  $\triangle ACD$ , we have

$$\tan 30^\circ = \frac{DC}{AC}$$

$$\Rightarrow \quad \quad \quad \frac{1}{\sqrt{3}} = \frac{28.5}{AC}$$

$$\Rightarrow \quad \quad \quad AC = 28.5\sqrt{3}$$

In  $\triangle BCD$ , we have

$$\tan 60^\circ = \frac{DC}{BC}$$

$$\Rightarrow \quad \quad \quad \sqrt{3} = \frac{28.5}{BC}$$

$$\Rightarrow \quad \quad \quad BC = \frac{28.5}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$$

$$\Rightarrow \quad \quad \quad BC = \frac{28.5\sqrt{3}}{3}$$

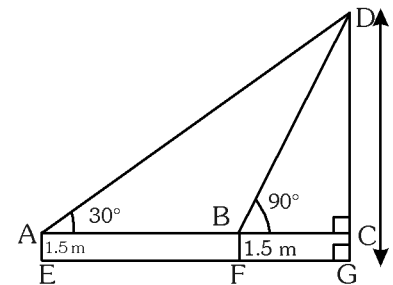
$\therefore$  The distance walked by the boy towards the building

$$= AC - BC$$

$$= 28.5\sqrt{3} - \frac{28.5\sqrt{3}}{3} = \frac{28.5\sqrt{3} \times 3 - 28.5\sqrt{3}}{3}$$

$$= \frac{28.5(3\sqrt{3} - \sqrt{3})}{3} = \frac{28.5 \times 2\sqrt{3}}{3} = \frac{57\sqrt{3}}{3} = 19\sqrt{3} \text{ m}$$

Hence, the distance walked by the boy towards the building is  $19\sqrt{3} \text{ m}$ .



29. A metallic right circular cone 20 cm high and whose vertical angle is  $60^\circ$  is cut into two parts at the middle of its height by a plane parallel to its base. If the frustum so obtained be drawn into a wire of diameter  $\frac{1}{16}$  cm, find the length of the wire.

**Sol.**

$$\tan 30^\circ = \frac{r_2}{10}$$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{r_2}{10}$$

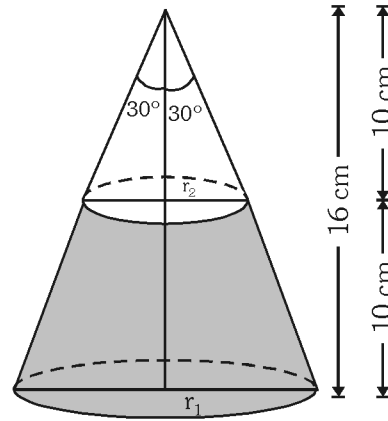
$$\Rightarrow r_2 = \frac{10}{\sqrt{3}} \text{ cm}$$

$$\tan 30^\circ = \frac{r_1}{20}$$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{r_1}{20}$$

$$\Rightarrow r_1 = \frac{20}{\sqrt{3}} \text{ cm}$$

$$h = 10 \text{ cm}$$



$$\therefore \text{Volume} = \frac{1}{3} \pi h (r_1^2 + r_2^2 + r_1 r_2)$$

$$= \frac{1}{3} \times \frac{22}{7} \times 10 \times \left\{ \left( \frac{20}{\sqrt{3}} \right)^2 + \left( \frac{10}{\sqrt{3}} \right)^2 + \left( \frac{20}{\sqrt{3}} \right) \left( \frac{10}{\sqrt{3}} \right) \right\}$$

$$= \frac{1}{3} \times \frac{22}{7} \times 10 \times \left\{ \frac{400}{3} + \frac{100}{3} + \frac{200}{3} \right\} = \frac{1}{3} \times \frac{22}{7} \times 10 \times \frac{700}{3} = \frac{22000}{9} \text{ cm}^3$$

$$\text{Diameter of the wire} = \frac{1}{16} \text{ cm}$$

$$\therefore \text{Radius of the wire (r)} = \frac{1}{2} \times \frac{1}{16} = \frac{1}{32} \text{ cm}$$

Let the length of the wire be  $l$  cm.

Then, volume of the wire =  $\pi r^2 l$

$$= \frac{22}{7} \left( \frac{1}{32} \right)^2 l = \frac{111}{3584} \text{ cm}^3$$

According to the question,

$$\frac{111}{3584} = \frac{22000}{9}$$

$$\Rightarrow l = \frac{22000 \times 3584}{11 \times 9} \Rightarrow l = \frac{2000 \times 3584}{9}$$

$$\Rightarrow l = \frac{7168000}{9} \text{ cm}$$

$$\Rightarrow l = 796444.44 \text{ cm}$$

$$\Rightarrow l = 7964.4 \text{ m}$$

Hence, the length of the wire is 7964.4 m.

30. The median of the following data is 52.5. Find the values of x and y if the sum of all frequency is 100.

Classes	Frequency
0 - 10	2
10 - 20	5
20 - 30	x
30 - 40	12
40 - 50	17
50 - 60	20
60 - 70	y
70 - 80	9
80 - 90	7
90 - 100	4

OR

During the medical checkup of 35 students of a class their weights were recorded as follows :

Weight (in kg)	38 - 40	40 - 42	42 - 44	44 - 46	46 - 48	48 - 50	50 - 52
Number of students	3	2	4	5	14	4	3

Draw a less than type and a more than type ogive from the given data on the same graph and hence obtain the median weight from the graph.

Sol.

Class Interval	Frequency	c.f.
0 - 10	2	2
10 - 20	5	7
20 - 30	x	7 + x
30 - 40	12	19 + x
40 - 50	17	36 + x
50 - 60	20	56 + x
60 - 70	y	56 + x + y
70 - 80	9	65 + x + y
80 - 90	7	72 + x + y
90 - 100	4	76 + x + y
		<b>100</b>

$$76 + x + y = 100$$

$$x + y = 24$$

... (1)

Given that median = 52.5

so median class = 50-60

$$l = 50, \frac{N}{2} = 50, h = 10, f = 20, F = 36 + x$$

$$\text{Median} = 1 + \frac{\frac{N}{2} - F}{f} \times h$$

$$52.5 = 50 + \frac{50 - 36 - x}{20} \times 10$$

$$x = 9, y = 24 - 9 = 15$$

**OR**

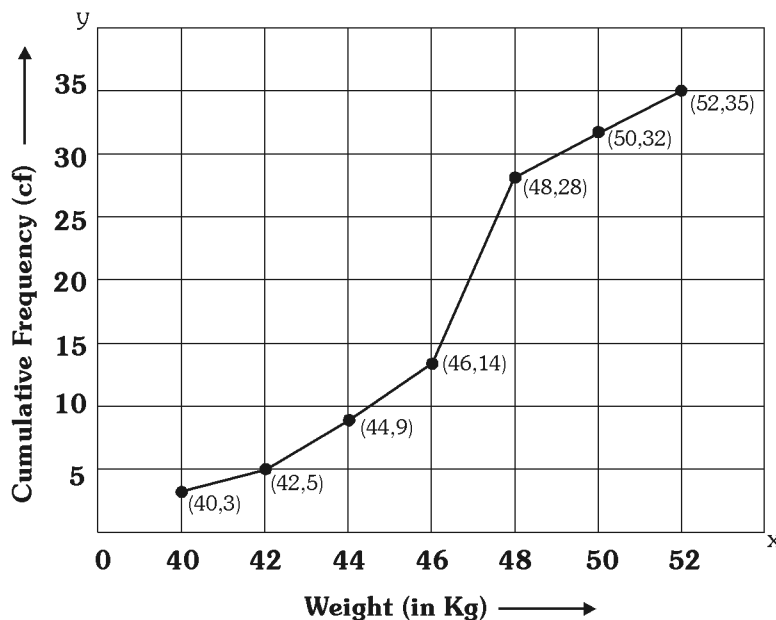
**Sol.** We first prepare the cumulative frequency table by 'less than' method as given below :

Weight (in Kg)	Cumulative Frequency
Less than 40	3
Less than 42	5
Less than 44	9
Less than 46	14
Less than 48	28
Less than 50	32
Less than 52	35

Now, we mark X-axis with upper limit

Y-axis with cumulative frequency.

Then, we plot the points (40, 3), (42, 5), (44, 9), (46, 14), (48, 28), (50, 32), (52, 35)



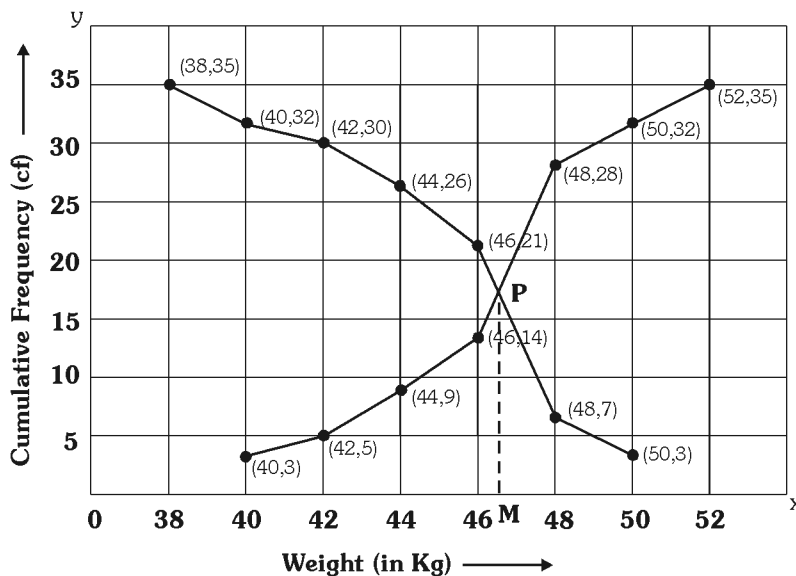
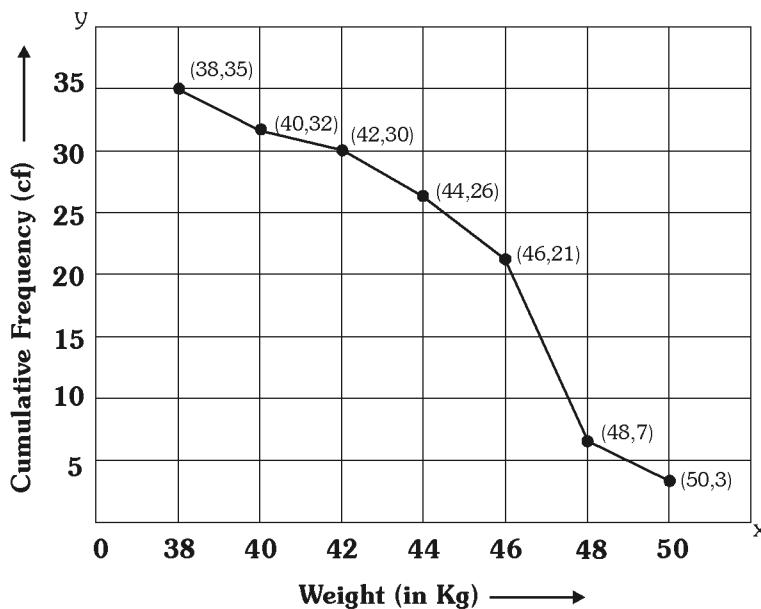
'More than' Method

Weight (in Kg)	Cumulative Frequency
More than 38	35
More than 40	32
More than 42	30
More than 44	26
More than 46	21
More than 48	7
More than 50	3

Now, we mark x-axis with lower class limit

y-axis with cumulative frequency

We plot the points (38, 35), (40, 32), (42, 30), (44, 26), (46, 21), (48, 7), (50, 3)



We find the two types of cumulative freq. curves intersect at point P. From point P perpendicular PM is drawn on X-axis. The value of height corresponding to M is 46.5.

Hence, Median is 46.5.

# TARGET CBSE

## CLASS-X BOARD 2019

### SCIENCE

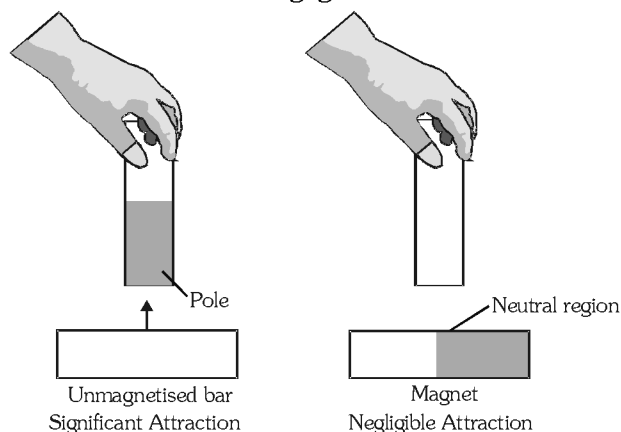
## SOLUTION COPY

### SECTION-A

1. Define life processes. [1]
- Sol.** The basic functions performed by living organisms to maintain their life on this earth are called life processes.
2. What are the function of lipase and amylase respectively? [1]
- Sol. Lipase:** Digestion of fat.  
**Amylase:** Digestion of starch.
3. (a) Why are the heating elements of electric toaster and electric iron made of an alloy rather than pure metal? [1 + 1 = 2]  
 (b) An electric iron of voltage 20 V takes a current of 5 A. Calculate the heat developed in 30 seconds.
- Sol.** (a) Resistivity of alloys are generally higher than that of their constituent metals, they produce more heat as compared to a pure metal. Also, they do not burn (oxidise) readily at high temperatures. Thus, they are used in electrical heating devices like geyser, electric iron, toasters, etc.  
 (b) As we know,  $H = Vit = 20 \times 5 \times 30 = 3000 \text{ J}$
4. One of the two identical bars is magnetised. How will you find out, without using any aid, which one of them is magnetised? [2]

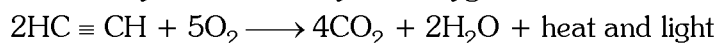
**Sol.** First, we will take one of the bar in our hand and touch its end with the centre of the other bar. If there is a significant attraction between them, then the bar in hand is magnetised. If there is a negligible attraction between the two bars, then the bar in hand is unmagnetised, this means the other one is magnetised.

This is because the central part of a magnet is its neutral region (the region where the attractive power is minimum). If the magnetised bar is in our hand, this means we are touching its pole to the unmagnetised bar, there will be a significant attraction between the two bars. If the unmagnetised bar is in our hand, this means we are touching its end with the neutral region of the magnetised bar, thus, the attraction between them will be negligible.



5. A mixture of oxygen and ethyne is burnt for welding. Can you tell why a mixture of ethyne and air is not used? [2]

**Ans.** Ethyne is an unsaturated hydrocarbon, therefore, combustion of ethyne in air produces a yellow flame with black smoke due to the presence of unburnt carbon in it. Due to this incomplete combustion heat produced is also low and a high temperature usually needed for welding cannot be attained. That is why mixture of ethyne & oxygen is used :



Ethyne    oxygen

6. On what factors does the magnetic field produced by a current carrying solenoid depend? [3]

**Sol.** Magnetic field produced by a current carrying solenoid depends on the following factors :

- magnitude of current
- number of turns per unit length of solenoid
- material of core

7. Differentiate between conventional and non-conventional sources. [3]

**Sol.**

<b>Conventional sources</b>	<b>Non-Conventional sources</b>
<ol style="list-style-type: none"> <li>Conventional sources are used extensively.</li> <li>They meet a major portion of our energy demand.</li> <li>Example – Fossil fuels               <ul style="list-style-type: none"> <li>– Hydro energy</li> <li>– Biomass and wind energy (as they are being used since ancient time)</li> </ul> </li> </ol>	<ol style="list-style-type: none"> <li>These are not used as extensively as conventional sources.</li> <li>Non-conventional sources meet are energy demand only on a limited scale.</li> <li>Examples – Solar energy               <ul style="list-style-type: none"> <li>– ocean energy</li> <li>– Geothermal energy</li> <li>– Nuclear energy</li> </ul> </li> </ol>

8. Give three uses of concave mirrors? [3]

**OR**

How can you distinguish between different types of mirrors (plane mirror, concave mirror, convex mirror) without actually touching them?

**Ans.** (i) It is used as a shaving mirror. When a man keeps his face near to mirror (i.e. between pole and focus of a large-aperture concave mirror), he observe an erect and magnified virtual image of his face in the mirror.

(ii) Concave mirrors are used as reflectors in head-lights, solar cookers etc.

(iii) Concave mirrors are used by ENT doctors to examine ear, nose and throat. The organs are illuminated by a convergent beam of light reflected from concave mirror and look through a hole at the centre of mirror.

**OR**

The mirrors can be distinguished by looking at the images formed by them.

(i) If the image is erect and of the same size whatever be the position of the object, then it will be a plane mirror.

(ii) If the image is inverted by placing the object far away from the mirror, then it is a concave mirror.

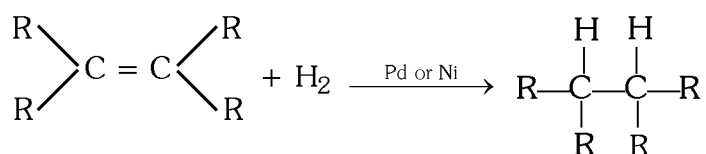
(iii) If the image is erect for all positions of object and size of image is smaller than size of object, then it is a convex mirror.

9. (a) Name two metals which will displace hydrogen from dilute acids, and two metals which will not.  
 (b) Define the following terms. [1 + 2 = 3]  
 (i) Ore (ii) Gangue

**Sol.** (a) Metals that are more reactive than hydrogen displace it from dilute acids. For example: sodium and potassium. Metals that are less reactive than hydrogen do not displace it. For example: copper and silver.  
 (b) (i) Ore : Minerals from which metals can be extracted profitably are known as ores.  
 (ii) Gangue: The impurities (sand, silt, soil, gravel, etc.) present in the ore are called gangue.

10. What is hydrogenation ? What is its industrial application? [3]

**Ans.** Unsaturated hydrocarbons add hydrogen in the presence of palladium or nickel. This reaction is called hydrogenation.



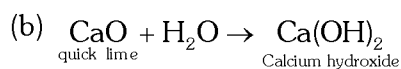
This reaction is industrially used in the conversion of vegetable oils into vegetable ghee.

11. A small amount of quick lime is added to water in a beaker. [3]  
 (a) Name and define the type of reaction that has taken place.  
 (b) Write balanced chemical equation for the above reaction. Write the chemical name of product obtained.  
 (c) State two observations that you will make in the reaction.

**Sol.** (a) It is a chemical combination reaction. Those chemical reactions which involve the combination of two or more substances to form a single new substance are called combination reactions.

**Combination reactions may involve either.**

- (i) Combination of two elements or,  
 (ii) Combination of an element and a compound or,  
 (iii) Combination of two compounds.



- (c) (i) A hissing sound accompanied by bubbles is noticed.  
 (ii) The beaker is heated up.

12. (i) How glucose is broken down in muscle cells during vigorous exercise? Explain.  
 (ii) What is the role of diaphragm during inhalation? [2 + 1 = 3]

**Sol.** (i) 
$$\begin{array}{c} \text{In} \\ \text{cytoplasm} \\ \text{Glucose (6-carbon molecule)} \end{array} \xrightarrow{\quad} \begin{array}{c} \text{Pyruvate (3-carbon molecule)} \\ + \\ \text{Energy} \end{array} \xrightarrow[\text{(in our muscle cells)}]{\text{Lack of oxygen}} \begin{array}{c} \text{Lactic acid + Energy (Total-2ATP)} \\ \text{(3-carbon molecule)} \end{array}$$

- (ii) During inhalation diaphragm become flat by contraction.

13. Name the plant/plant parts showing the following: [3]

- (i) Phototropism      (ii) Hydrotropism      (iii) Seismonasty

**Sol.** (i) Phototropism: Shoot of plant.  
(ii) Hydrotropism: Root of the plant.  
(iii) Seismonasty: Leaves of *Mimosa pudica*

14. What are homologous and analogous organs? Explain with suitable examples. [3]

**Sol. Homologous organs :** Those organs having similar basic structure but has been modified to perform different functions. e.g., forelimbs of reptiles, frog, lizard, bird and human are homologous organs.

**Analogous organs :** Those organs which have different origin and structural plan but appear similar and perform similar functions are called analogous organs.

**eg :** Wings of an insect, bird and bat.

15. Give the answers of following:

- (i) Name one mechanical method of contraception which is used by female. [1 + 2 = 3]  
(ii) What will happen in female body if fertilization does not occur after ovulation?

**Sol.** (i) IUCD (Copper-T)  
(ii) If egg is not fertilized the endometrium lining of uterus degenerates, rupturing the blood vessels. The degenerating blood vessels along with mucus get discharged through vagina as menstrual flow. This monthly cycle lasting 28 days is called the menstrual cycle.

16. (a) Find the size, nature and position of the image formed when an object of size 5 cm is placed at a distance of 15 cm from a concave mirror of focal length 10 cm. [1 + 1 + 1 = 3]

(b) No matter how far you stand from a spherical mirror, your image appears always erect. What is the kind of spherical mirror? [2]

**Ans.** (a) Height of object,  $h_0 = 5$  cm  
Position of object,  $u = -15$  cm  
Focal length,  $f = -10$  cm  
Height of image,  $h_i = -10$  cm  
Position of image,  $v = ?$

From mirror formula,  $\frac{1}{v} + \frac{1}{v} = \frac{1}{f}$

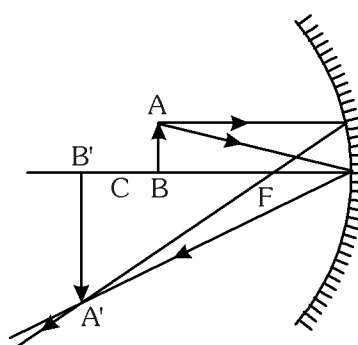
$$\frac{1}{v} + \frac{1}{-15} = -\frac{1}{10}$$

$$\frac{1}{v} = \frac{1}{15} - \frac{1}{10} = \frac{2-3}{30}$$

$$v = -30 \text{ cm}$$

$$\text{From, } m = \frac{-v}{u} = \frac{h_i}{h_0}$$

$$\frac{h_i}{5} = \frac{-(-30)}{(-15)}$$



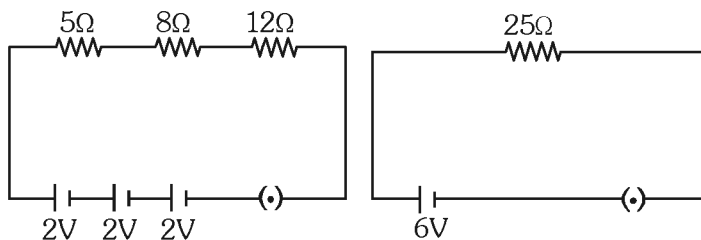
$$h_1 = -10 \text{ cm}$$

Since  $h_1$  is negative and  $v$  is also negative, the image formed is real and inverted.

(b) The spherical mirror is convex mirror.

17. (a) Draw a schematic diagram consisting of a battery of 3 cells of 2V each.  $5\Omega$  resistor an  $8\Omega$  resistor and a  $12\Omega$  resistor and a plug key all connected in series. [1+2+2=5]
- (b) Find the electric current and potential across  $8\Omega$  resistor?
- (c) Will current flow more easily through a thick wire or a thin wire of the same material, when connected to the same source? Why?

Sol. (a), (b)



$$I = \frac{V_{eq}}{R_s}; V_{eq} = 2+2+2 = 6 \text{ volt}$$

$$R_s = R_1 + R_2 + R_3 = 5\Omega + 8\Omega + 12\Omega = 25\Omega$$

$$I = \frac{6 \text{ volt}}{25 \Omega} = 0.24 \text{ Amp.}$$

$$V_{8\Omega} = I \times R = 0.24 \times 8 = 1.92 \text{ volt.}$$

(c) Current will flow more easily through thick wire because resistance of thick wire is less than that of thin wire.

18. (a) What property do all elements in the same column of the Periodic Table as boron have in common?
- (b) What property do all elements in the same column of the Periodic Table as fluorine have in common?
- (c) Compare and contrast the arrangement of elements in Mendeleev's Periodic Table and the Modern Periodic Table. **(Any three)** [1 + 1 + 3 = 5]

Ans. (a) All the elements in the same column as boron have the same number of valence electrons (3).

Hence, they all have valency equal to 3.

(b) All the elements in the same column as fluorine have the same number of valence electrons (7).

Hence, they all have valency equal to 1.

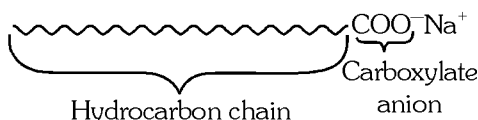
(c)	Mendeleev's Periodic Table	Modern Periodic Table
(i)	Elements have been arranged in order of increasing atomic mass.	(i) Elements have been arranged in the order of increasing atomic number.
(ii)	There are only eight vertical columns called groups.	(ii) There are eighteen vertical columns, called groups.
(iii)	All the transitional elements are arbitrarily placed together in single group VIII.	(iii) The transitional elements are placed in the middle of the long period.
(iv)	The inert gases not known at the time of Mendeleev.	(iv) The inert gases have been placed at the end of periods in group 18.
(v)	No proper places are assigned to isotopes of elements.	(v) Isotopes of elements are assigned the same place as their respective elements as they have same atomic number.
(vi)	Dissimilar metals are placed together.	(vi) Dissimilar metals are placed in separate groups.

19. (a) What is a homologous series of compounds? List any two characteristics of a homologous series.  
(b) Explain the mechanism of the cleansing action of soaps. [2 + 3 = 5]

**Ans.** (a) Homologous series is a family of organic compounds having the same functional group and can be represented by same general formula.

- (i) Each successive members differ by  $-\text{CH}_2$  group or a mass of 14 atomic mass units.  
(ii) Similarity of chemical properties due to common functional group.

(b) A molecule of soap has two dissimilar ends. At one end is the hydrocarbon chain, which is water repellent and the other end is carboxylate anion which is polar end.

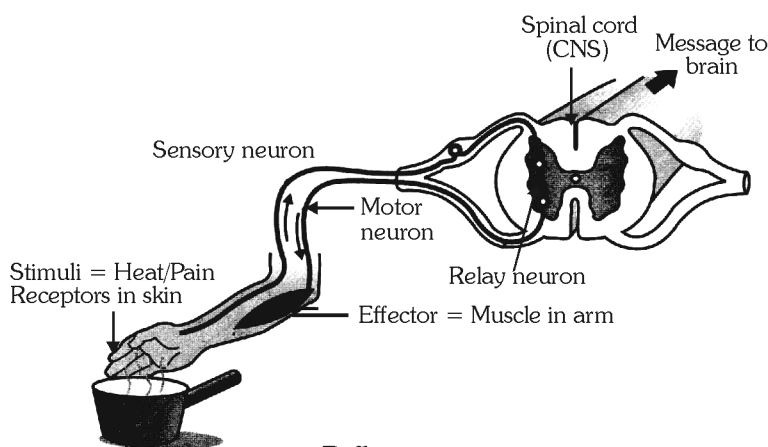


When soap is dissolved in water, many molecules come together and form a group called micelle. These micelles are formed because their hydrocarbon chains come together and the polar ends are projected outward.

When a cloth with a spot of oil is soaked into a soap solution, soap dissolves tiny oil droplets by the hydrophobic end in the middle of the micelle. Due to the outer polar ends, these micelles dissolve in water and are washed away. In this way cloth gets cleaned.

20. (i) Draw a well labelled diagram showing reflex-arc. [2.5 + 2.5 = 5]  
(ii) What is apical dominance and which hormone controls this? How farmer will overcome the disadvantage of it?

**Sol.** (i)



(ii) **Apical dominance:** Terminal or apical bud inhibits the development of lateral buds. Hormone responsible for this is **auxin**. Farmer removes apical part of fruiting plants to induce more branching.

21. (i) What is food chain? [1 + 3 + 1 = 5]  
(ii) Define biomagnification and explain it with suitable example.  
(iii) What are 3 R's?

**Sol.** (i) It is a sequence of organisms through which energy is transferred in the form of food by the process of one organism consuming the other.

(ii) The phenomenon that involves progressive increase in concentration of harmful non-biodegradable chemicals at different trophic levels in a food chain is called biomagnification.

For example in following food chain top carnivore is bird (Eg. Hawk) in which non-biodegradable chemicals are increasing)

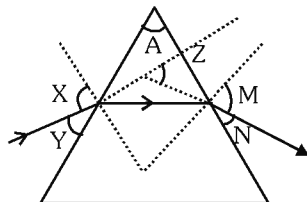
Grass → Insects → Frog → Birds  
 (Producer) (Herbivore) (Carnivore) (Top carnivores)

(iii) Reduce, Recycle, Reuse are 3 R's.

### SECTION-B

#### Practical based.

22. For a refraction of a ray of light through a glass prism. The path of a ray of light is shown below



Find (a) angle of incidence (b) angle of emergence [2]  
 (c) angle of deviation (d) prism angle

Respectively represented by

**Sol.** (a)  $\angle i = X$  (b)  $\angle e = M$  (c)  $\angle \delta = Z$  (d)  $\angle A = A$

23. Draw ray diagrams to represent the nature, position and size of the image formed by a convex lens for the object placed. [2]

- (a) at  $2F_1$   
 (b) Between  $F_1$  and the optical centre O of lens.

24. For making cake, baking powder is taken. If at home your mother uses baking soda instead of baking powder in cake, [2]

- (a) How will it affect the taste of the cake and why?  
 (b) How can baking soda be converted into baking powder?  
 (c) What is the role of tartaric acid added to baking soda?

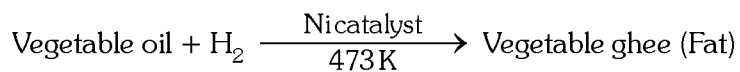
**Sol.** (a) Baking powder is a mixture of  $\text{NaHCO}_3$  and tartaric acid or citric acid. On heating,  $\text{NaHCO}_3$  decomposes to give out  $\text{CO}_2$  which causes the bread or cake to rise.  $\text{Na}_2\text{CO}_3$  is also formed which is neutralized by tartaric acid. If only baking soda ( $\text{NaHCO}_3$ ) is used,  $\text{Na}_2\text{CO}_3$  will not be neutralized and the cake will taste bitter.

- (b) Baking soda can be converted into baking powder by mixing it with an appropriate amount of tartaric acid or citric acid.  
 (c) Tartaric acid neutralizes the  $\text{Na}_2\text{CO}_3$  formed on heating  $\text{NaHCO}_3$ , so that the cake will not taste bitter.

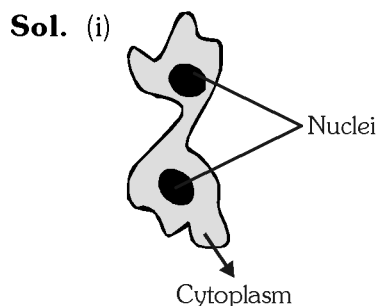
25. Simran and her mother went to a general store. There, her mother asked the shopkeeper to give a one kilogram pack of vanaspati ghee. Suddenly, Simran stopped her mother and suggested her not to buy vanaspati ghee. [2]

- (a) According to you, what could be the reason behind Simran's suggestion?  
 (b) How vegetable ghee is prepared from vegetable oil? What is this reaction called?  
 (c) What are the values shown by Simran?

- Sol.** (a) The vanaspati ghee is saturated fat which can increase the cholesterol in blood.  
(b) Vegetable ghee is prepared from vegetable oils by the process of hydrogenation. This can be shown by the following reaction.



- (c) Simran is a health conscious person.
- 26.** A student observed a dividing unicellular organism under microscope. He observed the stage in which nucleus was divided but not cytoplasm. [2]
- (i) Draw the diagram of observed stage. (ii) What is the name of process?



- (ii) It is binary fission.

- 27.** To explain the requirement of chlorophyll during photosynthesis which of the following are required? Mango plant, Croton plant, KOH solution, Iodine solution, Black strips to cover leaves, Alcohol, water. [2]

**Sol.** To explain the requirement of chlorophyll during photosynthesis following are required- Croton plant, Iodine solution, Alcohol and water.

\* \* \* \* \*