MATHEMATICS

SECTION-A

1. True

2. False

3. True

4. False

5. $4\pi r^2 = 154$

$$4 \times \frac{22}{7} \times r^2 = 154$$

$$r^2 = \frac{154 \times 7}{22 \times 4}$$

$$= \frac{14 \times 7}{8} = \frac{49}{4}$$

$$r = \frac{7}{2} \text{ cm}$$

$$\text{Volume} = \frac{4}{3}\pi r^3$$

$$= \frac{4}{3} \times \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} \times \frac{7}{2}$$

$$= \frac{11 \times 49}{3} = \frac{539}{3}$$

$$= 179\frac{2}{3} \text{ cm}^3 \text{ answer is none of these}$$

6. 52 complete work
1 day left
So, 1 day can be Sunday Monday, Tuesday, Wednesday, Thursday, Friday, Saturday.

So, $\frac{1}{7}$

7. Let n^{th} term be 0
$T_n = a + (n - 1) \times d$
$0 = 72 + (n - 1) \times (-9)$
$= 72 - 9n + 9$
$= 81 - 9n$
$9n = 81$
$n = 9$
9^{th} term is 0.

8. Number of cubes = $\frac{6 \times 6 \times 6}{2 \times 2 \times 2} = 3 \times 3 \times 3 = 27$

SECTION-A

1. સાચું

2. ખોટું

3. સાચું

4. ખોટું

5. $4\pi r^2 = 154$

$$4 \times \frac{22}{7} \times r^2 = 154$$

$$r^2 = \frac{154 \times 7}{22 \times 4}$$

$$= \frac{14 \times 7}{8} = \frac{49}{4}$$

$$r = \frac{7}{2} \text{ cm}$$

ઘનફળ $= \frac{4}{3}\pi r^3$

$$= \frac{4}{3} \times \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} \times \frac{7}{2}$$

$$= \frac{11 \times 49}{3} = \frac{539}{3}$$

$$= 179\frac{2}{3} \text{ cm}^3 \text{ જવાબ = એકપણ નહીં.}$$

6. 52 અઠવાડિયા એક વર્ષમાં લેતાં, $52 \times 7 = 364$
$\therefore$ એક દિવસ બાકી રહે જે રવિવાર, સોમવાર, મંગળવાર, બુધવાર..... શનિવાર પૈકી એક હોઇ શકે.

તેથી, સંભાવના, $\frac{1}{7}$

7. ધારો કે, n મું પદ શૂન્ય છે.
$T_n = a + (n - 1) \times d$
$0 = 72 + (n - 1) \times (-9)$
$= 72 - 9n + 9$
$= 81 - 9n$
$9n = 81$
$n = 9$
9 મું પદ 0 છે.

8. સમઘનની સંખ્યા = $\frac{6 \times 6 \times 6}{2 \times 2 \times 2} = 3 \times 3 \times 3 = 27$

9. $\frac{364}{365}$

10. Area of equilateral triangle = $\frac{\sqrt{3}}{4}\times(12)^2$

$= \frac{\sqrt{3}}{4} \times 12 \times 12$

$= 36\sqrt{3}\,cm^2$

$\frac{1}{2}$(base × height) = $36\sqrt{3}$

$\frac{1}{2}(12) \times h = 36\sqrt{3}$

$h = 6\sqrt{3}$ cm

11. $3 \times 2 + k \times 3 = 1$

$6 + 3k = 1 \Rightarrow 3k = 1 - 6 = -5 \Rightarrow k = -\frac{5}{3}$

12. P(E) = $\frac{3}{6}=\frac{1}{2}$

13. Diagonal of sohare = 12

$a\sqrt{2} = 12$

$a = \frac{12\times\sqrt{2}}{\sqrt{2}\times\sqrt{2}}=6\sqrt{2}$

Area = $\left(6\sqrt{2}\right)^2 = 72\ cm^2$

14. Radius = 1

Decreasing radius = $1 - 1 \times \frac{10}{100} = \frac{9}{10}$

Area = 1

New area = $\pi\left(\frac{81}{100}\right)$

Decrease in area = $\pi-\frac{81\pi}{100}=\frac{19\pi}{100}$

15. $\sin\theta = \frac{a}{b} \Rightarrow \cos\theta = \sqrt{1-\sin^2\theta}$

$= \sqrt{1-\frac{a^2}{b^2}}=\frac{\sqrt{b^2-a^2}}{b}$

9. $\frac{364}{365}$

10. સમભુજ ત્રિકોણનું ક્ષેત્રફળ = $\frac{\sqrt{3}}{4}\times(12)^2$

$= \frac{\sqrt{3}}{4} \times 12 \times 12$

$= 36\sqrt{3}\,cm^2$

$\frac{1}{2}$(પાયો × વેધ) = $36\sqrt{3}$

$\frac{1}{2}(12) \times h = 36\sqrt{3}$

$h = 6\sqrt{3}$ cm

11. $3 \times 2 + k \times 3 = 1$

$6 + 3k = 1 \Rightarrow 3k = 1 - 6 = -5 \Rightarrow k = -\frac{5}{3}$

12. P(E) = $\frac{3}{6}=\frac{1}{2}$

13. ચોરસના વિકર્ણની લંબાઈ = 12

$a\sqrt{2} = 12$

$a = \frac{12\times\sqrt{2}}{\sqrt{2}\times\sqrt{2}}=6\sqrt{2}$

ક્ષેત્રફળ = $\left(6\sqrt{2}\right)^2 = 72\ cm^2$

14. ત્રિજ્યા = 1

ત્રિજ્યામાં થતો ઘટાડો = $1 - 1 \times \frac{10}{100} = \frac{9}{10}$

ક્ષેત્રફળ = 1

નવું ક્ષેત્રફળ = $\pi\left(\frac{81}{100}\right)$

ક્ષેત્રફળમાં ઘટાડો = $\pi-\frac{81\pi}{100}=\frac{19\pi}{100}$

15. $\sin\theta = \frac{a}{b} \Rightarrow \cos\theta = \sqrt{1-\sin^2\theta}$

$= \sqrt{1-\frac{a^2}{b^2}}=\frac{\sqrt{b^2-a^2}}{b}$

16. $\pi r^2 = 220$

$$\frac{22}{7}\times r^2 = 220$$

$$r^2 = \frac{220\times 7}{22} = 70$$

$r = \sqrt{70}$ cm

$a\sqrt{2} = 2\sqrt{70}$

$$a = \frac{2\sqrt{70}\times\sqrt{2}}{\sqrt{2}\times\sqrt{2}} = \sqrt{140}$$

$a^2 = 140$ cm^2

SECTION-B

17. Let $3 + \sqrt{7}$ is rational number

$3 + \sqrt{7} = x$

$\sqrt{7} = (x - 3)$

We know that $\sqrt{7}$ is irration number.

So $(x - 3)$ is also irrational number. But we suppose that $(x - 3)$ is rational so our supposition is wrong

$\therefore 3 + \sqrt{7}$ is irrational number.

18. Other number $= \dfrac{\text{HCF}\times\text{LCM}}{\text{One number}}$

$$= \frac{162\times 27}{54} = 81$$

19. L.H.S. $\dfrac{\tan\theta}{1-\cot\theta}+\dfrac{\cot\theta}{1-\tan\theta}$

$$\frac{\frac{\sin\theta}{\cos\theta}}{1-\frac{\cos\theta}{\sin\theta}}+\frac{\frac{\cos\theta}{\sin\theta}}{1-\frac{\sin\theta}{\cos\theta}}$$

$$\frac{\sin^2\theta}{\cos\theta(\sin\theta-\cos\theta)}+\frac{\cos^2\theta}{\sin\theta(\cos\theta-\sin\theta)}$$

$$\frac{1}{(\sin\theta-\cos\theta)}\left[\frac{\sin^3\theta-\cos^3\theta}{\cos\theta\sin\theta}\right]$$

$$\frac{(\sin\theta-\cos\theta)(1+\sin\theta\cos\theta)}{(\sin\theta-\cos\theta)(\sin\theta\cos\theta)}$$

$$\frac{1}{\sin\theta\cdot\cos\theta}+\frac{\sin\theta\cdot\cos\theta}{\sin\theta\cdot\cos\theta}$$

$\text{cosec}\theta.\sec\theta + 1$

= R.H.S.

16. $\pi r^2 = 220$

$$\frac{22}{7}\times r^2 = 220$$

$$r^2 = \frac{220\times 7}{22} = 70$$

$r = \sqrt{70}$ cm

$a\sqrt{2} = 2\sqrt{70}$

$$a = \frac{2\sqrt{70}\times\sqrt{2}}{\sqrt{2}\times\sqrt{2}} = \sqrt{140}$$

$a^2 = 140$ cm^2

SECTION-B

17. *ધારો કે* $3 + \sqrt{7}$ *એ સંમેય સંખ્યા છે.*

$3 + \sqrt{7} = x$

$\sqrt{7} = (x - 3)$

આપણે જાણીએ છીએ કે, $\sqrt{7}$ *એ અસંમેય સંખ્યા છે તેથી* $(x - 3)$ *એ પણ અસંમેય સંખ્યા થશે પરંતુ આપણે ધારેલ છે કે* $(x - 3)$ *એ સંમેય છે. જેથી, આપણી ધારણા ખોટી છે. આથી,* $3 + \sqrt{7}$ *એ અસંમેય સંખ્યા છે.*

18. *બીજી સંખ્યા* $= \dfrac{\text{HCF}\times\text{LCM}}{\text{એક સંખ્યા}}$

$$= \frac{162\times 27}{54} = 81$$

19. *ડા.બા.* $= \dfrac{\tan\theta}{1-\cot\theta}+\dfrac{\cot\theta}{1-\tan\theta}$

$$\frac{\frac{\sin\theta}{\cos\theta}}{1-\frac{\cos\theta}{\sin\theta}}+\frac{\frac{\cos\theta}{\sin\theta}}{1-\frac{\sin\theta}{\cos\theta}}$$

$$\frac{\sin^2\theta}{\cos\theta(\sin\theta-\cos\theta)}+\frac{\cos^2\theta}{\sin\theta(\cos\theta-\sin\theta)}$$

$$\frac{1}{(\sin\theta-\cos\theta)}\left[\frac{\sin^3\theta-\cos^3\theta}{\cos\theta\sin\theta}\right]$$

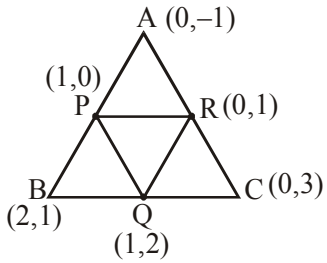
$$\frac{(\sin\theta-\cos\theta)(1+\sin\theta\cos\theta)}{(\sin\theta-\cos\theta)(\sin\theta\cos\theta)}$$

$$\frac{1}{\sin\theta\cdot\cos\theta}+\frac{\sin\theta\cdot\cos\theta}{\sin\theta\cdot\cos\theta}$$

$\text{cosec}\theta.\sec\theta + 1$

= *જ.બા.*

20.



Area of triangle (PQR)

$= \frac{1}{2}|1(1-2)+0+1(0-1)|$

$= \frac{1}{2}|-1-1| = 1$ sq unit

Area of triangle (ABC)

$= \frac{1}{2}|0+2(3+1)+1|$

$= \frac{1}{2}(8) = 4$ sq unit

Required ratio is 1 : 4

21. $3x^2 - 14x + 15 = 0$

$\alpha + \beta = -\frac{b}{a} = \frac{14}{3}$ ...(1)

$\alpha\beta = \frac{c}{a} = \frac{15}{3} = 5$ ...(2)

From (1) $(\alpha + \beta)^2 = \left(\frac{14}{3}\right)^2$

$\alpha^2 + \beta^2 + 2\alpha\beta = \frac{196}{9}$

$\alpha^2 + \beta^2 + 10 = \frac{196}{9}$

$\alpha^2 + \beta^2 = \frac{196}{9} - 10 = \frac{106}{9}$

OR

$x = 2 - \sqrt{3} \Rightarrow (x - 2 + \sqrt{3}) = 0$

$x = 2 + \sqrt{3} \Rightarrow (x - 2 - \sqrt{3}) = 0$

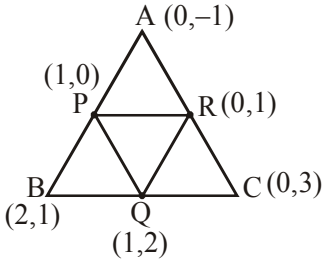
$(x - 2 + \sqrt{3})(x - 2 - \sqrt{3})$

$(x-2)^2 - (\sqrt{3})^2 = 0$

$x^2 + 4 - 4x - 3 = 0$

$x^2 - 4x + 1 = 0$

20.



Δ PQRQ ક્ષેત્રફળ

$= \frac{1}{2}|1(1-2)+0+1(0-1)|$

$= \frac{1}{2}|-1-1| = 1$ ચો. એકમ

Δ ABC નું ક્ષેત્રફળ

$= \frac{1}{2}|0+2(3+1)+1|$

$= \frac{1}{2}(8) = 4$ ચો.એકમ

ગુણોત્તર = 1 : 4

21. $3x^2 - 14x + 15 = 0$

$\alpha + \beta = -\frac{b}{a} = \frac{14}{3}$ ...(1)

$\alpha\beta = \frac{c}{a} = \frac{15}{3} = 5$ ...(2)

પરથી (1) $(\alpha + \beta)^2 = \left(\frac{14}{3}\right)^2$

$\alpha^2 + \beta^2 + 2\alpha\beta = \frac{196}{9}$

$\alpha^2 + \beta^2 + 10 = \frac{196}{9}$

$\alpha^2 + \beta^2 = \frac{196}{9} - 10 = \frac{106}{9}$

અથવા

$x = 2 - \sqrt{3} \Rightarrow (x - 2 + \sqrt{3}) = 0$

$x = 2 + \sqrt{3} \Rightarrow (x - 2 - \sqrt{3}) = 0$

$(x - 2 + \sqrt{3})(x - 2 - \sqrt{3})$

$(x-2)^2 - (\sqrt{3})^2 = 0$

$x^2 + 4 - 4x - 3 = 0$

$x^2 - 4x + 1 = 0$

22. PQ = PR

$(x - a - b)^2 + (y - b + a)^2 = (x - a + b)^2 + (y - a - b)^2$

$x^2 + a^2 + b^2 - 2ax + 2ab - 2bx + y^2 + b^2 + a^2 - 2by - 2ab + 2ay$

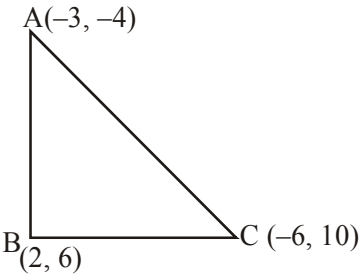
$= x^2 + a^2 + b^2 - 2ax - 2ab + 2bx + y^2 + a^2 + b^2 - 2ay + 2ab - 2by$

$- 2bx + 2ay = 2bx - 2ay$

$4bx = 4ay$

$bx = ay$

OR



$AB^2 = (-3 - 2)^2 + (-4 - 6)^2$

$= 25 + 100 = 125$

$BC^2 = (2 + 6)^2 + (6 - 10)^2$

$= 64 + 16 = 80$

$AC^2 = (-3 + 6)^2 + (-4 - 10)^2$

$= 9 + 196$

$= 205$

Yes these are the vertices of right angled Δ.

23. $\sqrt{3}x^2 - 2\sqrt{2}x - 2\sqrt{3} = 0$

$\sqrt{3}x^2 - 3\sqrt{2}x + \sqrt{2}x - 2\sqrt{3} = 0$

$\sqrt{3}x(x - \sqrt{6}) + \sqrt{2}(x - \sqrt{6})$

$(x - \sqrt{6})(\sqrt{3}x + \sqrt{2})$

24. L.H.S.

$$\sqrt{\frac{1+\sin A}{1-\sin A}}$$

$$\sqrt{\frac{(1+\sin A)^2}{(1-\sin A)(1+\sin A)}}$$

$$\sqrt{\frac{(1+\sin A)^2}{(1-\sin^2 A)}}$$

$$\sqrt{\frac{(1+\sin A)^2}{\cos^2 A}} = \sqrt{\frac{(1+\sin A)^2}{\cos^2 A}} = \frac{1+\sin A}{\cos A}$$

$$= \frac{1}{\cos A} + \frac{\sin A}{\cos A} = \sec A + \tan A$$

OR

22. PQ = PR

$(x - a - b)^2 + (y - b + a)^2 = (x - a + b)^2 + (y - a - b)^2$

$x^2 + a^2 + b^2 - 2ax + 2ab - 2bx + y^2 + b^2 + a^2 - 2by - 2ab + 2ay$

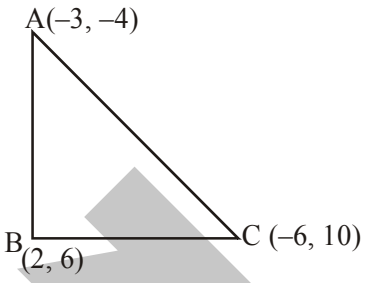
$= x^2 + a^2 + b^2 - 2ax - 2ab + 2bx + y^2 + a^2 + b^2 - 2ay + 2ab - 2by$

$- 2bx + 2ay = 2bx - 2ay$

$4bx = 4ay$

$bx = ay$

અથવા



$AB^2 = (-3 - 2)^2 + (-4 - 6)^2$

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$AC^2 = (-3 + 6)^2 + (-4 - 10)^2$

$= 9 + 196$

$= 205$

તેથી આપેલ શિરોબિંદુઓ કાટકોણ ત્રિકોણના શિરોબિંદુઓ છે.

23. $\sqrt{3}x^2 - 2\sqrt{2}x - 2\sqrt{3} = 0$

$\sqrt{3}x^2 - 3\sqrt{2}x + \sqrt{2}x - 2\sqrt{3} = 0$

$\sqrt{3}x(x - \sqrt{6}) + \sqrt{2}(x - \sqrt{6})$

$(x - \sqrt{6})(\sqrt{3}x + \sqrt{2})$

24. L.H.S.

$$\sqrt{\frac{1+\sin A}{1-\sin A}}$$

$$\sqrt{\frac{(1+\sin A)^2}{(1-\sin A)(1+\sin A)}}$$

$$\sqrt{\frac{(1+\sin A)^2}{(1-\sin^2 A)}}$$

$$\sqrt{\frac{(1+\sin A)^2}{\cos^2 A}} = \sqrt{\frac{(1+\sin A)^2}{\cos^2 A}} = \frac{1+\sin A}{\cos A}$$

$$= \frac{1}{\cos A} + \frac{\sin A}{\cos A} = \sec A + \tan A$$

અથવા

L.H.S.

$$\frac{\sin\theta(1+\cos\theta)}{(1-\cos\theta)(1+\cos\theta)}-\frac{\sin\theta(1+\cos\theta)}{(1-\cos^2\theta)}$$

$$=\frac{\sin\theta(1+\cos\theta)}{\sin^2\theta}=\frac{1+\cos\theta}{\sin\theta}$$

$$=\frac{1}{\sin^2\theta}+\frac{\cos\theta}{\sin\theta}$$

= cosecθ + cotθ

25. mode = 3 median – 2mean

12.4 = 3 mediuan – 2 × 10.5

12.4 + 21 = 3 mediun

$$33.4 = 3\text{median} \Rightarrow \text{median} = \frac{33.4}{3} = 11.1$$

OR

Total sum of all number = 18 × 30 = 540

30 number are increased by 2

∴ 30 × 2 = 60

540 + 60 = 600

$$\text{new mean} = \frac{600}{30} = 20$$

26. x + 2y = 2 .... (1)

x – 3y = 7 ....(2)

(1) – (2)

5y = – 5 ⇒ y = –1

Put in (1)

x – 2 = 2 ⇒ x = 4

x = 4 & y = –1 is the solution.

SECTION-C

27. $\text{Mode} = \ell + \left(\frac{f_1-f_0}{2f_1-f_0-f_2}\right)\times h$

given mode = 34.5

modal class = 32 – 41

ℓ = 32

h = 9

$f_1 = 53,\ f_0 = a,\ f_2 = b$

$$34.5 = 32 + \left(\frac{53-a}{2\times53-a-b}\right)\times9$$

$$2.5 = \left(\frac{53-a}{106-a-b}\right)\times9$$

$$\frac{25}{90}=\left(\frac{53-a}{106-a-b}\right)$$

139 – 56 = 424 ....(1)

Total of observation

5 + 11 + 9 + 53 + b + 16 + 10 = 165

95 + a + b = 165

a + b = 70 ....(2)

From equation (1) and (2)

a = 43

b = 27

L.H.S.

$$\frac{\sin\theta(1+\cos\theta)}{(1-\cos\theta)(1+\cos\theta)}-\frac{\sin\theta(1+\cos\theta)}{(1-\cos^2\theta)}$$

$$=\frac{\sin\theta(1+\cos\theta)}{\sin^2\theta}=\frac{1+\cos\theta}{\sin\theta}$$

$$=\frac{1}{\sin^2\theta}+\frac{\cos\theta}{\sin\theta}$$

= cosecθ + cotθ

25. *બહુલક* = 3 *મધ્યસ્થ* – 2 *મધ્યક*

12.4 = 3 *મધ્યસ્થ* – 2 × 10.5

12.4 + 21 = 3 *મધ્યસ્થ*

$$33.4 = 3\text{મધ્યસ્થ} \Rightarrow \text{મધ્યસ્થ} = \frac{33.4}{3} = 11.1$$

અથવા

અવલોકનનો કૂલ સરવાળો, = 18 × 30 = 540

30 *દરેકમાં* 2 *અવલોકનનો વધારો કરવામાં આવે છે. તેથી,*

∴ 30 × 2 = 60

540 + 60 = 600

$$\text{નવો મધ્યક} = \frac{600}{30} = 20$$

26. x + 2y = 2 .... (1)

x – 3y = 7 ....(2)

(1) – (2)

5y = – 5 ⇒ y = –1

(1) *માં મૂકતાં,*

x – 2 = 2 ⇒ x = 4

x = 4 & y = –1 *એ ઉકેલ છે.*

SECTION-C

27. $\text{Mode} = \ell + \left(\frac{f_1-f_0}{2f_1-f_0-f_2}\right)\times h$

આપેલ બહુલક = 34.5

બહુલક વર્ગલંબાઈ = 32 – 41

ℓ = 32

h = 9

$f_1 = 53,\ f_0 = a,\ f_2 = b$

$$34.5 = 32 + \left(\frac{53-a}{2\times53-a-b}\right)\times9$$

$$2.5 = \left(\frac{53-a}{106-a-b}\right)\times9$$

$$\frac{25}{90}=\left(\frac{53-a}{106-a-b}\right)$$

139 – 56 = 424 ....(1)

અવલોકનોનો સરવાળો

5 + 11 + 9 + 53 + b + 16 + 10 = 165

95 + a + b = 165

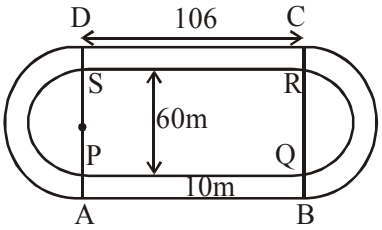
a + b = 70 ....(2)

સમીકરણ (1) *અને* (2) *પરથી*

a = 43

b = 27

28.



AB = DC = 106, PQ = RS = 106 m
AP = DS = 10 m, BQ = CR = 10m
ar(□ABQP) = 106 × 10 = 106 × 10 = 1060 m^2
ar(□RSDC) = 106 × 10 = 1060 m^2
Area of semi circle with diameter AD
AD = 60 + 10 + 10 = 80

$r = \frac{80}{2} = 40$

$A_1 = \frac{1}{2} \times \pi r^2 = \frac{1}{2} \times \frac{22}{7} \times 40 \times 40 = \frac{17600}{7} m^2$

Area of semicircle with diameter SP
SP = 60
r = 30

$A_2 = \frac{1}{2} \times \pi r^2 = \frac{1}{2} \times \frac{22}{7} \times 30 \times 30 = \frac{9900}{7} m^2$

Area of track = ar(□ABQP) + ar(□RSDC) + 2(Area of semicircle with diameter – Area of semicircle with diameter SP)

$= 1060 + 1060 + 2 \times \left(\frac{17600}{7} - \frac{9900}{7}\right)$

$= 2120 + 2 \times \frac{7700}{7}$

$= 4320\ m^2$

29. $S_p = q$

$\frac{P}{2}[2a + (p - 1)d] = q$

$2ap + p(p-1) = 2q$ ...(1)

$Sq = p$

$\frac{P}{2}[2a + (q - 1)d] = q$

$2ap + q(q-1) = 2p$ ...(2)

Equation (1) - (2)

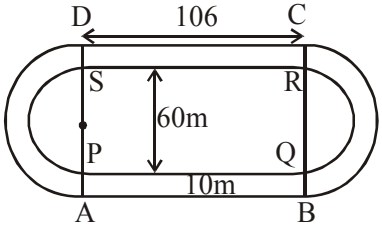
$2a(p - q) + [p(p - 1) - q(q - 1)]d = -2p + 2q$
$2a(p - q) + [p^2 - p - q^2 + q)]d = -2(p - q)$
$2a(p - q) + [p^2 - q^2 (p - q)]d = -2(p - q)$
$2a(p - q) + [(p - q)(p + q) - (p - q)]d = -2(p - q)$
$(p - q)[2a + (p + q - 1)d] = -2(p - q)$
$2a + p - q + 1 = -2$

$S_{p+q} = \frac{p+q}{2}[2a + (p + q - 1)d]$

$= \left(\frac{p+q}{2}\right)(-2)$

$S_{p+q} = -(p + q)$

28.



AB = DC = 106, PQ = RS = 106 m
AP = DS = 10 m, BQ = CR = 10m
ar(□ABQP) = 106 × 10 = 106 × 10 = 1060 m^2
ar(□RSDC) = 106 × 10 = 1060 m^2
AD વ્યાસવાળા અર્ધવર્તુળનું ક્ષેત્રફળ
AD = 60 + 10 + 10 = 80

$r = \frac{80}{2} = 40$

$A_1 = \frac{1}{2} \times \pi r^2 = \frac{1}{2} \times \frac{22}{7} \times 40 \times 40 = \frac{17600}{7} m^2$

SP વ્યાસવાળા અર્ધવર્તુળનું ક્ષેત્રફળ
SP = 60
r = 30

$A_2 = \frac{1}{2} \times \pi r^2 = \frac{1}{2} \times \frac{22}{7} \times 30 \times 30 = \frac{9900}{7} m^2$

ટ્રેકનું ક્ષેત્રફળ =
ar (□ABQP)નું ક્ષેત્રફળ + ar (□RSDC) નું ક્ષેત્રફળ + 2(AD વ્યાસવાળા અર્ધવર્તુળનું ક્ષેત્રફળ – SP વ્યાસવાળા અર્ધવર્તુળનું ક્ષેત્રફળ)

$= 1060 + 1060 + 2 \times \left(\frac{17600}{7} - \frac{9900}{7}\right)$

$= 2120 + 2 \times \frac{7700}{7}$

$= 4320\ m^2$

29. $S_p = q$

$\frac{P}{2}[2a + (p - 1)d] = q$

$2ap + p(p-1) = 2q$ ...(1)

$Sq = p$

$\frac{P}{2}[2a + (q - 1)d] = q$

$2ap + q(q-1) = 2p$ ...(2)

સમીકરણ (1) - (2) લેતાં,

$2a(p - q) + [p(p - 1) - q(q - 1)]d = -2p + 2q$
$2a(p - q) + [p^2 - p - q^2 + q)]d = -2(p - q)$
$2a(p - q) + [p^2 - q^2 (p - q)]d = -2(p - q)$
$2a(p - q) + [(p-q)(p + q) - (p - q)]d = -2(p-q)$
$(p - q)[2a + (p + q - 1)d] = -2(p - q)$
$2a + p - q + 1 = -2$

$S_{p+q} = \frac{p+q}{2}[2a + (p + q - 1)d]$

$= \left(\frac{p+q}{2}\right)(-2)$

$S_{p+q} = -(p + q)$

OR

$T_3 = 4$

$q + 2d = 4$ ...(1)

$T_9 = -8$

$a + 8d = -8$ ...(2)

From equation (1) & (2)

$$\begin{array}{r} a + 8d = -8 \\ a + 2d = \ \ 4 \\ \hline 6d = -12 \end{array}$$

$d = -2$ $\qquad a = 8$

$T_n = 0$

$a + (n - 1)\, d = 0$

$8 + (n - 1)(-2) = 0$

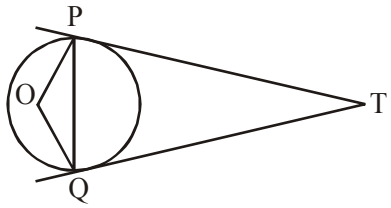
$8 = 2(n - 1)$

$n - 1 = \dfrac{8}{2}$

$n - 1 = 4$

$n = 5$

30.



In □OPTQ.

$\angle O + \angle P + \angle Q + \angle T = 360^\circ$

$\angle O + 90 + 90 + \angle T = 360^\circ$

$\angle O + \angle T = 360^\circ - 180^\circ = 180^\circ$

$\angle O = 180 - \angle T$

In ΔOPQ

$OP = OQ$

$\angle OPQ = \angle OQP$

$\angle OPQ + \angle OQP + \angle POQ = 180^\circ$

$2\angle OPQ + 180 - \angle PTQ = 180^\circ$

$2\angle OPQ = \angle PTQ$

31.

Cylinder	Cone
$r = 8$ cm	$R = 2$
$h = 40$ cm	$H = 12$
$y_{cy} = \pi r^2 h$	$y_c = \dfrac{\pi}{3} \times r^2 h$
$= p \times 8 \times 8 \times 40\ cm^3$	$= \dfrac{\pi}{3} \times (2)^2 \times 12$
	$= 16\pi\ cm^3$

Hemisphere $R^1 = 2$ cm

$V_H = \dfrac{2}{3}\pi(2)^3 \qquad = \dfrac{16\pi}{3}$

Total volume of ice-cream $= 16\pi + \dfrac{16\pi}{3} = \dfrac{64\pi}{3}$

અથવા

$T_3 = 4$

$q + 2d = 4$ ...(1)

$T_9 = -8$

$a + 8d = -8$ ...(2)

સમીકરણ (1) *અને* (2) *પરથી,*

$$\begin{array}{r} a + 8d = -8 \\ a + 2d = \ \ 4 \\ \hline 6d = -12 \end{array}$$

$d = -2$ $\qquad a = 8$

$T_n = 0$

$a + (n - 1)\, d = 0$

$8 + (n - 1)(-2) = 0$

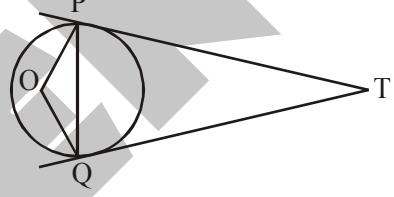
$8 = 2(n - 1)$

$n - 1 = \dfrac{8}{2}$

$n - 1 = 4$

$n = 5$

30.



□OPTQ *મા,*

$\angle O + \angle P + \angle Q + \angle T = 360^\circ$

$\angle O + 90 + 90 + \angle T = 360^\circ$

$\angle O + \angle T = 360^\circ - 180^\circ = 180^\circ$

$\angle O = 180 - \angle T$

ΔOPQ *મા,*

$OP = OQ$

$\angle OPQ = \angle OQP$

$\angle OPQ + \angle OQP + \angle POQ = 180^\circ$

$2\angle OPQ + 180 - \angle PTQ = 180^\circ$

$2\angle OPQ = \angle PTQ$

31.

નળાકાર માટે	*શંકુ માટે*
$r = 8$ cm	$R = 2$
$h = 40$ cm	$H = 12$
$y_{cy} = \pi r^2 h$	$y_c = \dfrac{\pi}{3} \times r^2 h$
$= p \times 8 \times 8 \times 40\ cm^3$	$= \dfrac{\pi}{3} \times (2)^2 \times 12$
	$= 16\pi\ cm^3$

અર્ધવર્તુળની ત્રિજ્યા $R^1 = 2$ cm

ઘનફળ $= V_H = \dfrac{2}{3}\pi(2)^3 \qquad = \dfrac{16\pi}{3}$

આઇસ્ક્રીમનું કુલ ઘનફળ $= 16\pi + \dfrac{16\pi}{3} = \dfrac{64\pi}{3}$

Number of ice-cream = $\frac{\text{Volume of cylinder}}{\text{Volume of ice}-\text{cream}}$

$= \frac{\pi \times 8 \times 8 \times 40}{\frac{64}{3}\pi} = 120$

OR

Total surface area of toy = C.S.A. of hemiphere + C.S.A. of cone

$= 2\pi R^2 + \pi r\ell$

R = 3.5 cm

r = 3.5 cm

H = 15.5 cm

Height of cone = 15.5 – 3.5 = 12 cm

$\ell^2 = h^2 + r^2$

$\ell^2 = 12^2 + (3.5)^2$

$\ell^2 = 144 + \frac{49}{4}$

$\ell^2 = \frac{62.5}{4}$

$\ell = \frac{25}{2}$

$A = 2 \times \frac{22}{7} \times (3.5) \times (3.5) + \frac{22}{7} \times 3.5 \times \frac{25}{2}$

$= 77 + 137.5$

$= 214.5\ cm^2$

32.

$$x^2 - 2x + k \overline{)x^4 - 6x^3 + 16x^2 - 25x + 10} \quad (x^2 - 4x + 8 - k$$

$$\underline{-\ x^4 \mp 2x^3 \pm kx^2}$$

$$-4x^3 + x^2(16 - k) - 25x + 10$$

$$\underline{+\ -4x^3 + 8x^2 - 4kx\ +}$$

$$x^2(8 - k) + x(4k - 25) + 10$$

$$\underline{-\ x^2(8 - k) - 16x + 2kx + 8k - k^2\ +}$$

$$x(2k - 9) + 10 - 8k + k^2$$

Remainder is = x + a

co-officient of x is 1

$2k - 9 = 1$

$k = 5$

$a = 10 - 8k + k^2$

$= 10 - 40 + 25$

$a = -5$

OR

આઇસ્ક્રીમની સંખ્યા = $\frac{\text{નળાકારનું ઘનફળ}}{\text{એક કોનનું ઘનફળ}}$

$= \frac{\pi \times 8 \times 8 \times 40}{\frac{64}{3}\pi} = 120$

અથવા

રમકડાનું કુલ ક્ષેત્રફળ = અર્ધવર્તુળની વક્રસપાટીનું ક્ષેત્રફળ + શંકુની વક્રસપાટીનું ક્ષેત્રફળ

$= 2\pi R^2 + \pi r\ell$

R = 3.5 cm

r = 3.5 cm

H = 15.5 cm

શંકુની ઊંચાઈ = 15.5 – 3.5 = 12 cm

$\ell^2 = h^2 + r^2$

$\ell^2 = 12^2 + (3.5)^2$

$\ell^2 = 144 + \frac{49}{4}$

$\ell^2 = \frac{62.5}{4}$

$\ell = \frac{25}{2}$

$A = 2 \times \frac{22}{7} \times (3.5) \times (3.5) + \frac{22}{7} \times 3.5 \times \frac{25}{2}$

$= 77 + 137.5$

$= 214.5\ cm^2$

32.

$$x^2 - 2x + k \overline{)x^4 - 6x^3 + 16x^2 - 25x + 10} \quad (x^2 - 4x + 8 - k$$

$$\underline{-\ x^4 \mp 2x^3 \pm kx^2}$$

$$-4x^3 + x^2(16 - k) - 25x + 10$$

$$\underline{+\ -4x^3 + 8x^2 - 4kx\ +}$$

$$x^2(8 - k) + x(4k - 25) + 10$$

$$\underline{-\ x^2(8 - k) - 16x + 2kx + 8k - k^2\ +}$$

$$x(2k - 9) + 10 - 8k + k^2$$

શેષ = x + a

x નો સહગુણક 1 છે.

તેથી, $2k - 9 = 1$

$k = 5$

$a = 10 - 8k + k^2$

$= 10 - 40 + 25$

$a = -5$

અથવા

$(x-2)(x+2) = x^2 - 4$

$$\begin{array}{r} 9x^2 - 6x + 1 \\ x^2 - 4 \overline{)9x^4 - 6x^3 - 35x^2 + 24x - 4} \\ \underline{-\ 9x^4 \ +\ 36x^2} \\ -6x^3 + x^2 + 24x - 4 \\ \underline{-6x^3 + 24x} \\ x^2 - 4 \\ \underline{x^2 - 4} \\ 0 \end{array}$$

$p(x) = q(x) \times g(x) + r(x)$

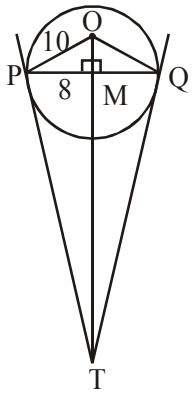
$g(x) = x^2 - 4$

$r(x) = 0$

$q(x) = 9x^2 - 6x + 1 = (3x - 1)^2$

$x = \frac{1}{3}, \frac{1}{3}$

33.



In ΔOMP

$OP^2 = PM^2 + OM^2$

$100 = 64 + OM^2$

$OM = 6$ cm

$PT = x$

$TM = y$

In ΔOPT

$(6 + y)^2 = 10^2 + x^2$ ....(1)

In ΔPMT

$x^2 = 8^2 + y^2$ ....(2)

from equation (1) & (2)

$(6 + y)^2 = 10^2 + 8^2 + y^2$

$y^2 + 12y + 36 = 100 + 64 + y^2$

$12y = 64 + 64$

$y = \frac{128}{12} = \frac{32}{3}$

$y = \frac{32}{3}$ cm

$(x-2)(x+2) = x^2 - 4$

$$\begin{array}{r} 9x^2 - 6x + 1 \\ x^2 - 4 \overline{)9x^4 - 6x^3 - 35x^2 + 24x - 4} \\ \underline{-\ 9x^4 \ +\ 36x^2} \\ -6x^3 + x^2 + 24x - 4 \\ \underline{-6x^3 + 24x} \\ x^2 - 4 \\ \underline{x^2 - 4} \\ 0 \end{array}$$

$p(x) = q(x) \times g(x) + r(x)$

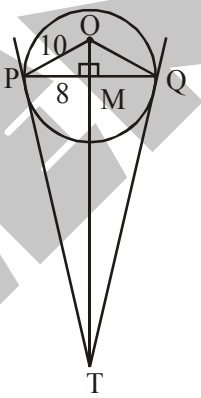
$g(x) = x^2 - 4$

$r(x) = 0$

$q(x) = 9x^2 - 6x + 1 = (3x - 1)^2$

$x = \frac{1}{3}, \frac{1}{3}$

33.



ΔOMP માં,

$OP^2 = PM^2 + OM^2$

$100 = 64 + OM^2$

$OM = 6$ cm

$PT = x$

$TM = y$

ΔOPT માં,

$(6 + y)^2 = 10^2 + x^2$ ....(1)

ΔPMT માં,

$x^2 = 8^2 + y^2$ ....(2)

સમીકરણ (1) અને (2) પરથી,

$(6 + y)^2 = 10^2 + 8^2 + y^2$

$y^2 + 12y + 36 = 100 + 64 + y^2$

$12y = 64 + 64$

$y = \frac{128}{12} = \frac{32}{3}$

$y = \frac{32}{3}$ cm

$x^2 = 8^2 + y^2 = 64 + \frac{1024}{9}$

$x^2 = \frac{1600}{9}$

$x = \frac{40}{3}$ cm

$PT = \frac{40}{3}$ cm

34.

M.Ex. (RS)	f	c.f.	
0-175	10	10	
175-350	14	24	
350-525	15	39	
525-700	21	60	→ Median – class
700-875	28	88	
875-1050	7	95	
1050-1125	5	100	

$N = 100$

$\frac{N}{2} = 50$

$\text{Median} = \ell + \left(\frac{\frac{N}{2} - cf}{f}\right)h$

$\ell = 525$
$f = 21$
$cf = 39$
$h = 175$

$M = 525 + \left(\frac{\frac{100}{2} - 39}{21}\right) \times 175$

$M = 616.66$

SECTION-D

35. Speed of bead = 18 km/hr
speed of streath = y km/hr
$t_{up} - t_{down} = 1$

$\frac{24}{18-y} - \frac{24}{18+y} = 1$

$24\left[\frac{18+y-18+y}{18^2-y^2}\right] = 1$

$24(2y) = 324 - y^2$
$y^2 + 48y - 324 = 0$
$(y + 54)(y - 6) = 0$
$y = 6$ km/hr

$x^2 = 8^2 + y^2 = 64 + \frac{1024}{9}$

$x^2 = \frac{1600}{9}$

$x = \frac{40}{3}$ cm

$PT = \frac{40}{3}$ cm

34.

M.Ex. (RS)	f	c.f.	
0-175	10	10	
175-350	14	24	
350-525	15	39	
525-700	21	60	→ મધ્યસ્થ વર્ગ
700-875	28	88	
875-1050	7	95	
1050-1125	5	100	

$N = 100$

$\frac{N}{2} = 50$

$\text{મધ્યસ્થ} = \ell + \left(\frac{\frac{N}{2} - cf}{f}\right)h$

$\ell = 525$
$f = 21$
$cf = 39$
$h = 175$

$M = 525 + \left(\frac{\frac{100}{2} - 39}{21}\right) \times 175$

$M = 616.66$

SECTION-D

35. બોટની ઝડપ = 18 km/hr
પ્રવાહની ઝડપ = y km/hr
$t_{up} - t_{down} = 1$

$\frac{24}{18-y} - \frac{24}{18+y} = 1$

$24\left[\frac{18+y-18+y}{18^2-y^2}\right] = 1$

$24(2y) = 324 - y^2$
$y^2 + 48y - 324 = 0$
$(y + 54)(y - 6) = 0$
$y = 6$ km/hr

OR

If the time taken by larger tap be x hr and time taken by smaller tap = (x + 10) hr

Fill by t_{up} in ore hr = total

$$\frac{1}{x}+\frac{1}{x+10}=\frac{8}{75}$$

$$\Rightarrow \frac{2x+10}{x^2+10x}=\frac{8}{75}$$

$\Rightarrow 8x^2 - 70x - 750 = 0$

$4x^2 - 60x + 25x - 375 = 0$

$(x - 75)(x + 25) = 0$

$x = 10$ hr

Smaller tap fill tank in 25 hr

larger top fill tank in 10 hr

36. State : In right angle triangle, square of hypotenuse is equal to sum of square of ther two sides

Given : In ΔABC $\angle B = 90^\circ$

To prove = $AC^2 = AB^2 + BC^2$

const draw $BD \perp AC$

Proof : In ΔABC & ΔADB

$\angle A = \angle A$ [common]

$\angle B = \angle ADB = 90^\circ$

by AA similunty

$\Delta ABC \sim \Delta ADB$

by CPCT

$$\frac{AB}{AD}=\frac{AC}{AB}$$

$AB^2 = AC.AD$ ....(1)

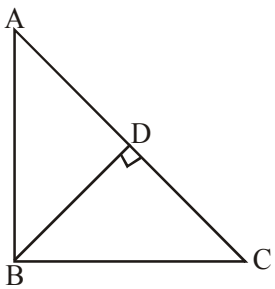
In ΔCAB and CBD

$\angle C = \angle C$ (common)

$\angle B = \angle D = 90^\circ$

by AA similanty

$\Delta CAB \sim \Delta CBD$



$$\frac{BC}{AC}=\frac{DC}{BC}$$

$BC^2 = AC.DC$ ....(2)

add (1) & (2)

$AB^2 + BC^2 = AC.DC + AC.AD$

$= AC(CD + AD)$

$= AC^2$

અથવા

ધારો કે મોટા નળ દ્વારા લાગતો સમય x કલાક જેટલો છે.

$\therefore$ નાના નળ દ્વારા લાગતો સમય = (x + 10) hr

એક કલાકમાં ભરાતુ પાણી = $\frac{1}{x}+\frac{1}{x+10}=\frac{8}{75}$

$$\Rightarrow \frac{2x+10}{x^2+10x}=\frac{8}{75}$$

$\Rightarrow 8x^2 - 70x - 750 = 0$

$4x^2 - 60x + 25x - 375 = 0$

$(x - 75)(x + 25) = 0$

$x = 10$ hr

આમ, નાના નળ દ્વારા પાણીની ટાંકી 25 કલાકમાં

અને મોટા નળ દ્વારા ટાંકી 15 કલાકમાં

36. પ્રમેય : કાટકોણ ત્રિકોણમાં કર્ણની લંબાઈનો વર્ગ એ બીજી બે બાજુના વર્ગના સરવાળા બરાબર હોય છે.

પક્ષ : ΔABC મા, $\angle B = 90^\circ$

સાધ્ય = $AC^2 = AB^2 + BC^2$

સાબિતી BD $\perp$ AC દોરો.

સાબિતી : ΔABC અને ΔADB મા,

$\angle A = \angle A$ [ખૂં ખૂ ખૂ]

$\angle B = \angle ADB = 90^\circ$

AA ની સમરૂપતા

$\Delta ABC \sim \Delta ADB$

CPCT દ્વારા

$$\frac{AB}{AD}=\frac{AC}{AB}$$

$AB^2 = AC.AD$ ....(1)

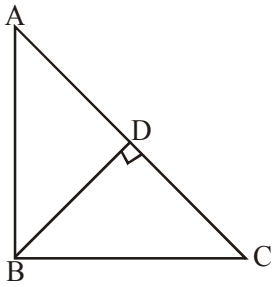
ΔCAB અને CBD મા,

$\angle C = \angle C$ (સામાન્ય)

$\angle B = \angle D = 90^\circ$

AA ની સમરૂપતા

$\Delta CAB \sim \Delta CBD$



$$\frac{BC}{AC}=\frac{DC}{BC}$$

$BC^2 = AC.DC$ ....(2)

(1) અને (2) ઉમેરતાં,

$AB^2 + BC^2 = AC.DC + AC.AD$

$= AC(CD + AD)$

$= AC^2$

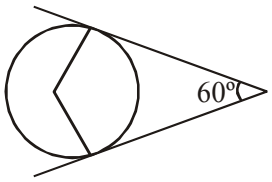
37. (1) Construct a circle with centre o and radius 6 cm

(2) Draw a radius OP

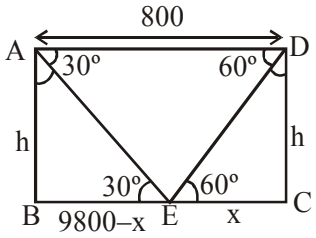
(3) Drawanother radius OQ inclined at 120° on OP

(4) Construct tangent perpendicular to radius

(5) Tangent meet at 60°



38.



In ΔABE

$$\tan 30^\circ = \frac{AB}{BE} = \frac{h}{800-x}$$

$$\frac{1}{\sqrt{3}} = \frac{h}{800-x}$$

$$h = \frac{800-x}{\sqrt{3}} \quad \ldots.(1)$$

In ΔDCE

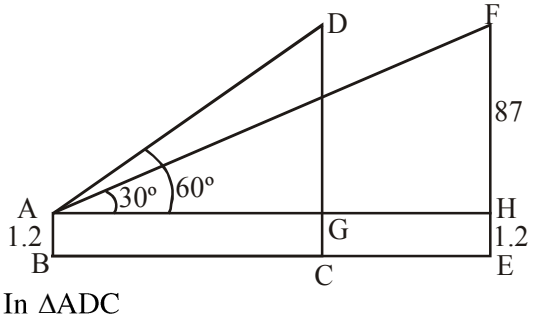
$$\tan 60^\circ = \frac{DC}{EC} = \frac{h}{x} = \sqrt{3}$$

$$x = \frac{h}{\sqrt{3}}$$

Solve (1) & (2)

$h = 200\sqrt{3}$

OR



In ΔADC

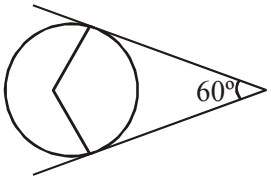
37. (1) o ને કેન્દ્ર રાખી 6 cm ત્રિજ્યાવાળું વર્તુળ દોરો.

(2) ત્રિજ્યા OP દોરો.

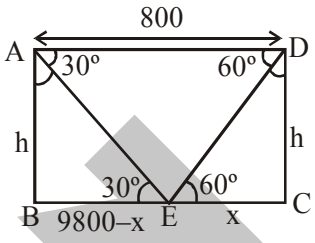
(3) OP પર 120° નો ખૂણો આંતરે એ રીતે બીજી ત્રિજ્યા OQ દોરો.

(4) બંને ત્રિજ્યાને લંબ સ્પર્શક દોરો.

(5) સ્પર્શકો 60° ના ખૂણા એકબીજાને મળે છે.



38.



ΔABE માં,

$$\tan 30^\circ = \frac{AB}{BE} = \frac{h}{800-x}$$

$$\frac{1}{\sqrt{3}} = \frac{h}{800-x}$$

$$h = \frac{800-x}{\sqrt{3}} \quad \ldots.(1)$$

ΔDCE માં,

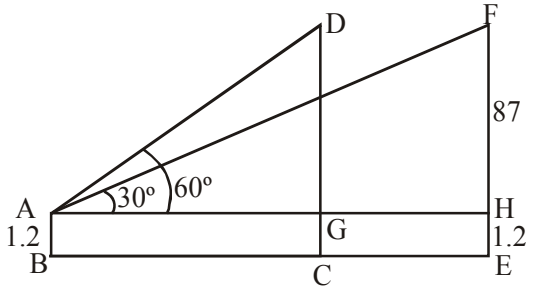
$$\tan 60^\circ = \frac{DC}{EC} = \frac{h}{x} = \sqrt{3}$$

$$x = \frac{h}{\sqrt{3}}$$

(1) અને (2) ને ઉકેલતાં,

$h = 200\sqrt{3}$

અથવા



ΔADC માં,

$\tan 60^\circ = \frac{DG}{AG} = \frac{87}{AG} = \sqrt{3}$

$AG = \frac{87}{\sqrt{3}}$

In ΔAFH

$\tan 30^\circ = \frac{FH}{AH} = \frac{87}{AG + GH} = \frac{1}{\sqrt{3}}$

$87\sqrt{3} = AG + GH$

$87\sqrt{3} - \frac{87}{\sqrt{3}} = GH$

$GH = \frac{87 \times 2}{\sqrt{3}} = \frac{174}{\sqrt{3}}$

39. $\frac{x+1}{x-1} + \frac{x+2}{x-2} = 4 - \frac{2x+3}{x-2}$

$\frac{(x+1)(x+2)+(x-2)(x-1)}{(x-1)(x+2)} = \frac{4(x-2)-2x-3}{x-2}$

$\frac{2x^2+4}{x^2+x-2} = \frac{2x-11}{x-2}$

$5x^2 + 19x - 30 = 0$

$(x + 5)(5x - 6) = 0$

$x = -5$ or $\frac{6}{5}$

$\tan 60^\circ = \frac{DG}{AG} = \frac{87}{AG} = \sqrt{3}$

$AG = \frac{87}{\sqrt{3}}$

ΔAFH માં,

$\tan 30^\circ = \frac{FH}{AH} = \frac{87}{AG + GH} = \frac{1}{\sqrt{3}}$

$87\sqrt{3} = AG + GH$

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39. $\frac{x+1}{x-1} + \frac{x+2}{x-2} = 4 - \frac{2x+3}{x-2}$

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$5x^2 + 19x - 30 = 0$

$(x + 5)(5x - 6) = 0$

$x = -5$ અથવા $\frac{6}{5}$