MATHEMATICS (CLASS-X) [GSEB] MAJOR TEST # 23

TIME : 3 Hrs. **MAX. MARKS : 80**

INSTRUCTIONS:

(1) All the questions are compulsory. Internal options are available in certain questions.
(2) In all 39 questions in this question paper are divided into four sections A, B, C and D.
(3) The numbers on the right side represent the marks of that question (Section).
(4) Draw figure, wherever necessary. Maintain the lines and arcs of the construction.
(5) Start writing a new section from a new page. Answer the questions in serial order.
(6) Use of calculator is not permitted

SOLUTION

SECTION - A

Answer according to instruction : (Question no.1 to 16) [1 mark each] [16]

State whether the following statements are True or False.

1. **False.** Yes it can be a quadratic polynomial because quadratic polynomials have atmost two zeroes but not exactly two zeroes. When the graph line of a polynomial intersects x axis at only one point, then it says that the polynomial has two equal roots (where Discriminant = 0). **[1]**

2. **True** **[1]**

Given $x^2 + x + 1 = 0$

Here a=1, b=1 and c=1

So, Discriminant $D = b^2 - 4ac$

$= (1)^2 - 4 \times 1 \times 1$

$= 1 - 4$

$= -3$

Here $D < 0$

$\therefore$ Equation has no real roots.

3. **True** **[1]**

Let S be the required sum.

Clearly, it is an Arithmetic Progression whose first term = 1, last term = n and number of terms = n.

Therefore, S = $\frac{n}{2}(n+1)$, [Using the formulas S = $\frac{n}{2}(a+l)$]

4. **False** **[1]**

LHS $= (\tan\theta + 2)(2\tan\theta + 1)$

$= 2\tan^2\theta + 4\tan\theta + \tan\theta + 2$

$= 2\tan^2\theta + 5\tan\theta + 2$

$= 2(\sec^2\theta - 1) + 5\tan\theta + 2$

$= 2\sec^2\theta - 2 + 5\tan\theta + 2$

$= 5\tan\theta + 2\sec^2\theta + =$ RHS

Select an option for correct statement.

5. (2) 2 **[1]**

6. (4) 1.5 **[1]**

7. (4) $\frac{3}{8}$ **[1]**

Let P(A) = 3x and $P(\overline{A})$ = 5x

P(A) + $P(\overline{A})$ = 1

$3x + 5x = 1$

$8x = 1$

$x = \frac{1}{8}$

$\therefore P(A) = 3x = \frac{3}{8}$

8. (1) $\frac{5}{7}$ **[1]**

Let the required fraction be $\frac{x}{y}$. Then,

$x + y = 12$ ....(1)

and, $\frac{x}{y+3} = \frac{1}{2}$

$2x - y = 3$ ....(2)

On adding (1) and (2), we get, $x = 5$

On putting $x = 5$ in equation (1), we get, $y = 7$

Therefore, the required fraction is $\frac{5}{7}$

Fill in the blanks for correct statement.

9. Put $x = 2$ and $y = 3$ in $5x - 3y = k$ **[1]**

$5(2) - 3(3) = k$

$k = 10 - 9 = 1$

10. $\Delta = \left|\frac{1}{2}\left[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\right]\right|$ **[1]**

Here, $x_1 = 3$, $y_1 = 0$, $x_2 = 7$, $y_2 = 0$, $x_3 = 8$ and $y_3 = 4$

$\therefore \Delta = \left|\frac{1}{2}\left[3(0-4) + 7(4-0) + 8(0-0)\right]\right|$

$= \left|\frac{1}{2}(-12 + 28 + 0)\right| = \left|\frac{1}{2}(16)\right| = 8$ sq. units

11. Given, $\cos A = \frac{4}{5}$ **[1]**

$\sin A = \sqrt{1 - \cos^2 A}\ \left[\because \sin^2 A + \cos^2 A = 1\right]$

$= \sqrt{1 - \left(\frac{4}{5}\right)^2} = \sqrt{1 - \left(\frac{16}{25}\right)} = \sqrt{\frac{9}{25}} = \frac{3}{5}$

Now, $\tan A = \frac{\sin A}{\cos A} = \frac{\frac{3}{5}}{\frac{4}{5}} = \frac{3}{4}$

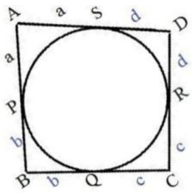
Hence, the required value of tan A is $\frac{3}{4}$

12. $AB + CD = BC + AD$

$7.2 + CD = 8.5 + 6.9$

$CD = 15.4 - 72$

$CD = 8.2$ cm



Answer in one word or one number or one sentence:

13. It is given that the area of a circle is 220 cm^2. **[1]**

$A = \pi r^2$

$220 = \frac{22}{7} \times r^2$

$r^2 = 70$

$r = \sqrt{70}$

The diameter of circle is

$2r = 2\sqrt{70}$

The diameter of the circle is diagonal of square. Let the side of square be a.

Using Pythagoras theorem

$a^2 + a^2 = d^2$

$2a^2 = \left(2\sqrt{70}\right)^2$

$2a^2 = 4 \times 70$

$a^2 = 2 \times 70$

$a^2 = 140$

The area of square is a^2, therefore the area of square is 140 cm^2.

14. Diameter of the base of a right circular cylinder, d = 28 cm **[1]**

Base radius = r = $\frac{d}{2} = \frac{28}{2}$ = 14 cm.

Height of the cylinder = h = 21 cm.

Now, Area of base (area of each end) = $\pi r^2 = \frac{22}{7} \times (14)^2 = 616 cm^2$

Curved surface are = $2\pi rh = 2 \times \frac{22}{7} \times 14 \times 21 = 1848 cm^2$.

15. Number of favorable cases = 1 (marks scored = 35) **[1]**

Total Number of cases = 51 (marks can be scored from 0 to 50)

Probability = 1/51

16. Let A be the event that there are 5 Sundays in February **[1]**

The number of days in February = 29

That is, 4 weeks and 1 more day

There are 4 Sundays in 4 weeks.

However, the last day can be any day in the last week

So, the probability of the event A = $\frac{1}{7}$

SECTION - B

Do as directed (Q-17 to Q-26) [2 marks each] **[20]**

17. The length of the room = 7m 50cm = 700 + 50 = 750cm **[1/2]**

The breadth of the room = 6 m = 600 cm

The height of the room = 3 m 75 cm = 300 + 75 = 375 cm

Now, in order to find the length of the longest rod that can measure all the three dimensions of the room exactly, we will find the H.C.F of 750, 600 & 375

$750 = 2 \times 3 \times 5^3$ **[1]**

$600 = 2^3 \times 3 \times 5^2$

$375 = 3 \times 5^3$

$HCF = 3 \times 5^2 = 75\ cm$

Hence, the length of the longest rod that can measure all the three dimensions of the room exactly is 75 cm. **[1/2]**

18. As per Euclid's Division Lemma **[1/2]**

If a and b are 2 positive integers, then

$a = bq + r$ where $0 \le r < b$

Let positive integer be a and b = 3

Hence $a = 3q + r$ where $(0 \le r < 3)$ **[1]**

If r = 0	If r = 1	If r =2
Our equation becomes	Our equation becomes	Our equation becomes
$a = 3q + r$	$a = 3q + r$	$a = 3q + r$
$a = 3q + 0$	$a = 3q + 1$	$a = 3q + 2$
$a = 3q$	Squaring both sides	Squaring both sides
Squaring both sides	$a^2 = (3q + 1)^2$	$a^2 = (3q + 2)^2$
$a^2 = (3q)^2$	$a^2 = (3q)^2 + 2^2 + 2(2)(3q)$	$a^2 = (3q)^2 + 2^2 + 2(2)(3q)$
$a^2 = 9q^2$	$a^2 = 9q^2 + 6q + 1$	$a^2 = 9q^2 + 12q + 3 + 1$
$a^2 = 3(3q)^2$	$a^2 = 3(3q^2 + 2q) + 1$	$a^2 = 3(3q^2 + 4q + 1) + 1$
$a^2 = 3m$	$= 3m + 1$	$a^2 = 3m + 1$
Where $m = 3q^2$	Where $m = 3q^2 + 2q$	Where $m = 3q^2 + 4q + 1$

Hence the square of any positive integer is either of the form 3 m or 3m + 1 for some integer m. **[1/2]**

19. $x^2 + 7x + 10 = (x + 2)(x + 5)$ **[1]**

So, the value of $x^2 + 7x + 10$ is zero when $x + 2 = 0$ or $x + 5 = 0$

Therefore, the zeroes of $x^2 + 7x + 10$ are –2 and –5.

Sum of zeroes = –7 = –(Coefficient of x) / (Coefficient of x^2) **[1]**

Product of zeroes = 10 = Constant term / Coefficient of x^2

20. $\frac{x+1}{2} + \frac{y-1}{3} = 9$ **[1/2]**

or, $3(x + 1) + 2(y – 1) = 54$

or, $3x + 2y = 53$ ....(i)

and $\frac{x-1}{3} + \frac{y+1}{2} = 8$

or, $2(x-1)+3(y+1)=48$

or, $2x+3y=47$ ....(ii)

Multiply eqn. (i) by 3, $9x+6y=159$ **[1]**

Multiply equn.(ii) by 2, $4x+6y=94$

$-\quad -\qquad -$

On subtracting $5x = 65$

$$x=\frac{65}{5}=13$$

Substitute the value of x in eqn. (ii), **[1/2]**

$2(13)+3y=47$

$3y=47-26=21$

$\therefore\quad y=\frac{21}{3}=7$

Hence $x=13, y=7$.

OR

20. Let the speeds of the cars be x km/hr and y km/hr **[1/2]**

Case 1 : When the cars are going in the same direction **[1]**

Relative speed $= x-y$

Distance = 100 km

Time $= 100/(x-y) = 5$ hrs.

$x-y = 100/5 = 20$

$x-y=20$ .....(1)

Case 1 : When the cars are going in the opposite direction

Relative speed $= x+y$

Time $= 100/(x+y) = 1$ hrs.

$x+y=100$ .....(2)

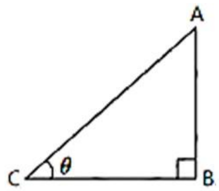
Solving the equations (1) and (2).

$x=60,\ y=40$ **[1/2]**

Hence the speeds of the cars are 60 km/hr and 40 km/hr.

21. **Solution-1** **[1/2]**

Let us consider a right triangle ABC, right-angled at point B.



$$\cot\theta=\frac{7}{8}$$

If BC is 7k, then AB will be 8k, where k is a positive integer.

Applying Pythagoras theorem in ΔABC, we obtain **[1]**

$AC^2 = AB^2 + BC^2$

$= (8k)^2 + (7k)^2$

$= 64k^2 + 49k^2$

$= 113k^2$

$AC = \sqrt{113}k$

$\sin\theta = \frac{8}{\sqrt{113}k} = \frac{8}{\sqrt{113}}$

$\cos\theta = \frac{7k}{\sqrt{113}k} = \frac{7}{\sqrt{113}}$

$\frac{(1+\sin\theta)(1-\sin\theta)}{(1+\cos\theta)(1-\cos\theta)} = \frac{1-\sin^2\theta}{1-\cos^2\theta}$

$= \frac{1-\left(\frac{8}{\sqrt{113}}\right)^2}{1-\left(\frac{8}{\sqrt{113}}\right)^2} = \frac{1-\frac{64}{113}}{1-\frac{49}{113}}$ **[1/2]**

$= \frac{\frac{49}{113}}{\frac{64}{113}} = \frac{49}{64}$

Solution – 2 **[1/2]**

$\cot\theta = \frac{7}{8}$

$\frac{(1+\sin\theta)(1-\sin\theta)}{(1+\cos\theta)(1-\cos\theta)}$

$= \frac{1-\sin^2\theta}{1-\cos^2\theta}\left[\because (a+b)(a-b) = a^2-b^2\right]$

$= \frac{\cos^2\theta}{\sin^2\theta}\left[\because \sin^2\theta+\cos^2\theta=1\right]$ **[1]**

$= \cot^2\theta$

$= (\cot\theta)^2 = \left[\frac{7}{8}\right]^2$ **[1/2]**

$= \frac{49}{64}$

22. Sin 2x = sin 60º cos 30º – cos 60º sin 30º **[1/2]**

We know that, sin 60º = $\frac{\sqrt{3}}{2}$

cos 30º = $\frac{\sqrt{3}}{2}$

sin 30º = $\frac{1}{2}$

$\cos 60° = \frac{1}{2}$

Putting all values **[1]**

$\sin 2x = \sin 60° \cos 30° - \cos 60° \sin 30°$

$$\sin 2x = \left(\frac{\sqrt{3}}{2}\right)\times\left(\frac{\sqrt{3}}{2}\right)-\left(\frac{1}{2}\right)\times\left(\frac{1}{2}\right)$$

$$\sin 2x = \frac{\sqrt{3}\times\sqrt{3}}{2\times 2}-\frac{1}{2\times 2}$$

$\sin 2x = 1 = \sin 30°$ **[1/2]**

$2x = 30°$

$x = 15°$

OR

Given $\sin\theta + \sin^2\theta = 1$ **[2]**

$\Rightarrow \sin\theta = 1 - \sin^2\theta$

$\Rightarrow \sin\theta = \cos^2\theta$

$\Rightarrow \sin^2\theta = \cos^4\theta$

$\Rightarrow 1 - \cos^2\theta = \cos^4\theta$

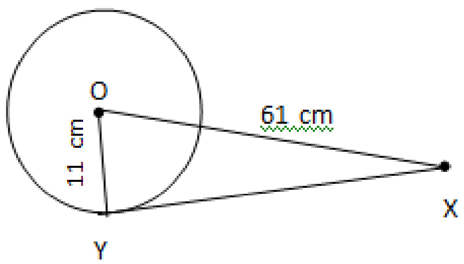
$\Rightarrow \cos^2\theta + \cos^4\theta = 1$

23. Given Diameter of a circle = 22 cm **[1/2]**

$\Rightarrow$ Radius of a circle = OY = $\frac{22}{2}$ = 11 cm

$OY \perp XY$

[$\because$ Tangent to the circle is perpendicular to radius] **[1]**



In ΔOYX, by applying Pythagoras theorem,

$OY^2 + XY^2 = OX^2$

$(11)^2 + XY^2 = (61)^2$

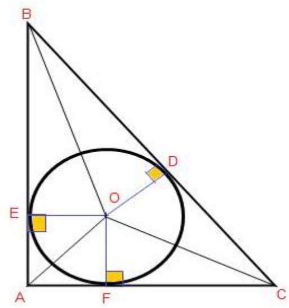
$121 + XY^2 = 3721$

$XY^2 = 3721 - 121$

$\therefore XY = 60$ cm **[1/2]**

OR

23. Given AB, BC and AC are tangents to the circle at E, D and F. **[1/2]**



BD = 30 cm and DC = 7 cm and $\angle BAC = 90°$

Recall that tangents drawn from an exterior point to a circle are equal

Hence BE = BD = 30 cm

Also FC = DC = 7 cm

Let AE = AF = x $\rightarrow$ (1)

Then AB = BE + AE = (30 + x)

AC = BD + DC = 30 + 7 = 37 cm

Consider right ΔABC, by Pythagoras theorem we have

$BC^2 = AB^2 + AC^2$ **[1/2]**

$\Rightarrow (37)^2 = (30 + x)^2 + (7 + x)^2$

$\Rightarrow 1369 = 900 + 60x + x^2 + 49 + 14x + x^2$

$\Rightarrow 2x^2 + 74x + 949 - 1369 = 0$

$\Rightarrow 2x^2 + 74x - 420 = 0$

$\Rightarrow x^2 + 37x - 210 = 0$

$\Rightarrow x^2 + 42x - 5x - 210 = 0$

$\Rightarrow x (x + 42) - 5(x + 42) = 0$

$\Rightarrow (x - 5)(x + 42) = 0$

$\Rightarrow (x - 5) = 0$ or $(x + 42) = 0$

$\Rightarrow x = 5$ or $x = -42$

$\Rightarrow x = 5$[Since x cannot be negative]

$\therefore$ AF = 5 cm [From (1)]

Therefore AB = 30 + x = 30 + 5 = 35 cm **[1/2]**

AC = 7 + x = 7 + 5 = 12 cm

Let 'O' be the centre of the circle and 'r' the radius of the circle.

Join point O, F; points O, D and points O, E.

From the figure,

Area of (ΔABC) = Area (ΔAOB) + Area (ΔBOC) + Area(ΔAOC) **[1/2]**

$= \frac{1}{2} \times AC \times AB = \frac{1}{2} \times AB \times OE + \frac{1}{2} \times BC \times OD + \frac{1}{2} \times AC \times OC$

$\Rightarrow AC \times AB = AB \times OE + BC \times OD + AC \times OC$

$\Rightarrow 12 \times 15 = 35 \times r + 37 \times r + 12 \times r$

$\Rightarrow 12 \times 35 = 84r$

$\therefore r = 5$

Thus the radius of the circle is 5 cm

24. **[1/2]**

Lifetimes (in hours)	Frequency
0 – 20	10
20 – 40	35
40 – 60	52 f_0
60 – 80	61 f_1
80 – 100	38 f_2
100 – 120	29

Mode = $l + \frac{f_1 - f_0}{2f_1 - f_0 - f_2} \times h$ **[1]**

Modal class = interval with highest frequency

= 60 – 80

where l = lower limit of modal class = 60

h = class interval = 20 – 0 = 20

f_1 = frequency of the modal class = 61

f_0 = frequency of the class before modal class = 52

f_2 = frequency of the class after modal class = 38

Mode $= 60 + \frac{61-52}{2(61)-52-38} \times 20$

$= 60 + \frac{9}{122-90} \times 2$

$= 60 + \frac{9}{32} \times 20$

$= 60 + \frac{9}{8} \times 5$

= 60 + 5.625

= 65.625 **[1/2]**

25. Given : Lamp post (AB) = 3.6 m **[1/2]**

Height of girl (CD) = 90 cm = $\frac{90}{100}$ m = 0.9 m

Speed = 1.2 m/sec.

To Find : Length of her shadow i.e. DE

Solution :

The girl walks BD distance in 4 seconds

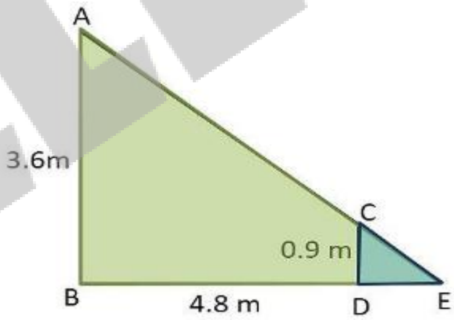
We know that,

Speed = $\frac{\text{Distance}}{\text{Time}}$

$1.2 = \frac{BD}{4}$

1.2 × 4 = BD

BD = 4.8 m



Now, in $\triangle$ABE and $\triangle$CDE

$\angle E = \angle E$ (Common)

$\angle B = \angle D$ (Both 90° because lamp post as well a the girl ar standing vertical to the ground)

Therefore, using AA similarity criterion

So, $\triangle ABE \sim \triangle CDE$

We know that if two triangles are similar, their sides are in proportion **[1]**

$\frac{BE}{DE} = \frac{AB}{CD}$

$\frac{BD+DE}{DE} = \frac{AB}{CD}$

$\frac{4.8+DE}{DE} = \frac{3.6}{0.9}$

$\frac{4.8+DE}{DE} = \frac{36}{9} \times \frac{10}{10}$

$\frac{4.8+DE}{DE} = 4$

4.8 + DE = 4 × DE

4.8 = 4DE – DE

4.8 = 3DE

3DE = 4.8

$$DE = \frac{4.8}{3}$$

$$DE = \frac{48}{3 \times 10}$$

$$DE = \frac{16}{10} = 1.6m$$

Hence, Shadow is 1.6 long **[1/2]**

26. Let x be one's digit and y be tenth digit.

Then, number is 10y + x

So, if the digits interchange their places, then the number is 10x + y **[1]**

now, xy = 20 ....(i)

10y + x – 9 = 10x + y

or 9y – 9x = 9

or y – x = 1 ......(ii)

From (i) and (ii), we get

x (x + 1) = 20 = 4 × 5 = 4 × (4 + 1)

On comparing we get x = 4, then from (i), y = 5 **[1/2]**

Thus, the number is 54.

[**Note :** you can also find roots by solving quadratic equation $x^2 + x - 20 = 0$]

OR

26. $\frac{1}{3}x^2 - \sqrt{11}x + 1 = 0$ **[1/2]**

Multiply by 3 on both sides, we have

$\Rightarrow 3x^2 - 3\sqrt{11}x + 3 = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$ **[1]**

$$= \frac{-\left(-3\sqrt{11}\right) \pm \sqrt{\left(-3\sqrt{11}\right)^2 - 4(3)(3)}}{2(3)}$$

$$= \frac{3\sqrt{11} \pm \sqrt{99 - 16}}{6}$$

$$= \frac{3\sqrt{11} \pm \sqrt{83}}{6}$$

$\therefore$ Roots are $\frac{3\sqrt{11} + \sqrt{83}}{6}, \frac{3\sqrt{11} - \sqrt{83}}{6}$ **[1/2]**

SECTION - C

Do as directed : (Q-27 to Q-34) [3 marks each] **[24]**

27. Given polynomial is $P(x) = 2x^3 + 9x^2 - 15$ **[1]**

Let a is the number which must be added to above polynomial so that it is exactly divisible by 2x + 3

So, by remainder theorem

$$P\left(\frac{-3}{2}\right) + a = 0$$ **[1]**

$$\Rightarrow 2\left(\frac{-3}{2}\right)^3 + 9\left(\frac{-3}{2}\right)^2 - 15 + a = 0$$

$$\Rightarrow \frac{-27}{4} + \frac{81}{4} + a = 0$$ **[1]**

$$\therefore \frac{54}{4} + a = 0 \Rightarrow a = \frac{-27}{2}$$

Hence $\frac{-27}{2}$ must be added the polynomial $2x^3 + 9x^2 - 15$ so that the resulting polynomial is exactly divisible by 2x + 3

28. Let the larger number be x and smaller be y **[1]**

Then according to question $x^2 - y^2 = 180$ ...(1)

and $y^2 = 8x$ ...(2)

From (1) and (2)

$x^2 - 8x - 180 = 0$ **[1]**

$\Rightarrow x^2 - 18x + 10x - 180 = 0$

$\Rightarrow x(x - 18) + 10(x - 18) = 0$

$(x - 18)(x + 10) = 0$

$\therefore x = 18$ or $x = -10$ (Rejected)

$\therefore y^2 = 18 \times 8 = 144$ [From (2)] **[1]**

$y = \pm 12$

Hence the numbers are 18, 12, and 18, –12

29. 3, 8, 13 ...., 253 **[1]**

To find 20^{th} term form the last term we consider the sequence as follows

$a = 253$ and $d = 3 - 8 = -5$ **[1]**

$a_n = a + (n - 1)d$

$a_{20} = 253 + (20 - 1)(-5)$

$= 253 + 19 \times (-5)$

$= 253 - 95$

$= 158$ **[1]**

Hence, the 20^{th} term from the last term is 158.

OR

Multiples of 8 are : 8, 16, 24, 32.... which form an A.P. **[1]**

It is given that, a = 8, d = 16 – 8 = 8, here n = 15

$$S_n = \frac{n}{2}\left[2a + (n-1)d\right]$$ **[1]**

$$\Rightarrow \quad S_n = \frac{15}{2}\left[2\times8 + (15-1)\times8\right]$$

We know that, $= \frac{15}{2}[16 + 14\times8]$

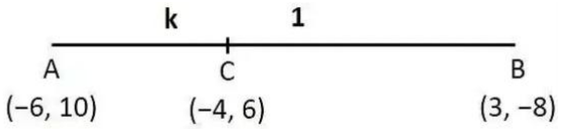
$$= \frac{15}{2}[16 + 112]$$

$$= \frac{15}{2}\times128 = 15\times64 = 960$$ **[1]**

Hence, the sum of first 15 multiples of 8 = 960.

30. Given points A(–6, 10) & B(3, –8) **[1]**

Let point C(–4, 6)



We need to find ratio between AC & CB

Let the ratio be k : 1

Hence, $m_1 = k$, $m_2 = 1$

Also, $x_1 = -6$, $y_1 = 10$

$x_2 = 3$, $y_2 = -8$

& $x = -4$, $y = 6$

Using section formula **[1]**

$$x = \frac{m_1x_2 + m_2x_1}{m_1 + m_2}$$

$$-4 = \frac{3k - 6}{k + 1}$$

$-4(k + 1) = 3k - 6$

$-4k - 4 = 3k - 6$

$-4k - 3k = -6 + 4$

$-7k = -2$

$$k = \frac{-2}{-7}$$

$$k = \frac{2}{7}$$

Hence, then ratio is k : 1 **[1]**

$$= \frac{2}{7} : 1$$

Multiplying 7 both sides

$$= 7\times\frac{2}{7} : 7\times1$$

= 2 : 7

So, the ratio is 2 : 7

31. **[1]**

Dail pocket allowance (in Rs)	Number of workers (f_i)	Class mark (x_i)	$f_i\ x_i$
11 – 13	7	$\frac{11+13}{2} = 12$	7 × 12 = 84
13 – 15	6	$\frac{13+15}{2} = 14$	6 × 14 = 84
15 – 17	9	$\frac{15+17}{2} = 16$	9 × 16 = 144
17 – 19	13	$\frac{17+19}{2} = 18$	13 × 18 = 234
19 – 21	f	$\frac{19+21}{2} = 20$	f × 20 = 20f
21 – 23	5	$\frac{21+23}{2} = 22$	5 × 22 = 110
23 – 25	4	$\frac{23+25}{2} = 24$	4 × 24 = 96
$\sum f_i = 44 + f$		$\sum f_i x_i = 752 + 20f$	

Mean $\left(\bar{x}\right) = \frac{\sum f_i x_i}{\sum f_i}$ **[1]**

$18 = \frac{752 + 20f}{44 + f}$

18 (44 + f) = 752 + 20f

18(44) + 18(f) = 752 + 20f

792 + 18f = 752 + 20f

792 – 752 = 20f – 18f

40 = 2f

$f = \frac{40}{2}$

f = 20 **[1]**

OR

31. Here, modal class = 32 – 41 **[1]**

$l = 32$, $f_0 = a$, $f_1 = 53$, $f_2 = b$ and h = 9

$$\text{Mode} = l + \frac{f_1 - f_0}{2f_1 - f_0 - f_2} \times h$$

$$\Rightarrow 34.5 = 32 + \frac{53 - a}{2 \times 53 - a - b} \times 9$$

$$\Rightarrow \frac{2.5}{9} = \frac{53 - a}{106 - a - b}$$

$\Rightarrow$ 265 – 2.5a – 2.5b = 477 – 9a

$\Rightarrow$ $7.5a - 2.5b = 212$

$\Rightarrow$ $75a - 25b = 2120$

$\therefore$ $13a - 5b = 424$ ...(1)

As total observation are 165 **[1]**

$\Rightarrow$ So, $5 + 11 + a + 53 + b + 16 + 10 = 165$

$\Rightarrow$ $95 + a + b = 165$

$a + b = 70$ ...(2)

$\therefore$ $b = 70 - a$

Substitute $b = 70 - a$ in equation (1) **[1]**

$\Rightarrow$ $13a - 5(70 - a) = 424$

$\Rightarrow$ $13a - 350 + 5a = 424$

$\Rightarrow$ $18a = 424 + 350$

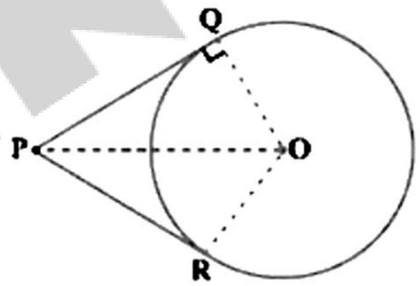
$\Rightarrow$ $18a = 774$

$\therefore$ $a = 43$

$\Rightarrow$ $b = 70 - 43 = 27$

$\therefore$ Frequency $a = 43$ and $b = 27$

32. Given : Let circle be with centre O and P be a point outside circle. PQ and PR are two tangents to circle interscting at point Q and R respectively **[1]**



To prove : Lengths of tangents are equal

i.e. PQ = PR

Construction : Join OQ, OR and OP

Proof : As PQ is a tangent **[1]**

$OQ \perp PQ$ (Tangent at any point of circle is perpendicular to the radius through point of contact)

So, $\angle OQP = 90^\circ$

Hence ΔOQP is right triangle

Similarly,

PR is a tangent

$OR \perp PR$ (Tangent at any point of circle is perpendicular to the radius through point of contact)

So, $\angle ORP = 90^\circ$

In ΔOQP and ΔORP **[1]**

$\angle OQP = \angle ORP$ (Both 90º)

$OP = OP$ (Common)

$OQ = OR$ (Both radius)

$\therefore$ $\Delta OQP \cong \Delta ORP$ (R.H.S. congruency)

Hence, PQ = PR (by CPCT)

Hence both tangents from external point are equal in length

33. Area of the shaded region **[2]**

= (Area of circle with OD (=7 cm) as diameter) + Area of Semi-Circle with AB as diameter – Area of ΔABC

$$= \pi\times\left(\frac{7}{2}\right)^2+\frac{1}{2}\times\pi\times(7)^2-\frac{1}{2}\times AB\times OC$$

$$=\left(\frac{\pi}{4}\times 49+\frac{\pi}{2}\times 49-\frac{1}{2}\times 14\times 7\right)cm^2$$

$$=\left(\frac{3\pi}{4}\times 49-49\right)cm^2$$

$$=\left(\frac{3}{4}\times\frac{22}{7}\times 49-49\right)cm^2 = \frac{231-98}{2}cm = 66.5\ cm^2$$ **[1]**

34. Internal diameter of the pipe 20 cm = $\frac{20}{100}=\frac{1}{5}m$ **[1]**

Internal radius = $\frac{1}{2}\times\frac{1}{5}=\frac{1}{10}m$

Rate of flow of water = 3km/h = 3000 m/h

Let the pipe take t hours to fill up the tank.

Volume of the water that flows in t hours from the pipe **[1]**

= Area of cross section × speed × time

= $\pi r^2 \times$ speed × time

$$= \pi\left(\frac{1}{100}\right)\times 3000\times t = 30\pi t$$

Diameter of the cylinder = 10m $\Rightarrow$ Radius = 5m

Depth = 2m

So, Volume of the tank = $\pi r^2 h = \pi(25)2 = 50\pi\ m^3$

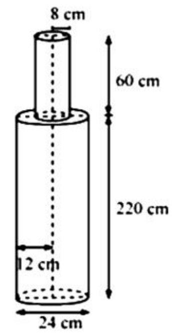
Now volume of the water that flows from the pipe in t hours = volume of the tank **[1]**

$\therefore$ $30\pi t = 50\pi$

$\therefore$ t = $\frac{50}{30}$ hours = $\frac{50}{30}\times 60$ = 100 mins.

OR

34. Form the figure we have **[1]**



Height (h_1) of larger cylinder = 220 cm

Radius (r_1) of larger cylinder = $\frac{24}{2}$ = 12 cm

Height (h_2) of smaller cylinder = 60 cm

Radius (r_2) of smaller cylinder = 8 cm

Total volume of pole

= volume of larger cylinder + volume of smaller cylinder **[1]**

$$= \pi r_1^2 h_1 + \pi r_2^2 h_2$$

$= \pi(12)^2 \times 220 + \pi(8)^2 \times 60$

$= \pi[144 \times 220 + 64 \times 60]$

$= 35520 \times 3.14 = 1,11,532.8 \text{ cm}^3$

Mass of 1 cm^3 iron = 8 gm **[1]**

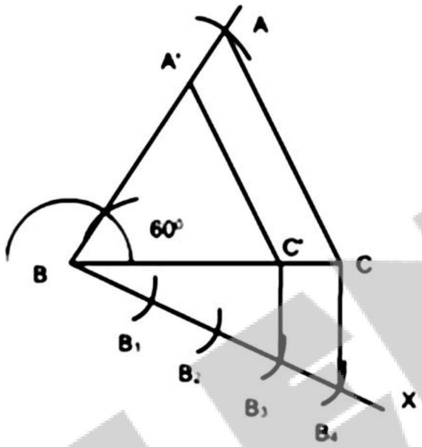
Mass of 111532.8 cm^3 iron = 111532.8 × 8

= 892262.4 gm = 892.262 kg

SECTION - C

Do as directed : (Q-35 to Q-39) [4 marks each] **[20]**

35. **[3]**



Steps of construction: **[1]**

(i) Draw a line segment BC with measurement of 6 cm.

(ii) Now construct angle 60º from point B and draw AB = 5 cm.

(iii) Join the point C with point A. Thus ABC is the required triangle.

(iv) Draw a line BX which makes an acute angle with BC and is opposite of vertex A.

(v) Cut four equal parts of line BX namely BB_1, BB_2, BB_3, BB_4.

(vi) Now join B_4 to C. Draw a line B_3C' parallel to B_4C.

(vii) And then draw a line C´A´ parallel to CA.

Hence

A´BC´ is the required triangle.

OR

35. **Steps of construction:** **[1]**

(i) Taking point O as centre draw a circle of radius 6 cm.

(ii) Now, name a point P which is 10 cm away from point O. Join OP.

(iii) Draw a perpendicular bisector of OP and name the intersection point of bisector and OP as O´.

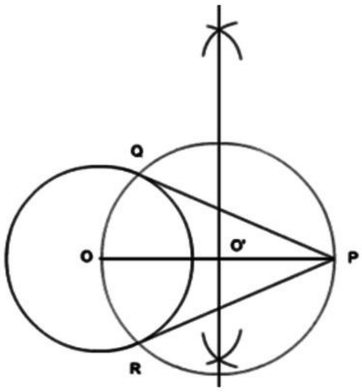
(iv) Now draw a circle considering O´ as centre and O´P as the radius.

(v) Name the intersection point of circles as Q and R.

(vi) Join PQ and PR. These are the required tangents.

(vii) Measure lengths of PQ = 8cm and PR = 8cm

[3]



36. Let the speed of train and bus be u km/h and v km/h respectively. **[1]**

According to the question,

$\frac{60}{u}+\frac{240}{v}=4$ ...(1)

$\frac{100}{u}+\frac{200}{v}=4+\frac{10}{60}=\frac{25}{6}$ ...(2)

Let $\frac{1}{u}=p$ and $\frac{1}{v}=q$

The given equations reduce to: **[1]**

60p + 240q=4 ...(3)

$100p + 200q = \frac{25}{6}$

600p + 1200q = 25 ....(4)

Multiplying equation (3) by 10, we obtain: **[1]**

600p + 2400q =40 ... (5)

Subtracting equation (4) from equation (5), we obtain:

1200q = 15

$q = \frac{15}{1200}=\frac{1}{80}$

Substituting the value of q in equation (3), we obtain: **[1]**

60p + 3 = 4

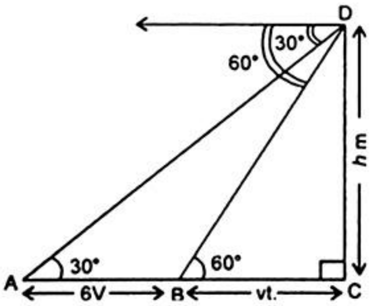
60p = 1

$\therefore\ p = \frac{1}{u}=\frac{1}{60}, q=\frac{1}{v}=\frac{1}{80}$

u = 60 km/h, v = 80 km/h

Thus, the speed of train and the speed of bus are 60 km/h and 80 km/h respectively.

37. Fig. **[1]**



It is given that the angle of depression at A and B from the top of a tower be 30° and 60° respectively.

Let the speed of the car be v m/s. Then AB = distance travelled by the car in 6 s.

= (6 x v) m (Dist = speed x time)

= 6v m

Let the car takes t sec. to reach the tower CD from B. Then,

BC = distance travelled by car in t sec.

= (v x t) m = vt m

In right triangle BCD, we have **[1]**

$\tan 60^o = \frac{CD}{BC}$

$\Rightarrow \sqrt{3} = \frac{h}{vt}$

$\Rightarrow h = \sqrt{3}\ vt$ .....(i)

In right triangle ACD, we have **[1]**

$\tan 30^o = \frac{CD}{AC}$

$\Rightarrow \frac{1}{\sqrt{3}} = \frac{h}{6v+vt}$

$\Rightarrow 6v + vt = \sqrt{3}h$

$\Rightarrow h = \frac{6v+vt}{\sqrt{3}}$ .....(ii)

Comparing (i) and (ii), we get **[1]**

$\sqrt{3}\ vt = \frac{6v+vt}{\sqrt{3}}$

$\Rightarrow \sqrt{3}\times\sqrt{3}vt = 6v+vt$

$\Rightarrow 3vt = 6v + vt$

$\Rightarrow 3vt - vt = 6v$

$\Rightarrow vt\ (3 - 1) = 6v$

$\Rightarrow t \times 2 = 6$

$\Rightarrow t = 3$ seconds

Hence, the time taken by the car to reach the foot of the tower is 3 sec.

38. Volume of the tank $= \frac{2}{3}\pi r^3$ **[1]**

$= \frac{2}{3} \times \frac{22}{7} \times \left(\frac{3}{2}\right)^3$

$= \frac{99}{14} m^3$

$= \frac{99}{14} \times 1000$ liters

$= \frac{99000}{14}$ liters

Volume of water to be emptied $= \frac{1}{2} \times$ volume of tank **[1]**

$= \frac{1}{2} \times \frac{99000}{14}$

$= \frac{99000}{28}$ litres

Time taken to empty $\frac{25}{7}$ litres = 1 sec **[1]**

Time taken to empty 1 litres $= \frac{7}{25}$ sec

Time taken to empty $\frac{99000}{8}$ litres $= \frac{7}{25} \times \frac{99000}{28}$ sec **[1]**

$= \frac{693000}{700}$

= 990 sec

$= \frac{990}{60}$ min

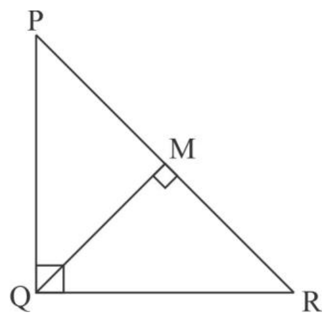
= 16.5 min

39. Given: In ΔPQR, $\angle Q = 90^o$. **[1]**

To prove : $PQ^2 + QR^2 = PR^2$.

Construction : Draw $QM \perp PR$

Proof: **[1]**



In ΔPQM and ΔPQR,

$\angle PMQ = \angle PQR = 90^o$

$\angle QPM = \angle RPQ$ (Common)

$\therefore$ $\Delta PQM \sim \Delta PRQ$ (By AA Similarity)

$\frac{PQ}{PR} = \frac{MR}{PQ}$

$\Rightarrow$ $PQ^2 = PM \times PR$ ...(i)

Similarly, **[1]**

$\therefore$ In ΔQMR and ΔPQR,

$\angle QMR = \angle PQR = 90^o$

$\angle QRM = \angle QRP$ (Common)

$\therefore \angle QRM \sim \angle PQR$ (By AA similarity)

$\therefore \dfrac{PQ}{PR} = \dfrac{MR}{PQ}$

$\Rightarrow PQ^2 = PM \times PR$ ...(ii)

Adding the relations obtained in (i) and (ii), we get, **[1]**

$PQ^2 + QR^2 = PM \times PR + PR \times MR$

$= PR(PM + MR)$

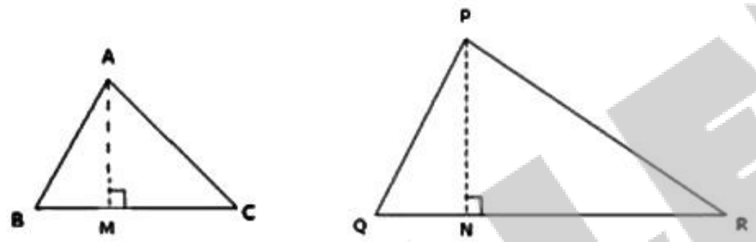
$= PR \times PR$

$= PR^2$

OR

39. Statement : Ratio of the areas of two similar triangles is equal to the square of the ratio of their corresponding sides. **[1]**

Given : Two triangles ABC and PQR such that $\Delta ABC \sim \Delta PQR$



To prove : $\dfrac{ar(ABC)}{ar(PQR)} = \left(\dfrac{AB}{PQ}\right)^2 = \left(\dfrac{BC}{QR}\right)^2 = \left(\dfrac{CA}{RP}\right)^2$

Proof : For finding the areas of the two triangles, we draw altitudes AM and PN of the triangles. **[1]**

Now, $ar(ABC) = \dfrac{1}{2} BC \times AM$

And $ar(PQR) = \dfrac{1}{2} QR \times PN$

So, $\dfrac{ar(ABC)}{ar(PQR)} = \dfrac{\frac{1}{2} \times BC \times AM}{\frac{1}{2} \times QR \times PN} = \dfrac{BC \times AM}{QR \times PN}$ ...(1)

Now, in ΔABM and ΔPQN. **[1]**

$\angle B = \angle Q$ (As $\Delta ABC \sim \Delta PQR$)

And $\angle M = \angle N$ (Each = 90^o)

So, $\Delta ABM \sim \Delta PQN$ (AA similarity criterion)

Therefore, $\dfrac{AM}{PN} = \dfrac{AB}{PQ}$ ...(2)

Also, $\Delta ABC \sim \Delta PQR$

So, $\frac{AB}{PQ} = \frac{BC}{QR} = \frac{CA}{RP}$ ...(3)

Therefore, $\frac{ar(ABC)}{ar(PQR)} = \frac{AB}{PQ} \times \frac{AM}{PN}$ [from(1) and (3)] **[1]**

$= \frac{AB}{PQ} \times \frac{AB}{PQ}$ [From(2)]

$= \left(\frac{AB}{PQ}\right)^2$

Now using (3), we get

$$\frac{ar(ABC)}{ar(PQR)} = \left(\frac{AB}{PQ}\right)^2 = \left(\frac{BC}{QR}\right)^2 = \left(\frac{CA}{RP}\right)^2$$