PART-1 : PHYSICS SET-B SOLUTION

SECTION-I

1. Ans. (B)

2. Ans. (D)

Sol. Particles can have same magnitude of momentum but different directions.

3. Ans. (B)

Sol. EM Wave propagates in the direction parallel to $(\vec{E}\times\vec{B})$

4. Ans. (C)

Sol. $T = 2\pi\sqrt{\dfrac{L}{g}}$

$$\frac{\Delta T}{T} = \frac{1}{2}\frac{\Delta L}{L} + \frac{1}{2}\frac{\Delta g}{g} = 2\% + 1\% = 3\%$$

5. Ans. (B)

Sol. Velocity of particle must be $\dfrac{E_0}{B_0}$

$$P = \sqrt{2mK.E.} = m\frac{E_0}{B_0}$$

$$\sqrt{2m(QV)} = m\frac{E_0}{B_0}$$

$$V = \frac{mE_0^2}{2QB_0^2}$$

6. Ans. (C)

Sol. $F_{min} = \dfrac{\mu mg}{\sqrt{1+\mu^2}} = 24N$

7. Ans. (C)

Sol. Net electric field inside the dielectric

$$E = \frac{Q}{kA\epsilon_0}$$

$$\frac{Q}{4\times\left(\frac{32}{100}\right)^2\times 9\times 10^{-12}} = 20\times 10^6$$

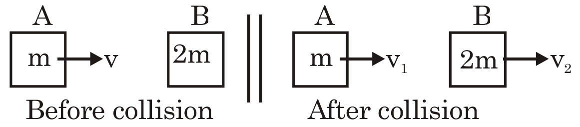
$\Rightarrow Q = 73.7\ \mu C$

8. Ans. (B)

Sol. $K_{eq} = 4K$

9. Ans. (A)

Sol. First collision (between A & B)

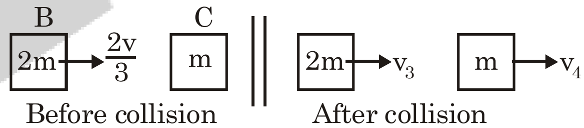


$$mv = mv_1 + 2mv_2$$

$$\frac{1}{2}mv^2 = \frac{1}{2}mv_1^2 + \frac{1}{2}2mv_2^2$$

$$v_1 = -\frac{v}{3},\ v_2 = \frac{2v}{3}$$

Second collision (between B & C)



$$2m\cdot\left(\frac{2v}{3}\right) = 2mv_3 + mv_4$$

$$\frac{1}{2}2m\left(\frac{2v}{3}\right)^2 = \frac{1}{2}(2m)v_3^2 + \frac{1}{2}mv_4^2$$

$$v_3 = \frac{2v}{9},\ v_4 = \frac{8v}{9}$$

10. Ans. (B)

Sol. For PQRU

$$\vec{\mu} = -Ia^2\hat{j}$$

For RUTS

$$\vec{\mu} = -Ia^2\hat{i}$$

$$\mu_{net} = -\left(\frac{\hat{i}+\hat{j}}{\sqrt{2}}\right)$$

11. Ans. (D)

Sol. On increasing resistance R_1, the balancing length should increase. So the given situation is not possible.

12. Ans. (B)

Sol. Let as complete the sphere by taking another hemesphere of charge Q. Total potential of A & B due to each hemisphere

$$(v_A + v_B) + (v_A + v_B) = \frac{K(2Q)}{R} + K\left(\frac{2Q}{R}\right)$$

Due to one hemisphere

$$\Rightarrow v_A + v_B = \frac{2KQ}{R}$$

13. Ans. (A)

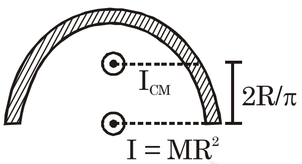
Sol. For polarisation

$$\tan i = \mu_{rel} = \frac{3/2}{1}$$

$$\tan i = \frac{3}{2} \Rightarrow \sin i = \frac{3}{\sqrt{13}}$$

14. Ans. (B)

Sol.



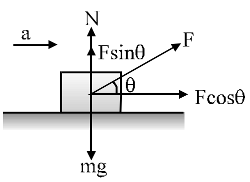
$$I = I_{cm} + Md^2$$

$$MR^2 = I_{cm} + M\left(\frac{2R}{\pi}\right)^2$$

$$\Rightarrow I_{cm} = MR^2 - M\left(\frac{2R}{\pi}\right)^2$$

15. Ans. (C)

Sol.



$$F\cos\theta = ma$$

$$\Rightarrow \frac{mg}{3}\cos\theta = m\frac{dv}{dt}$$

$$\Rightarrow \frac{mg}{3}\cos(ks) = m\frac{dv}{ds}\frac{ds}{dt}$$

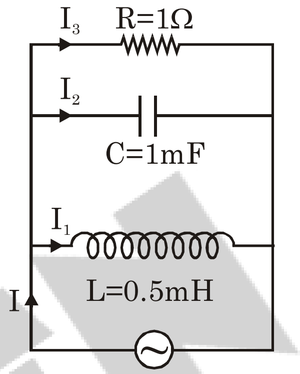
$$\Rightarrow \frac{mg}{3}\cos(ks) = mv\frac{dv}{ds}$$

$$\Rightarrow vdv = \frac{g}{3}\cos(ks)ds$$

$$\Rightarrow \frac{v^2}{2} = \frac{g}{3k}\sin(ks) \Rightarrow v = \sqrt{\frac{2g}{3k}\sin\theta}$$

16. Ans. (C)

Sol.



$V = V_0\sin(1000t + 30°)$

$$R = 1\Omega,\ x_C = \frac{1}{\omega C} = 1\Omega,\ X_L = \omega L = 0.5$$

$$I_1 = \frac{v_0}{x_L}\sin(1000t + 30° - 90°)$$

$$= 2v_0\sin(1000t + 30° - 90°)$$

$$I_2 = \frac{v_0}{x_C}\sin(1000t + 30° + 90°)$$

$$= v_0\sin(1000t + 30° + 90°)$$

$$I_3 = \frac{v_0}{R}\sin(1000t + 30°) = v_0\sin(1000t + 30°)$$

$I_{net} = I_1 + I_2 + I_3 = v_0 2\sin(1000t + 30° - 45°)$

Phase difference between v & I_{net} is 45°

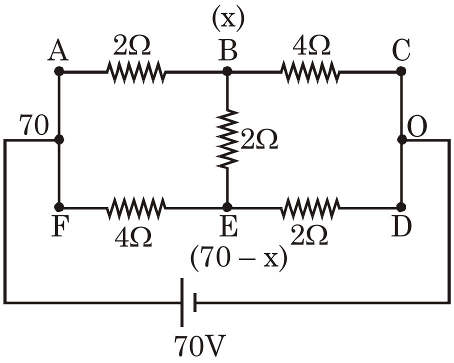
17. Ans. (C)

Sol. $S = ut_R + \dfrac{u^2}{2a}$

$$= 4 \times 0.2 + \frac{(4)^2}{2 \times 2} = 4.8 \text{ m}$$

18. Ans. (B)

Sol.



Applying nodal analysis at B we get

$$\frac{x-70}{2}+\frac{x-0}{4}+\frac{x-(70-x)}{2}=0$$

$\Rightarrow x = 40$

$\Rightarrow$ current in BE = 5A

19. Ans. (C)

Sol. $F=-\frac{dU}{dx}$

K + U = E

20. Ans. (C)

Sol. String free at one end vibrates in odd hermonics only.

$\therefore$ 300 Hz can be the option.

SECTION-II

1. Ans. 6.40

Sol. $\Delta v=-\left[\int E_x dx+\int E_y dy\right]$

$$= =-\left[\int_1^3 2xdx+\int_2^4 3y^2dy\right]$$

$|\Delta v| = 64$

2. Ans. 9.50

Sol. At point P, $v = \sqrt{2g\ell}$

$\Rightarrow a_C = \frac{v^2}{R}$ **= 2g**

$a_t = g \sin 60 = \frac{g\sqrt{3}}{2}$

$a_{net} = \sqrt{a_c^2+a_t^2}$

$= g\frac{\sqrt{19}}{2} \Rightarrow \alpha = 19, \beta = 2$

3. Ans. 5.00

Sol. $K=\frac{\pi}{2}=\frac{2\pi}{\lambda} \Rightarrow \lambda = 4$

Second overtone $\Rightarrow \frac{5\lambda}{4}=L \Rightarrow L = 5m$

4. Ans. –3.00

Sol. $W=\frac{\Delta Q}{u}$

$\Rightarrow \Delta u = \frac{3}{4}\Delta Q$

$nC_v\Delta T = \frac{3}{4}nC\Delta T$

for process PV^x = constant

$\Rightarrow C = \frac{4C_V}{3} = C_v + \frac{R}{1-x}$

$\frac{R}{1-x}=\frac{C_V}{3}=\frac{R}{2}$

$\Rightarrow x = -1$

$\Rightarrow PV^{-1}$ = constant

$\Rightarrow P^3V^{-3} = K$

5. Ans. 25.00

Sol. $\varepsilon=\left(\vec{v}\times\vec{B}\right).\vec{\ell}$

$= \left[1\hat{i}\times\left(3\hat{i}+4j+5\hat{k}\right)\right].5\hat{j}$

$\varepsilon = 25$ v

PART–2 : CHEMISTRY SOLUTION

SECTION-I

1. Ans. (B)
2. Ans. (C)
3. Ans. (A)
4. Ans. (A)
5. Ans. (B)
6. Ans. (C)
7. Ans. (C)
8. Ans. (B)
9. Ans. (A)
10. Ans. (C)
11. Ans. (C)
12. Ans. (D)
13. Ans. (D)
14. Ans. (A)
15. Ans. (A)
16. Ans. (D)
17. Ans. (D)
18. Ans. (A)
19. Ans. (C)
20. Ans. (C)

SECTION-II

1. Ans. (1.40)
2. Ans. (0.02)
3. Ans. (4.00)
   (i), (ii), (iii), (v)
4. Ans. (5.00)
5. Ans. (3.00)

PART-3 : MATHEMATICS SOLUTION

SECTION–I

1. Ans. (C)

Sol. $f(x) = |x|\sin x + |(x-\pi)(x+\pi)|\cos x$ points to be checked $x = 0, \pi, -\pi$ at $x = 0$ function is differentiable and not differentiable at $x = \pi, -\pi$

2. Ans. (B)

Sol. $f'(x) = x^2 - (m-3)x + m \geq 0$

$D \leq 0$

$(m-3)^2 - 4m \leq 0$

$m \in [1, 9]$

3. Ans. (A)

Sol. $\dfrac{12!}{(3!)^4 4!} \times 4! \times \dfrac{40!}{(10!)^4 4!} \times 4!$

$\Rightarrow \dfrac{22(40!)}{(3!)^3(10!)^3}$

4. Ans. (D)

Sol. $z = x + iy$ and $x = |x - 2 + iy|$

$x^2 = (x-2)^2 + y^2$

$4x = y^2 + 4$

Hence $4x_1 = y_1^2 + 4$ ...(i)

$4x_2 = y_2^2 + 4$ ...(ii)

Also $\arg(x_1 - x_2) + i(y_1 - y_2) = \dfrac{\pi}{3}$

$\dfrac{y_1 - y_2}{x_1 - x_2} = \sqrt{3}$

$y_1 - y_2 = \sqrt{3}(x_1 - x_2)$ ...(iii)

on solving (i), (ii), (iii) $y_1 + y_2 = \dfrac{4}{\sqrt{3}}$

5. Ans. (D)

Sol. Sum of roots = tr(A) = 5

product of roots = $|A|$

Characterstic equation is $x^2 - 5x + |A|$

since $f(A) = A^2 + aA + 3I$

$\therefore a = -5$, and $|A| = 3$

consider $A = \begin{bmatrix} p & s \\ r & q \end{bmatrix}$ where $p + q = 5, pq - rs = 3$

$\text{adj}A = \begin{bmatrix} q & -s \\ -r & p \end{bmatrix}$

$(\text{adj}A)^2 = \begin{bmatrix} q & -s \\ -r & p \end{bmatrix}\begin{bmatrix} q & -s \\ -r & p \end{bmatrix} = \begin{bmatrix} q^2 + rs & -rs + p^2 \\ -r(p+q) & p^2 + rs \end{bmatrix}$

$\text{tr}((\text{adj}A)^2 + (a+1)\text{adj}A + 3I)$

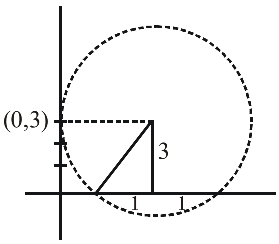
$= p^2 + q^2 + 2rs - 4[p + q] + 3(1 + 1)$

$(p + q)^2 - 2(pq - rs) - 4(p + q) + 6$

$\Rightarrow 25 - 6 - 20 + 6 = 5$

6. Ans. (C)

Sol.



R= Radius = $\sqrt{10}$ centre $\left(\sqrt{10},3\right)$

S = Equation of circle

$=\left(x-\sqrt{10}\right)^2+(y-3)^2=10$

Intercept made by circle S = 0 on line

$\sqrt{10}x-3y=1$

Perpendicular distance $P=\left|\frac{10-9-1}{\sqrt{19}}\right|=0$

Length of intercept $=2\sqrt{R^2-P^2}$

= 2R

$=2\sqrt{10}$

7. Ans. (B)

Sol. $(\vec{r}-\vec{p})\times(\vec{p}\times\vec{q})=0$

$\vec{r}-\vec{p}=\lambda(\vec{p}\times\vec{q})$

$\vec{r}=\vec{p}+\lambda(\vec{p}\times\vec{q})$

$(\vec{p}\times\vec{q}).\vec{r}=12$

$(\vec{p}\times\vec{q}).(\vec{p}+\lambda(\vec{p}\times\vec{q}))=12$

$0+\lambda|\vec{p}\times\vec{q}|^2=12$

$\lambda=\frac{4}{3}$

$|\vec{r}-\vec{p}|=3\lambda=4$

8. Ans. (C)

Sol. $\frac{dy}{dx}-y=e^x$

Integrating factor $e^{\int -1.dx}=e^{-x}$

$\therefore\ ye^{-x}=\int e^x e^{-x}\,dx$

$ye^{-x} = x + c$

$y = xe^x + ce^x$

$1 = 0 + c \quad \therefore c = 1$

$y = (x + 1)e^x$

$f(x) = (x + 1)e^x$

$f'(x) = (x + 1)e^x + e^x = e^x(x + 2)$

$f''(x) = e^x(x + 3)$

$f''(0) = 3$

9. Ans. (A)

Sol. $(1 + x^5)(1 + 5x + 10x^2 + 10x^3 + 5x^4 + x^5)(1 + 3x^3 + 3x^6 + x^9)(1 + 5x + 10x^2 + 10x^3 + 5x^4 + x^5)(1 + 3x^3 + x^5 + ...)$

$\Rightarrow (1 + 1 + 30)x^5$

$32x^5$

10. Ans. (B)

Sol.

$\sim p$	p	q	$p' \to q$	$q \vee \sim p$	$(p\to q)\leftrightarrow q\vee\sim p$
F	T	T	T	T	T
F	T	F	F	F	T
T	F	T	T	T	T
T	F	F	T	T	T

Hence tautology

11. Ans. (A)

Sol. E_1 : failed in physics $P(E_1)=\frac{3}{10}$

E_2 : failed in maths $P(E_2)=\frac{2}{10}$

$P(E_1\cap E_2)=\frac{1}{10}$

$P\left(\frac{E_1}{E_2}\right)=\frac{P(E_1\cap E_2)}{P(E_2)}=\frac{\frac{1}{10}}{\frac{2}{10}}=\frac{1}{2}$

12. Ans. (B)

Sol. Direction of line through (a, 2, 2) and (6, 11, –1) is 6 – a, 9, –3

Direction ratio of line of intersection of planes $\begin{vmatrix} i & j & k \\ 2 & -1 & -3 \\ 1 & 2 & -4 \end{vmatrix}$

= 10i – (–5)j + (5)k

= 10i + 5j + 5k

Since lines are perpendicular

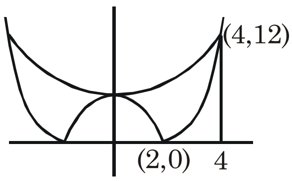
10(6 – a) + 5(9) + 5(–3) = 0

12 – 2a + 6 = 0

a = 9

13. Ans. (A)

Sol.



$$\text{Area} = 2\left[\int_0^4\left(\frac{x^2}{2}+4\right)dx-\int_0^2\left(4-x^2\right)dx-\int_2^4\left(x^2-2\right)dx\right]$$

$$= \frac{64}{3}$$

14. Ans. (B)

Sol. $\dfrac{x+y+\frac{z}{2}+\frac{z}{2}}{4} \ge \left(\dfrac{xyz^2}{4}\right)^{\frac{1}{4}}$

$\dfrac{4}{4} \ge \left(\dfrac{xyz^2}{4}\right)^{\frac{1}{4}}$

$xyz^2 \le 4$

15. Ans. (B)

Sol. $y = mx - 2am - am^3$

$y = x - 2 - 1$

$y = x - 3$

Point of intersection with $y^2 = 4x$

$(1, -2)$ and $(9, 6)$

Length $= 8\sqrt{2}$

16. Ans. (C)

Sol. If each observation is increased by a same quantity the variance remains unchanged.

$\therefore \sigma_A : \sigma_B : \sigma_C = 1 : 1 : 1$

17. Ans. (A)

Sol. $f(x) = \displaystyle\int \frac{2x+5}{(x+1)(x+2)(x+3)(x+4)+1}$

$x^2 + 5x = t$

$\displaystyle\int \frac{dt}{(t+4)(t+6)+1} = \int \frac{dt}{(t+5)^2} = \frac{-1}{t+5} + C$

$\Rightarrow \dfrac{-1}{x^2+5x+5} + C$

18. Ans. (A)

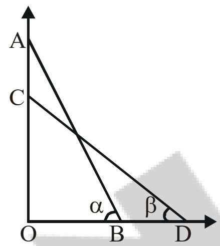
Sol. $\alpha + \beta + \gamma = 0$, $\alpha\beta\gamma = \dfrac{1}{5}$, $5\alpha^3 - 2\alpha - 1 = 0$

$\dfrac{\alpha^3 - 3}{\alpha\beta\gamma} + \dfrac{\beta^3 - 3\beta}{\alpha\beta\gamma} + \dfrac{\gamma^3 - 3\gamma}{\alpha\beta\gamma}$

$\dfrac{\alpha^3 + \beta^3 + \gamma^3 - 3(0)}{\alpha\beta\gamma} = \dfrac{3\alpha\beta\gamma}{\alpha\beta\gamma} = 3$

19. Ans. (B)

Sol.



Let length of ladder is $= \ell \; \therefore \; AB = CD = \ell$

$AC = 3$, $BD = 2$

In ΔAOB $\quad AO = \ell \sin\alpha, \quad OB = \ell\cos\alpha$

In ΔCOD $\quad CO = \ell \sin\beta, \quad OD = \ell\cos\beta$

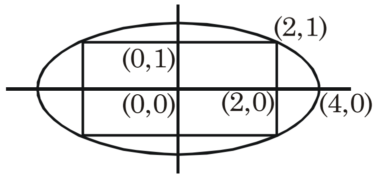
$\dfrac{AC}{BD} = \dfrac{AO - CO}{OD - OB} = \dfrac{\sin\alpha - \sin\beta}{\cos\beta - \cos\alpha}$

$$\frac{AC}{BD} = \frac{2\cos\left(\frac{\alpha+\beta}{2}\right)\sin\left(\frac{\alpha-\beta}{2}\right)}{2\sin\left(\frac{\alpha+\beta}{2}\right)\sin\left(\frac{\alpha-\beta}{2}\right)} = \cot\left(\frac{\alpha+\beta}{2}\right)$$

$\therefore \tan\left(\dfrac{\alpha+\beta}{2}\right) = \dfrac{BD}{AC} = \dfrac{2}{3}$

20. Ans. (B)

Sol.



Let equation be $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$

$\dfrac{16}{a^2} + 0 = 1 \quad \therefore a^2 = 16$

and $\frac{4}{a^2}+\frac{1}{b^2}=1$

$$\frac{1}{b^2}=\frac{3}{4}$$

$$b^2=\frac{4}{3}$$

$$\frac{x^2}{16}+\frac{y^2}{4/3}=1$$

$$e=\sqrt{1-\frac{4}{3\times 16}}=\sqrt{\frac{11}{12}}=\sqrt{\frac{11}{2\sqrt{3}}}$$

SECTION–II

1. Ans. 1.25

Sol. Point of intersection of family

$2x-y-9+\lambda(3x+y-2)=0$ is B(1, 3)

d = maximum distance is $AB=\sqrt{25}=5$

$$\therefore\ \frac{d}{4}=1.25$$

2. Ans. 1.57

Sol. $x+\pi=t$

$dx=dt$

$$I=\int_{\frac{-\pi}{2}}^{\frac{\pi}{2}}\left[t^3+\cos^2(2\pi+t)\right]dt$$

$$I=\int_{\frac{-\pi}{2}}^{\frac{\pi}{2}}t^3+\int_{\frac{-\pi}{2}}^{\frac{\pi}{2}}\cos^2 t\,dt$$

$$I=0+2\int_{0}^{\frac{\pi}{2}}\cos^2 t\,dt$$

$$I=2.\frac{\pi}{4}=\frac{\pi}{2}=1.57$$

3. Ans. 2.50

Sol. $\frac{f(3)-f(2)}{3-2}=f'(4)$

$f(3)\le 5$ ...(1)

$\frac{f(5)-f(3)}{5-3}=f'(2)$

$f(3)\ge 5$ ...(2)

Hence $f(3)=5$

4. Ans. 0.25

Sol. $\Delta=\begin{vmatrix}3&4&5\\4&5&6\\5&6&7\end{vmatrix}=0$

Hence $\Delta_x=\Delta_y=\Delta_z=0$

$\begin{vmatrix}a&4&5\\b&5&6\\c&6&7\end{vmatrix}=0\ \therefore\ a+c=2b\ \therefore\ \frac{a+c}{8b}=\frac{1}{4}$

5. Ans. 0.00

Sol. $\frac{1}{\cos 55°}+\frac{1}{\cos 65°}+\frac{\cos 175°}{\cos 55°\cos 65°}$

$$\frac{\cos(60+5)+\cos(60-5)-\cos 5°}{\cos 55°\cos 65°}=0.00$$