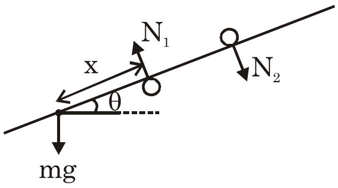
SET-B

PART-1 : PHYSICS SOLUTION

SECTION-I

1. Ans. (A, C)

Sol. Velocity will be maximum where a = 0



$$mg\cos\theta = N_2 \times 10$$

$$mg\sin\theta = \mu(N_1 + N_2)$$

$$mg\cos\theta + N_2 = N_1$$

$$mg\sin\theta = \mu(mg\cos\theta + 2N_2)$$

$$mg\sin\theta = \mu\ mg\cos\theta\left(1+\frac{2\times x}{10}\right)$$

$$1 = \frac{4}{5}\left(1+\frac{2x}{10}\right)$$

$$x = \frac{5}{4}$$

Initially N_2 will be zero as torque about the first peg is zero.

2. Ans. (A,B,D)

Sol. Power leaving the system = P

$$P = \sigma(4\pi C^2)T_C^4$$

Net power leaving B = P

$$P = \sigma(4\pi b^2)T_B^4 - \sigma(4\pi b^2)T_C^4$$

3. Ans. (B,D)

Sol. As 3H inductor is short circuited $\frac{di}{dt}$ across it will be zero and it will retain its energy.

4. Ans. (B,D)

Sol. $V = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{\ell T}{m}}$

$$T = \frac{\ell}{V} = \sqrt{\frac{M\ell}{T}}$$

5. Ans. (A,B,D)

Sol. Curve is symmetric near minimum deviation.

$$\mu = \frac{\sin\left(\frac{\delta_{min}+A}{2}\right)}{\sin\left(\frac{A}{2}\right)} \Rightarrow \mu = \sqrt{2}$$

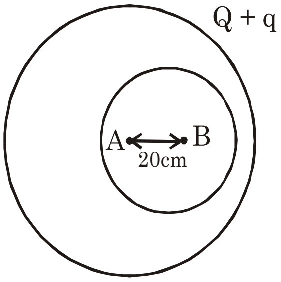
Critical angle is 45°. If $\angle r_1 < 15°$ then TIR will happen on second face.

6. Ans. (C, D)

Sol. Since there is no parallax image formed in the same place so MI = 2f. We need position of M & I.

7. Ans. (B,D)

Sol.



The charge distribution on the inner surface of the sphere will be non-uniform, while charge distribution on the outer surface will be uniform.

$$\therefore\ V_B = \frac{k(Q+q)}{R} + \frac{k(-q)}{r} + \frac{kq}{r'}$$

$$= \frac{9\times10^2\times9\times10^{-6}}{1}$$

$$+\frac{k(-5)\times10^{-6}}{0.5} + \frac{k(5\times10^{-6})}{0.2} = 216\text{ kV}$$

Also outside the sphere field is due to charge on the outer surface which doesnot change on changing the position of charge in the cavity. Hence field outside the sphere will not change.

8. Ans. (A,B,D)

Sol. Just before point A,

$N_1 = mg$

Just after point A,

$$N_2 - mg = \frac{mv^2}{R} \Rightarrow N_2 = mg + \frac{mv^2}{R}$$

$$\therefore\ N_1 = 40\ \&\ N_2 = 40 + \frac{4\times16}{1} = 104$$

Applying work energy theorem,

$$-mgh = \frac{1}{2}mv'^2 - \frac{1}{2}mv^2$$

$$\Rightarrow -4\times10\times\frac{1}{2} = \frac{1}{2}\times4\times V'^2 - \frac{1}{2}\times4\times16$$

$$\therefore\ V' = \sqrt{6}\ \text{m/s}$$

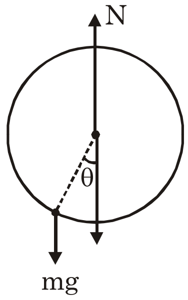
From geometry, $R - R\cos\theta = \frac{R}{2}$

$$\Rightarrow \cos\theta = \frac{1}{2} \Rightarrow \theta = 60°$$

Note : At maximum height, horizontal component of velocity will remain non-zero.

9. Ans. (B)

Sol.



Writing torque equation about centre point,

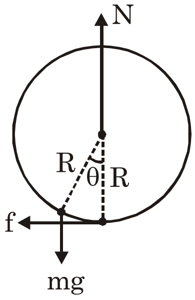
$\tau = mgR\sin\theta$

$$\therefore \alpha = \frac{mg\,R\sin\theta}{mR^2 + mR^2}$$

$$\Rightarrow \alpha = \frac{g}{2R}\theta\ \ (\theta \text{ is very small})$$

10. Ans. (D)

Sol. Writing torque equation of out lower-most point,



$\tau = mg\,R\sin\theta$

$I \approx 2\,mR^2$

11. Ans. (C)

Sol. Intensity of light is given by $I = \frac{\dot{n}\,hC}{A\lambda}$

Also when position of jockey are interchanged, polarity of the plates changes.

12. Ans. (D)

Sol. Since $V_0 >>> \phi$ (work function)

$\Rightarrow$ all the electrons ejected due to PEE reaches the +ve plate.

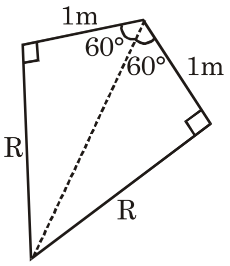
On increasing V_0, K.E. of is reaching at +ve plate increases thus decreasing the cut off wavelength of X-rays.

Changing the plate material will change atomic no. thus nature of characteristic X-ray.

SECTION-II

1. Ans. 1.10 to 1.15

Sol.



$$\frac{\pi}{3} = \omega t$$

$$\tan 60 = \frac{R}{1} = \sqrt{3} = \frac{mv}{qB}$$

$$B = \frac{9\times10^{-31}}{1.6\times10^{-19}}\times\frac{\sqrt{3}}{2} = \frac{9}{3.2}\times10^{-12}$$

$$= 4\pi \times 10^{-5} \times 200\ i$$

$$i = \frac{9}{3.2\times8\pi}\times10^{-7}$$

2. Ans. 5.00

Sol. $v\cos\theta = 5$

$$v\cos(53 - \theta) = 3$$

$$v\left[\frac{3}{5}\cos\theta + \frac{4}{5}\times\sin\theta\right] = 3$$

$$\frac{4}{5}v\sin\theta = 0$$

$$\theta = 0^\circ$$

$$\Rightarrow v = 5$$

3. Ans. 960.00

Sol. Net work done is zero.

$$(\rho - 800) \times gV \times 1.5 = (1200 - \rho)gV \times 1$$

$$\rho = \frac{4800}{5} = 960\,kg/m^3$$

4. Ans. 22.22

Sol. $\lambda_1 = \frac{h}{m(v_1 - v_2)} = 25\mu$

$$\lambda_2 = \frac{h}{m(v_1 + v_2)} = 20\mu m$$

$$\Rightarrow \frac{v_1 + v_2}{v_1 - v_2} = \frac{5}{4} \Rightarrow v_2 = \frac{v_1}{9}$$

again

$$\lambda_1 = \frac{h}{m\left(v_1 - \frac{v_1}{9}\right)} = 25\mu m \Rightarrow \frac{h}{mv_1} = \frac{25\times8}{9}\mu m$$

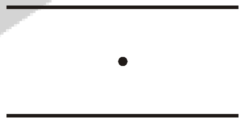
$$\lambda = \frac{h}{mv_1} = \frac{200}{9}\mu m = 22.22\mu m$$

5. Ans. 0.40

Sol. $-1 \times 0.04 \times \frac{3}{4} + \frac{1}{2} \times 2 \times v^2 = 0$

6. Ans. 7.35 to 7.38

Sol.



$$v = v_0 e^{-t/RC} = 20e^{-\frac{t}{2}}$$

$$qE = mg$$

$$q \times \frac{20}{d} = mg$$

$$mg - qE = ma$$

$$mg\,(1 - e^{-t/2}) = ma = \frac{dv}{dt}$$

$$v = 10\,[t + 2e^{-t/2} - 2]$$

$$v = \frac{20}{e} \text{ at } t = 2$$

PART–2 : CHEMISTRY SOLUTION

SECTION-I

1. **Ans.(B)**

2. **Ans.(B)**

3. **Ans.(D)**

4. **Ans.(A,B,C)**

For liquid → gas, ΔH_{sys} + ve, ΔS_{sys} + ve

As it is equilibrium condition, process is thermodynamically reversible and hence, $\Delta G_{sys} = 0$, $\Delta S_{univ} = 0$

5. **Ans.(A,B,D)**

$$Z_A = 8\times\frac{1}{8}+6\times\frac{1}{2}=4$$

$$Z_B = 1 + 12 \times \frac{1}{4} = 4$$

Hence, simplest formula is $A_4B_4 \equiv AB$, and there is 4 AB formula units per unit cell. As 'B' atoms are smaller than 'A' atoms, they are not in contact but as $\frac{r_B}{r_A}=(\sqrt{2}-1)$ = ideal value of octahedral void, 'B' atoms are in contact.

6. **Ans.(A,C,D)**

7. **Ans.(C)**

8. **Ans.(A,C,D)**

9. **Ans.(D)**

10. **Ans.(B)**

11. **Ans.(A)**

12. **Ans.(B)**

SECTION-II

1. **Ans.(5.50)**

$$\left(P+\frac{an^2}{V^2}\right)(V-nb)=nRT$$

$$\text{or}, (35.5+\frac{a\times5^2}{5^2}(5-5\times0.1)=5\times0.082\times450$$

$\therefore$ a = 5.50 atm L^2 mol^{-2}

2. **Ans.(–4.20)**

left electrode : $H_2(g) + 2A^-(aq.) \rightleftharpoons 2HA(aq.) + 2e^-$

Right electrode : $2HB(aq.) + 2e^-(aq.) \rightleftharpoons H_2(g) + 2B^-(aq.)$

Net cell reaction : $2HB(aq.) + 2A^-(aq.) \rightleftharpoons 2HA(aq.) + 2B^-(aq.)$

$$K_{eq.} = \frac{[HA]^2[B^-]^2}{[HB]^2[A^-]^2}=\frac{K_a^2(HB)}{K_a^2(HA)}$$

$$\text{Now, } E^0_{cell}=\frac{0.06}{n}.\log K_{eq.}=\frac{0.06}{2}.\log\frac{K_a^2(HB)}{K_a^2(HA)}$$

$$=\frac{0.06}{1}.\log\frac{4\times10^{-6}}{2\times10^{-5}}=-0.042V$$

3. **Ans.(1650.00)**

$$\Delta t = \frac{t_{1/2}}{\ln 2}.\ln\frac{r_A}{r_B}=\frac{5775}{\ln 2}\times\ln\frac{5000}{4000} = 1650 \text{ yrs.}$$

4. **Ans.(5.00)**

Angular nodes = 2 = l ⇒ d - orbital ⇒ five in number.

5. **Ans.(3.00)**

6. **Ans.(3.00)**

PART-3 : MATHEMATICS SOLUTION

SECTION–I

1. Ans. (A,C)

$$\operatorname{cosec}\left(\frac{\pi}{4}+x\right)+\operatorname{cosec}\left(\frac{\pi}{4}-x\right)=2\sqrt{2}$$

$$\frac{\sqrt{2}}{\sin x+\cos x}+\frac{\sqrt{2}}{\cos x-\sin x}=2\sqrt{2}$$

$$\frac{2\cos x}{\cos^2 x-\sin^2 x}=2$$

$\cos 2x = \cos x$

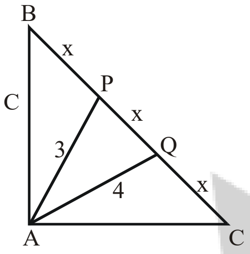
$$2\sin\frac{3x}{2}.\sin\frac{x}{2}=0$$

$x = 2n\pi, \dfrac{2n\pi}{3}$

$x \in [0, 4\pi]$ is 7

$x \in (0, 2\pi)$ is 2

2. Ans. (C)



$9 = c^2 + x^2 - 2cx \cos B$

$$\cos B=\frac{c}{3x}$$

$$9=c^2+x^2-2cx\left(\frac{c}{3x}\right)=\frac{c^2}{3}+x^2 \qquad ...(1)$$

Similarly in ΔACQ

$$16=\frac{b^2}{3}+x^2 \qquad ...(2)$$

Adding (1) + (2)

We get

$$25=\left(\frac{b^2+c^2}{3}\right)+2x^2$$

$25 = 5x^2$, $(b^2 + c^2 = a^2 = 9x^2) \Rightarrow x=\sqrt{5}$

$\ell(BC) = 3\sqrt{5}$

3. Ans. (A,B,D)

$ty = x + 2t^2 \xrightarrow[(6,8)]{} 8t = 6t2t^2$

$t^2 - 4t + 3 = 0 \Rightarrow t = 1,3 \qquad (2t^2,4t) \equiv Q \ \& \ R$

$Q = (2,4) \quad P = (6,8)$

$R \equiv (18,12)$

$$\text{area } =\frac{(64-48)^{3/2}}{4}=\frac{64}{4}=16$$

$PS : 2x - y - 4 = 0$

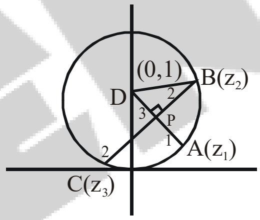
Equation of circle circumscribing the ΔPQR is normal at Q and R intersect at $(30,-24) = S$

Equation of circle on PS as diameter

$(x - 30)(x - 6) + (y + 24)(y - 8) = 0$

$x^2 + y^2 - 36x + 16y - 12 = 0$

4. Ans. (A,C)



$$\frac{3z_1+i}{4}=\frac{2z_2+2z_3}{4}$$

$AD = 1$

$$DP=\frac{3}{4}$$

$$r^2 = BP^2 + DP^2 \Rightarrow BP=1-\frac{9}{16}=\frac{\sqrt{7}}{4}$$

$$BC=\frac{\sqrt{7}}{2}$$

$$\text{Area of quad ABCD }=\frac{1}{2}\times\frac{\sqrt{7}}{2}\times 1=\frac{\sqrt{7}}{4}$$

5. Ans. (A,C)

$$\text{Let } S=\sum_{k=1}^{\infty}\frac{P_k}{3^k}=\frac{P_1}{3}+\frac{P_2}{3^2}+\frac{P_3}{3^3}+\frac{P_4}{3^4}+\frac{P_5}{3^5}$$

$$S=\frac{1}{3}+\frac{1}{3^2}+\frac{2}{3^3}+\frac{3}{3^4}+\frac{5}{3^5}+\frac{8}{3^6}+....+\frac{P_n}{3^n}$$

$$\frac{S}{3}=\frac{1}{3^2}+\frac{1}{3^3}+\frac{2}{3^4}+\frac{3}{3^5}+....+\frac{P_{n-1}}{3^n}+....$$

$$\frac{S}{9}=\frac{1}{3^3}+\frac{1}{3^4}+\frac{2}{3^5}+....+\frac{P_{n-2}}{3^n}$$

$$S-\left(\frac{S}{3}+\frac{S}{9}\right)=\frac{1}{3}\Rightarrow S=\frac{3}{5}$$

6. Ans. (B,C,D)

Let H = event that married man watches the show

W = event that married women watches the show

$$P(H) = 0.4, \quad P(W) = 0.5, \quad P\left(\frac{H}{W}\right) = 0.7$$

$$P(H \cup W) = 0.35$$

$$P\left(\frac{W}{H}\right) = \frac{0.35}{0.4} = \frac{7}{8}, \ P(H \cup W) = 0.55$$

7. Ans. (A,C)

Directly expand put –x = t

$\Delta = t(t^2 - ab) - a(bt - a^2) + b(b^2 - at)$

$\Delta = t^3 + a^3 + b^3 - 3abt = (t + a + b)$

$(t + a + b)(t^2 + a^2 + b^2 - at - bt - ab)$

$(a + b - x)(x^2 + x(a + b) + a^2 + b^2 - ab)$

If $a = b \quad \Delta = (2a - x)(x^2 + 2ax + a^2)$

8. Ans. (B,C)

$A.\text{adj}(2B) = 16I$

$A(4\text{adj}B) = 16I \Rightarrow A.\text{adj}B = 4I$

$\Rightarrow A|B|.B^{-1} = 4I$

$AB^{-1} = 2I \Rightarrow A = 2B$

$B.\text{adj}A = B.\text{adj}(2B) = 4B.\text{adj}B = 4|B|I_3 = 8I_3$

$A.\text{adj}B = 4I \Rightarrow A^{-1}A.\text{adj}B$

$\Rightarrow \text{adj}B = 4I \Rightarrow A^{-1}.A.\text{adj}B = 4A^{-1}$

$\Rightarrow \text{adj}B = 4A^{-1}$

$A^{-1}.\text{adj}B = A^{-1}.(4A^{-1}) = 4(A^{-1})^2$

$(A^{-1}(\text{adj}B))^{-1} = (4A^{-1}A^{-1})^{-1}$

$$= \frac{1}{4}A^2 = \frac{1}{4}(2B)^2 = B^2$$

Solution Q.9 & Q. 10

9. Ans. (A)

10. Ans. (B)

$$I = \int_{e^{-x}}^{1} (-\ell nt)^n \, dt$$

$-\ell nt = u \Rightarrow t = e^{-u} \Rightarrow dt = -e^{-u}du$

$$I = \int_0^x u^n e^{-u} \, du$$

Hence,

$$I_n = \int_0^\infty \underbrace{u^n}_{I} \underbrace{e^{-u}}_{II} du = \left|-u^n e^{-u}\right|_0^\infty + \int_0^\infty n u^{n-1} e^{-u} \, du$$

$\Rightarrow I_n = 0 + nI_{n-1} \Rightarrow I_n = n(n-1)I_{n-2}$

$\therefore I_n = n(n-1)(n-2).....2.I_1$

$$\Rightarrow I_1 = \int_0^\infty u e^{-u} \, du = 1$$

$\Rightarrow I_n = n!$

Let

$$J = \int_0^{1/2} \left(\frac{1}{4} - x^2\right)^4 dx = \int_0^{1/2} \left(\frac{1}{4} - \left(\frac{1}{2} - x\right)^2\right)^4 dx$$

(Using King property)

$$\therefore J = \int_0^{1/2} (x - x^2)^4 \, dx = \int_0^{1/2} x^4 (1-x)^4 \, dx \quad ...(i)$$

$\Rightarrow J = K$, So $J - K = 0$

Put $x = 1 - y$

$$\text{Also, } J = -\int_1^{1/2} (1-y)^4 y^4 \, dx$$

$$\Rightarrow J = \int_{1/2}^{1} (1-x)^4 x^4 \, dx \qquad ...(ii)$$

$\therefore$ (i) + (ii)

$$\Rightarrow J = \frac{1}{2}\int_0^1 x^4 (1-x)^4 \, dx$$

(Using Queen property)

Put $x = \sin^2\theta$,

$$J = \int_0^{\pi/2} \sin^9\theta.\cos^9\theta \, d\theta$$

$$= \frac{(8 \times 6 \times 4 \times 2)(8 \times 6 \times 4 \times 2)}{(18 \times 16 \times 14 \times 12 \times 10 \times 8 \times 6 \times 4 \times 2)} = \frac{1}{1260}$$

Solution Q.11 & Q. 12

11. Ans. (A)

12. Ans. (B)

$L_2 : x - 3y - 4 = 0 \ \& \ 4y - z + 5 = 0$

$P \equiv (4, 0, 5) \qquad \overrightarrow{DR} = \vec{n} = \overrightarrow{n_1} \times \overrightarrow{n_2}$

$$\vec{n} = 3\hat{i} + \hat{j} + 4\hat{k} \qquad \begin{pmatrix} \vec{n}_1 = \hat{i} - 3\hat{j} \\ \vec{n}_2 = 4\hat{j} - \hat{k} \end{pmatrix}$$

$$L_2 : \frac{x-4}{3} = \frac{y}{1} = \frac{z-5}{4} \qquad ...(1)$$

$P_1 : x + y - z = 5, \ L_2 : (3\lambda + 4, \lambda, 4\lambda + 5)$

$$\frac{x-(3\lambda+4)}{1}=\frac{y-\lambda}{1}=\frac{z-(4\lambda+5)}{-1}=-2\left(\frac{-6}{3}\right)=4$$

$(x, y, z) \equiv (3\lambda + 8, \lambda + 4, 4\lambda + 1)$

Image of line $L_2 : \frac{x-8}{3}=\frac{y-4}{1}=\frac{z-1}{4}$

$P_1 = 0; P_3 = 0 ; P_1 + \lambda P_3 = 0$

$(x + y - z - 5) + \lambda(x - 3y - 4) = 0$

$x(1 + \lambda) + y(1 - 3\lambda) - z - (5 + 4\lambda) = 0$

$p_1 = p_2$ from (4, 0, 0)

$$\Rightarrow \left|\frac{1}{\sqrt{(\lambda+1)^2+(1-3\lambda)^2+1}}\right|=\left|\frac{1}{\sqrt{3}}\right|$$

$$\lambda = 0, \frac{2}{5}$$

$P : 7x - y - 5z - 33 = 0$

$L_1 : x + y - z = 5, 2x - y + \lambda z = 3$

$$\text{Point} \equiv \left(\frac{8}{3}, \frac{7}{3}, 0\right)$$

$$\vec{n}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 1 & 1 & -1\\ 2 & -1 & \lambda\end{vmatrix}=(\lambda-1)\hat{i}-(\lambda+2)\hat{j}-3\hat{k}$$

If line L_1 and L_2 are coplanor

$$\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ \frac{4}{3} & \frac{-7}{3} & 5\\ 3 & 1 & 4\\ (\lambda-1) & -(\lambda+2) & -3\end{vmatrix}=0$$

$$\lambda = -\frac{5}{4}$$

SECTION–II

1. **Ans. 4.00**

$-2 \le f'(x) \le 2$, Apply LMVT in $x \in [1, 2]$ and $x \in [2, 4]$

$f(2) - f(1) = f'(x) \Rightarrow f(2) - 2 \in [-2, 2]$

$f(2) \in [0, 4]$ ...(1)

$$\frac{f(4)-f(2)}{2} \in [-2,2] \Rightarrow f(2) - f(4) \in [-4, 4]$$

$f(2) \in [4, 12]$ ...(2)

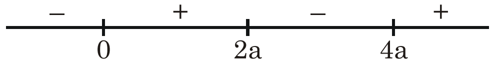
So from (1) and (2), $f(2) = 4$

2. **Ans. 5.00**

$$f'(x)=\frac{(4a-2x)}{(4ax-x^2)\log a} \ge 0$$

$$f'(x)=\frac{(x-2a)}{x(x-4a)\log a} \ge 0$$

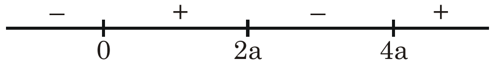
C-1 : $a \in (1, \infty) \Rightarrow \frac{(x-2a)}{x(x-4a)} \ge 0$



$x \in (0, 2a)$ so always ↑

C-2 : $a \in (0, 1)$

$$\frac{(x-2a)}{x(x-4a)} < 0$$



$x \in (2a, 4a)$

$$2a \le \frac{3}{2} \Rightarrow a \le \frac{3}{4} \text{ and } 4a > 2 \Rightarrow a > \frac{1}{2}$$

$$\text{So } a \in \left(\frac{1}{2}, \frac{3}{4}\right] \cup [1,\infty)$$

3. **Ans. 15.00**

$$I=\int\frac{(\sec x-\tan x)}{\sqrt{\sin^2 x-\sin x}}\,dx$$

$$I=\int\frac{(\sec x-\tan x)\sec x}{\sqrt{\tan^2 x-\sec x\tan x}}\,dx$$

Put $-\sec x + \tan x = t$

$$= -\sec x \tan x + \sec^2 x = \frac{dt}{dx}$$

also $(\tan x - \sec x)^2 = t^2$

$2\tan^2 x - 2\tan x \sec x = t^2 - 1$

$$I=\int\frac{dt}{\sqrt{\frac{t^2-1}{2}}}=\sqrt{2}\int\frac{dt}{\sqrt{t^2-1}}$$

$$I=\sqrt{2}\log_e\left|t+\sqrt{t^2-1}\right|$$

$$I=\sqrt{2}\log_e\left|(\tan x-\sec x)+\sqrt{2}\sqrt{\tan^2 x-\tan x\sec x}\right|+c$$

$$k=\sqrt{2}$$

$$f\left(\frac{4\pi}{3}\right)=\sqrt{3}+2$$

4. Ans. 5.00

$$dy = \frac{(1-x)}{e^x}dx \qquad \text{Integrate}$$

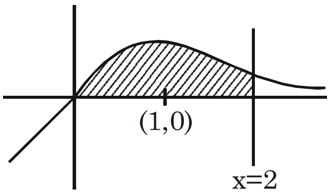
$y = x\,e^{-x} + c$, $f(0) = 0 \Rightarrow c = 0$

$y = xe^{-x}$

$$\frac{d^2y}{dx^2} = (x-2)e^{-x}$$

Point of inflection $\equiv \left(2, \frac{2}{e^2}\right)$

$$A = \int_0^2 xe^{-x}\ dx$$



$$A = 1 - \frac{3}{e^2}$$

$p + q = 5$

5. Ans. 7.00

$(x - c)^2 + (y - c)^2 = c^2$, $c > 0$

$p = r$, $\left|\frac{7c-12}{5}\right| = c \Rightarrow c = 1, 6$

Sum = 7

6. Ans. 6.00

$$f'(0^+) = \lim_{h\to 0^+} \frac{f(0+h) - f(0)}{h}$$

$$f'(0^+) = \lim_{h\to 0^+} \frac{h^{p-2}\sin\frac{1}{h} + h|\tanh|^{q-3}}{h}$$

$$f'(0^+) = \lim_{h\to 0^+} h^{p-3}\sin\frac{1}{h} + (\tanh)^{q-3}$$

It exist only when $p > 3$ and $q \geq 3$, $p + q > 6$

$[p + q]_{\min} = 6$