SET-2

General Instructions:

(i) All the questions are compulsory.
(ii) The question paper consists of 36 questions divided into 4 sections A, B, C, and D.
(iii) Section A comprises of 20 questions of 1 mark each. Section B comprises of 6 questions of 2 marks each. Section C comprises of 6 question of 4 marks each. Section D comprises of 4 questions of 6 marks each.
(iv) There is no overall choice. However, an internal choice has been provided in three questions of 1 mark each, two questions of 2 marks each, two questions of 4 marks each, and two questions of 6 marks each. You have to attempt only one of the alternative in all such questions.
(v) Use of calculators is not permitted

Section - A

Q1-Q10 are multiple choice type questions. Select the correct option.

1. The value of the determinant $\begin{vmatrix} 1+a & 1 & 1 \\ 1 & 1+a & 1 \\ 1 & 1 & 1+a \end{vmatrix}$ is zero, then value of a is

(A) –3 (B) 0 (C) 1 (D) 3

2. If $\begin{bmatrix} x+y \\ x-y \end{bmatrix} = \begin{bmatrix} 2 & 1 \\ 4 & 3 \end{bmatrix} \times \begin{bmatrix} 1 \\ -2 \end{bmatrix}$, then the value of (x,y) is which of the following ?

(A) (1, 1) (B) (1, –1) (C) (–1,1) (D) (–1,–1)

3. The unit vector in the direction of $\hat{i}+\hat{j}+\hat{k}$ is :

(A) $\frac{1}{\sqrt{3}}(\hat{i}+\hat{j}+\hat{k})$ (B) $\sqrt{3}(\hat{i}+\hat{j}+\hat{k})$ (C) $\frac{1}{\sqrt{2}}(\hat{i}+\hat{j}+\hat{k})$ (D) $\sqrt{2}(\hat{i}+\hat{j}+\hat{k})$

4. If a die is thrown and a card is selected at random from a deck of 52 playing cards, then the probability of getting an even number on the die and a spade card is :

(A) $\frac{1}{2}$ (B) $\frac{1}{4}$ (C) $\frac{1}{8}$ (D) $\frac{3}{4}$

5. LPP theory states that the optimal solution to any problem will be at

(A)the origin

(B) a corner point of feasible region

(C) the highest point of the feasible region

(D) the lowest point of the feasible region

6. If $\tan^{-1}\frac{1}{2}+\tan^{-1}\frac{2}{11}=\tan^{-1}a$, then the value of a is ___________ .

(A) $\frac{1}{4}$ (B) $\frac{1}{2}$ (C) $\frac{3}{4}$ (D) 1

7. Two events E and F are independent. If P(E) = 0.3 and P(E ∪ F) = 0.5, then P(E/F) – P(F/E) equal to :

(A) 2/7 (B) 3/35 (C) 1/70 (D) 1/7

8. $\int \frac{x^3}{x+1}dx$ is equal to :

(A) $x+\frac{x^2}{2}+\frac{x^3}{3}-\log|1-x|+C$ (B) $x+\frac{x^2}{2}-\frac{x^3}{3}-\log|1-x|+C$

(C) $x-\frac{x^2}{2}-\frac{x^3}{3}-\log|1+x|+C$ (D) $x-\frac{x^2}{2}+\frac{x^3}{3}-\log|1+x|+C$

9. The distance between the parallel planes $2x-2y-z+3=0$ and $4x-4y-2z+5=0$ is :

(A) 1 (B) $\frac{5}{6}$ (C) $\frac{1}{6}$ (D) $\frac{1}{2}$

10. The perpendicular distance of the plane $3x-6y+5z=12$ from origin is :

(A) $\frac{-\sqrt{70}}{12}$ (B) $\frac{-12}{\sqrt{70}}$ (C) $\frac{12}{\sqrt{70}}$ (D) $\frac{\sqrt{70}}{12}$

(Q11-Q15) Fill in the blanks.

11. If $f:R\to R$ is given by $f(x)=(3-x^3)^{1/3}$. Then fof(x) is ....

12. If $f(x)=\begin{cases}\frac{1-\cos 4x}{8x^2}, & x\neq 0\\ k, & x=0\end{cases}$, is continuous at x = 0, then the value of k is....................

13. If A and B are symmetric matrices, then BA – 2AB is a .... matrix.

14. For all real values of x, the function $f(x)=e^x-e^{-x}$ is ....

OR

A particle is moving in a straight line. Its displacement is given by $s=4t-2t^2$, where t is in seconds. Then the particle will come to rest after .... second.

15. If $\vec{a}, \vec{b}, \vec{c}$ are coplanar then $[\vec{a}\ \vec{b}\ \vec{c}]$ will be ....

OR

The vectors $2\hat{i}-3\hat{j}+5\hat{k}$ and $-\lambda\hat{i}+2\hat{j}+2\hat{k}$ are mutually perpendicular if λ = ...

(Q16-Q20) Answer the following questions.

16. For what values of k, the system of linear equations

$x+y+z=2$

$2x+y-z=3$

$3x+2y+kz=4$

has a unique solution ?

17. Evaluate : $\int_2^4 \frac{x}{x^2+1}dx$

18. Evaluate $\int \frac{(1+\cos x)}{x+\sin x}dx$

OR

Evaluate $\int \frac{dx}{\sqrt{16-9x^2}}$

19. Evaluate $\int \frac{(x^2+2)}{x+1}dx$

20. Find the order and degree of the differential equation $\frac{d^2y}{dx^2}+\left(\frac{dy}{dx}\right)^{1/4}+x^{1/5}=0$.

Section - B

21. Solve for x : $\tan^{-1}(x-1)+\tan^{-1}x+\tan^{-1}(x+1)=\tan^{-1}3x$.

OR

If A = {1, 2, 3, ..., 9} and R be the relation in A × A defined by (a, b) R (c, d) if a + d = b + c for a, b, c, d ∈ A is an equivalence relation, then find the equivalence class [(2,5)].

22. Differentiate $\tan^{-1}\left[\frac{\sqrt{1+x^2}-1}{x}\right]$ with respect to x

23. The length x, of a rectangle is decreasing at the rate of 5 cm/minute and the width y, is increasing at the rate of 4 cm/minute. When x = 8 cm and y = 6 cm, find the rate of change of the area of the rectangle.

24. Show that the four points A,B,C and D with position vectors $4\hat{i}+5\hat{j}+\hat{k}, -(\hat{j}+\hat{k}), 3\hat{i}+9\hat{j}+4\hat{k}$ and $4(\hat{i}+\hat{j}+\hat{k})$, respectively are coplanar.

OR

If $\vec{a}=\hat{i}+\hat{j}+\hat{k}, \vec{b}=4\hat{i}-2\hat{j}+3\hat{k}$ and $\vec{c}=\hat{i}-2\hat{j}+\hat{k}$, find a vector of magnitude 6 units which is parallel to the vector $2\vec{a}-\vec{b}+3\vec{c}$.

25. Find the points on the line $\frac{x+2}{3}=\frac{y+1}{2}=\frac{z-3}{2}$ at a distance of 5 units from the point P(1,3,3).

26. A black and a red die are rolled together. Find the conditional probability of obtaining the sum 8, given that the red die resulted in a number less than 4.

Section - C

27. Consider $f : R_+ \to [4, \infty)$ given by $f(x) = x^2 + 4$. Show that f is invertible with inverse f^{-1} of f given by $f^{-1}(y)=\sqrt{y-4}$, where R_+ is the set of all non-negative real numbers.

28. If $x = a\cos^3\theta$ and $y = a\sin^3\theta$, then find the value of $\frac{d^2y}{dx^2}$ at $\theta=\frac{\pi}{6}$.

OR

If $(ax + b)e^{y/x} = x$, then show that $x^3\left(\frac{d^2y}{dx^2}\right)=\left(x\frac{dy}{dx}-y\right)^2$

29. Solve the following differential equation : $x^2\frac{dy}{dx}=y^2+2xy$. Given that y = 1, when x = 1.

30. Evaluate : $\int_{-2}^{2} \frac{x^2}{1+5^x} dx$

31. Two cards are drawn simultaneously (without replacement) from a well-shuffled pack of 52 cards: Find the mean and variance of the number of red cards

OR

There are three coins. One is a two-headed coin (having head on both faces), another is a biased coin that comes up heads 75% of the times and third is also a biased coin that comes up tails 40% of the times. One of the three coins is chosen at random and tossed, and it shows heads. What is the probability that it was the two-headed coin ?

32. A diet for a sick person must contain at least 4000 units of vitamins, 50 units of minerals and 1400 units of calories. Two foods A and B are available at a cost of Rs. 5 and Rs. 4 per unit respectively. One unit of the food A contains 200 units of vitamins, 1 unit of minerals and 40 units of calories, while one unit of the food B contains 100 units of vitamins, 2 units of minerals and 40 units of calories. Find what combination of the foods A and B should be used to have least cost, but it must satisfy the requirements of the sick person. Form the equation as LPP and solve it graphically.

Section - D

33. If $A = \begin{bmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{bmatrix}$, find A^{-1}. Use it to solve the system of equations

$2x - 3y + 5z = 11$

$3x + 2y - 4z = -5$

$x + y - 2z = -3$

OR

Using elementary row transformations, find the inverse of the matrix $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 5 & 7 \\ -2 & -4 & -5 \end{bmatrix}$

34. Using the method of integration, find the area of the triangular region whose vertices are (2, –2), (4, 3) and (1, 2).

35. Show that the altitude of the right circular cone of maximum volume that can be inscribed in a sphere of radius r is $\frac{4r}{3}$. Also show that the maximum volume of the cone is $\frac{8}{27}$ of the volume of the sphere.

OR

Show that the height of the cylinder of maximum volume that can be inscribed in a sphere of radius R is $\frac{2R}{\sqrt{3}}$. Also find the maximum volume.

36. Find the coordinates of the point where the line through the points (3, –4, –5) and (2, –3, 1), crosses the plane determined by the points (1, 2, 3), (4, 2, –3) and (0, 4, 3).