MATHEMATICS SAMPLE PAPER # 2

ANSWER AND SOLUTIONS

SECTION-A

1. Option (3)
$\frac{1}{2}$

2. Option (3)
$36°$

3. Option (2)
-1

4. Option (1)
(3, 1)

5. Option (2)
$k \le 4$

6. Option (3)
28

7. Option (4)
$4\sqrt{2}\,\text{cm}$

8. Option (2)
1

9. Option (1)
$60°$

10. Option (4)
9 units

11. 3 cm

12. 7.8

13. 162
OR
$\frac{3}{4}$

14. $-\frac{9}{4}$

15. 0

16. Since, given rational number $= \frac{441}{2^2.5^7.7^2}$

$= \frac{7^2 \times 3^2}{2^2.5^7.7^2} = \frac{3^2}{2^2.5^7}$

$\Rightarrow$ Rational number has a terminating decimal expansion.

[$\because$ Denominator is of form 2^n5^m]

OR

Since denominator $= 2^4 \times 5^3$

highest power of 2 and 5 = 4

So, it will terminate after 4 decimal places

17. $\frac{\sec(90°-\theta)\operatorname{cosec}\theta - \tan(90°-\theta)\cot\theta + \cos^2 25° + \cos^2 65°}{3\tan 27°.\tan 63°}$

$= \frac{\operatorname{cosec}\theta \cdot \operatorname{cosec}\theta - \cot\theta \cdot \cot\theta + \cos^2(90°-65°) + \cos^2 65°}{3\tan(90°-63°)\tan 63°}$

$= \frac{(\operatorname{cosec}^2\theta - \cot^2\theta) + (\sin^2 65° + \cos^2 65°)}{3\cot 63° \cdot \tan 63°}$

$= \frac{1+1}{3 \times \frac{1}{\tan 63°} \times \tan 63°} = \frac{2}{3}$

[$\because$ $\operatorname{cosec}^2\theta - \cot^2\theta = 1$, $\sin^2\theta + \cos^2\theta = 1$]

18. $a = 11$, $d = -3$, $a_n = -150$

$a_n = a + (n-1)d$

$-150 = 11 + (n-1)(-3)$

$-150 = 11 - 3n + 3$

$3n = 164$

$\Rightarrow n = \frac{164}{3} = 54.66$

Hence -150 is not a term of the given AP.

19. Given, $px^2 - 2\sqrt{5}\,px + 15 = 0$

Since roots are equal

$\therefore$ Discriminant D = 0

i.e., $(-2\sqrt{5}p)^2 - 4 \times p \times 15 = 0 \Rightarrow 20p^2 - 60p = 0$

$\Rightarrow p^2 - 3p = 0 \Rightarrow p(p-3) = 0$

$\Rightarrow p = 0$ or $p = 3 \Rightarrow p = 3$ ($\because$ p cannot be zero)

20. Given $\Delta ABC \sim \Delta PQR$

$\therefore \frac{ar(\Delta ABC)}{ar(\Delta PQR)} = \frac{AB^2}{PQ^2}$ ($\because$ ratio area of two similar triangles is equal to square of ratio of their corresponding sides)

$= \left(\frac{1}{3}\right)^2 = \frac{1}{9}$

SECTION-B

21. Number divisible by 8 between 200 and 500 are 208, 216, 224, ...496 which forms an A.P.

$\therefore$ First term (a) = 208, common difference (d) = 8

n^{th} term of an A.P. is $a_n = a + (n-1)d$

$496 = 208 + (n-1)8$

$\Rightarrow 288 = (n-1)8$

$\Rightarrow n - 1 = 36$

$\Rightarrow n = 37$

OR

Here, $a = 3$, $\ell = 24$

$$S_n = \frac{n}{2}(a + \ell)$$

$$= \frac{8}{2}(3 + 24)$$

$= 4 \times 27$

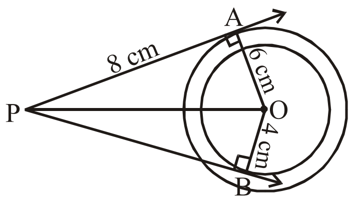
$= 108$

22. Total possible outcomes = $6 \times 6 = 36$

Favourable outcomes are {(1, 6), (2, 3), (3, 2), (6, 1)} i.e. 4 in number.

$\therefore$ P(getting the product 6) = $\frac{4}{36} = \frac{1}{9}$

23.



OA = 6 cm, OB = 4 cm, AP = 8 cm

In ΔAPO, by pythagores theorem

$OP^2 = OA^2 + AP^2 = 36 + 64 = 100$

$\Rightarrow$ OP = 10 cm

In ΔBPO, by pythagores theorem

$BP^2 = OP^2 - OB^2 = 100 - 16 = 84$

$\Rightarrow$ BP = $2\sqrt{21}$ cm

24. If height is 40 cm

circumference of base of cylinder = 22 cm

$$2 \times \frac{22}{7} \times r = 22$$

$$r = \frac{7}{2} \text{ cm}$$

25. Any number which ends in zero must have at least 2 and 5 as prime factors.

$6 = 2 \times 3$

$6^n = (2 \times 3)^n$

$= 2^n \times 3^n$

Hence, prime factor of 6 are 2 and 3

Thus, 6^n can never end with digit 0.

OR

$90 = 2 \times 3^2 \times 5$

$144 = 2^4 \times 3^2$

HCF = $2 \times 3^2 = 18$

LCM = $2^4 \times 3^2 \times 5 = 720$

26. Let P(x, y) is equidistant from A(–5, 3) and B(7, 2)

AP = BP

$$\Rightarrow \sqrt{((x+5)^2 + (y-3)^2)} = \sqrt{((x-7)^2 + (y-2)^2)}$$

$\Rightarrow x^2 + 10x + 25 + y^2 - 6y + 9$

$= x^2 - 14x + 49 + y^2 - 4y + 4$

$10x - 6y + 34 = -14x - 4y + 53$

$10x + 14x - 6y + 4y = 53 - 34$

$24x - 2y = 19$

$24x - 2y - 19 = 0$

is the required relation 1

SECTION-C

27. Given $\sqrt{2}$ and $-\sqrt{2}$ are two zeroes of $2x^4 + 7x^3 - 19x^2 - 14x + 30$ then $(x^2 - 2)$ is a factor of $2x^4 + 7x^3 - 19x^2 - 14x + 30$

$$\therefore\ x^2 - 2 \overline{\big)\, 2x^4 + 7x^3 - 19x^2 - 14x + 30 \,\big(}\, 2x^2 + 7x - 15$$

$$\begin{array}{l} 2x^4 \qquad\qquad -4x^2 \\ \underline{-\qquad\qquad\qquad +} \\ \qquad 7x^3 - 15x^2 - 14x + 30 \\ \qquad 7x^3 \qquad\qquad -14x \\ \underline{\qquad -\qquad\qquad\quad +\qquad\qquad} \\ \qquad\quad -15x^2 \qquad\quad +30 \\ \qquad\quad -15x^2 \qquad\quad +30 \\ \underline{\qquad\quad +\qquad\qquad\quad -\qquad} \\ \qquad\qquad\qquad 0 \end{array}$$

$\therefore\ 2x^4 + 7x^3 - 19x^2 - 14x + 30$

$= (x^2 - 2)(2x^2 + 7x - 15)$

$= (x^2 - 2)(2x^2 + 10x - 3x - 15)$

$$= (x - \sqrt{2})(x + \sqrt{2})(2x - 3)(x + 5)$$

Therefore, the zeroes of the given polynomial are

$$\sqrt{2}, -\sqrt{2}, \frac{3}{2}, -5$$

28. Radius of the cylinder (r) = 3.5 cm

Height of the cylinder (h) = 10 cm

Curved surface area of cylinder = $2\pi rh$

$$= 2 \times \frac{22}{7} \times \frac{35}{10} \times 10 \text{ cm}^2$$

$= 220 \text{ cm}^2$

Curved surface area of a hemisphere = $2\pi r^2$

Curved surface area of both hemispheres

$= 2 \times 2\pi r^2 = 4\pi r^2 = 4 \times \frac{22}{7} \times \frac{35}{10} \times \frac{35}{10}\ cm^2$

$= 154\ cm^2$

Total surface area of the remaining solid

= (Curved surface area of cylinder + curved surface area of 2 hemispheres)

$= (220 + 154)\ cm^2 = 374\ cm^2$.

OR

Given : d = 24 m, h = 3.5 m

r = 12 m

Volume of rice $= \frac{1}{3}\pi\ 12^2 \times 3.5 = 528\ m^3$

Canvas cloth required to cover heap

$= \pi r\ell$ .... (1)

$\ell = \sqrt{12^2 + 3.5^2} = 12.50$

From (1)

Cloth required $= \frac{22}{7} \times 12 \times 12.5 = 471.43\ m^2$

29.

Salary (₹ in thousand)	Number of Persons	c.f.
5 – 10	49	49
10 – 15	133	182
15 – 20	63	245
20 – 25	15	260
25 – 30	6	266
30 – 35	7	273
35 – 40	4	277
40 – 45	2	279
45 – 50	1	280

$n = 280,\ \frac{n}{2} = 140$

So, median class is 10 – 15

$\ell = 10$, cf = 49, f = 133, h = 5

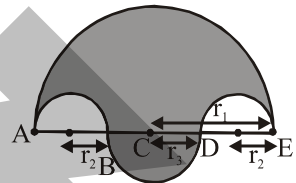
Median $= \ell + \frac{\frac{n}{2} - cf}{f} \times h$

$= 10 + \frac{140 - 49}{133} \times 5$

$= 10 + 3.42$

$= 13.42$

30. Let the radii of the largest semicrcle, the smallest semicircle and the circle with diameter BD be r_1, r_2 and r_3 respectively.



Given, AE = 14 cm $\Rightarrow r_1 = 7$ cm

and DE = AB = 3.5 cm $\therefore r_2 = \frac{3.5}{2}$ cm

$r_3 = r_1 - 2r_2 = 7 - 2 \times \frac{3.5}{2} = 7 - 3.5 = 3.5$ cm

Area of the shaded region = Area of semicircle with radius r_1 + Area of semicircle with radius r_3 – 2 × Area of semicircle with radius r_2

$= \frac{1}{2}\pi(r_1)^2 + \frac{1}{2}\pi(r_3)^2 - 2 \times \frac{1}{2}\pi(r_2)^2$

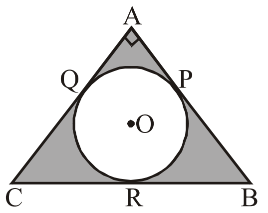
$= \frac{1}{2}\pi\{(r_1)^2 + (r_3)^2 - 2(r_2)^2\}$

$= \frac{1}{2} \times \frac{22}{7}\left\{(7)^2 + (3.5)^2 - 2\left(\frac{3.5}{2}\right)^2\right\}$

$= \frac{11}{7}\left\{49 + 12.25 - \frac{12.25}{2}\right\} = \frac{11}{7}(49 + 6.125)$

$= \frac{11}{7} \times 55.125 = 86.625\ cm^2$

OR



Given, AB = 6 cm and BC = 10 cm

By pythagoras theorem, in ΔABC, we get

$AC^2 = BC^2 - AB^2 = (10)^2 - (6)^2 = 64$

$\Rightarrow AC = 8$ cm

Let the radius of the incircle be r.

Let the circle touch side AB at P, side AC at Q and side BC at R.

Join OP, OQ and OR.

We know that the radius from the centre of the circle is perpendicular to the tangent through the point of contact.

$\therefore$ $OP \perp AB$, $OQ \perp AC$ and $OR \perp BC$

Also, the tangents drawn from an external point to the circle are equal.

$\therefore$ AP = AQ, BP = BR, CR = CQ

Now, in quadrilateral

AQ = AP and $\angle AQO = \angle APO = \angle PAQ = 90°$

OPAQ is a square.

$\therefore$ OP = AQ = AP = OQ = r

$\therefore$ $PB = 6 - r \Rightarrow BR = 6 - r$

$CQ = 8 - r \Rightarrow CR = 8 - r$

Now, BC = BR + CR

$\Rightarrow 10 = 6 - r + 8 - r \Rightarrow 10 = 14 - 2r$

$\Rightarrow r = 2$ cm

Now, area of shaded region

= Area of ΔABC – Area of circle

$$= \frac{1}{2} \times AB \times AC - \pi r^2 = \frac{1}{2} \times (8) \times (6) - 3.14(2)^2$$

$= 24 - 12.56 = 11.44 \text{ cm}^2$

31. Two solutions of each linear equation

$x + 3y = 6$ ...(i)

and $2x - 3y = 12$ ...(ii)

are given below.

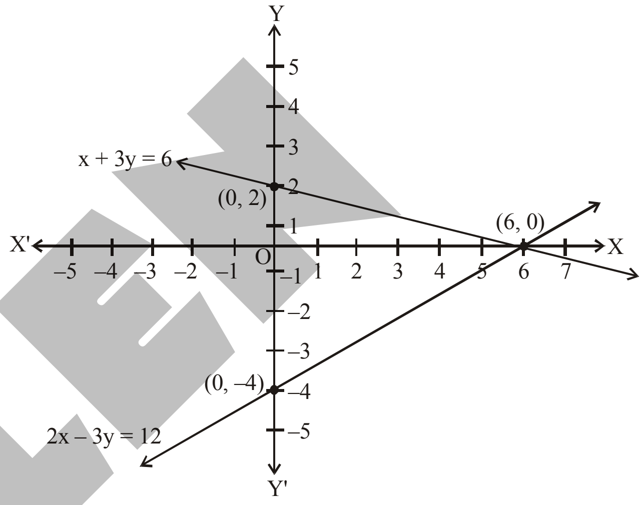
(i)

x	6	0
y	0	2

(ii)

x	6	0
y	0	– 4

The graphical representation of the given pair of linear equations is as follows :



Thus, the coordinates of point where the line $x + 3y = 6$ intersects the y-axis at (0, 2) and the line $2x - 3y = 12$ intersects the y-axis at (0, – 4).

OR

Let the fraction be $\frac{x}{y}$.

According to question

$\therefore x + y = 2x + 4 \Rightarrow x = y - 4$

Also, $\frac{x+3}{y+3} = \frac{2}{3}$

$\Rightarrow \frac{y-4+3}{y+3} = \frac{2}{3}$

$\Rightarrow \frac{y-1}{y+3} = \frac{2}{3}$

$\Rightarrow 3y - 3 = 2y + 6 \Rightarrow y = 9$

Substituting the value of y in (i), we get

x = 5

Thus, the required fraction is $\frac{5}{9}$.

32. L.H.S. = (1 + cotA – cosecA) (1 + tanA + secA)

$$= \left(1+\frac{\cos A}{\sin A}-\frac{1}{\sin A}\right)\left(1+\frac{\sin A}{\cos A}+\frac{1}{\cos A}\right)$$

$$= \left(\frac{(\sin A+\cos A-1)(\sin A+\cos A+1)}{\sin A\cos A}\right)$$

$$= \frac{(\sin A+\cos A)^2-1^2}{\sin A\cos A}$$

$$= \frac{\sin^2 A+\cos^2 A+2\sin A\cos A-1}{\sin A\cos A}$$

$$= \frac{1+2\sin A\cos A-1}{\sin A\cos A} \quad [\because \sin^2 A + \cos^2 A = 1]$$

= 2 = R.H.S.

Hence proved

33. $$\text{LHS} = \frac{\sin A-2\sin^3 A}{2\cos^3 A-\cos A}$$

$$= \frac{\sin A(1-2\sin^2 A)}{\cos A(2\cos^2 A-1)}$$

$$= \frac{\tan A[1-2(1-\cos^2 A)]}{\cos A(2\cos^2 A-1)}$$

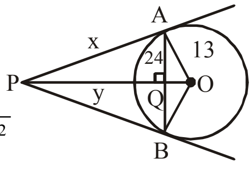
$$\left[\because \tan A = \frac{\sin A}{\cos A},\ \sin^2 A = 1-\cos^2 A\right]$$

$$= \frac{\tan A[1-2+2\cos^2 A]}{(2\cos^2 A-1)}$$

$$= \frac{\tan A[2\cos^2 A-1]}{(2\cos^2 A-1)}$$

= tanA = RHS.

34. BQ = 12 cm,



OB = 13 cm

$\therefore$ OQ = $\sqrt{13^2-12^2}$

= $\sqrt{169-144}$ = $\sqrt{25}$

OQ = 5 cm

Let PQ = y and PA = x

In Δ POA : $x^2 + 13^2 = (y + 5)^2$

$x^2 + 169 = y^2 + 10y + 25$

: $x^2 - y^2 + 169 - 25 = 10y$ ... (1)

In ΔPQA : $x^2 = 12^2 + y^2$

$x^2 - y^2 = 144$ ... (2)

Put (2) in (1) $144 + 169 - 25 = 10y$

$10y = 288 \Rightarrow y = 28.8$

PA = x = $\sqrt{144+(28.8)^2}$ = $\sqrt{973.44}$

= 31.2 cm

SECTION-D

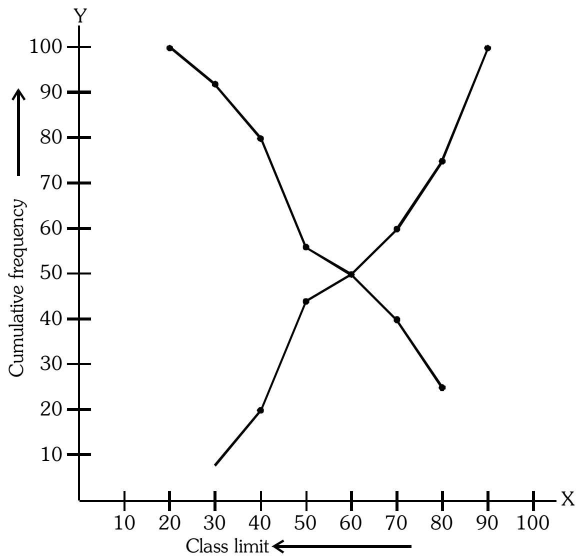
35. Less than type cumulative frequency table :

Class	Upper class limit	Frequency	Cumulative frequency
20 – 30	30	8	8
30 – 40	40	12	8 + 12 = 20
40 – 50	50	24	20 + 24 = 44
50 – 60	60	6	44 + 6 = 50
60 – 70	70	10	50 + 10 = 60
70 – 80	80	15	60 + 15 = 75
80 – 90	90	25	75 + 25 = 100

More than type cumulative frequency table :

Class	Lower class limit	Frequency	Cumulative frequency
20 – 30	20	8	100
30 – 40	30	12	100 – 8 = 92
40 – 50	40	24	92 – 12 = 80
50 – 60	50	6	80 – 24 = 56
60 – 70	60	10	56 – 6 = 50
70 – 80	70	15	50 – 10 = 40
80 – 90	80	25	40 – 15 = 25

The less than type and more than type ogives can be drawn on graph paper as follows :



Since, graph intersect at (60, 50)

$\therefore$ Median is 60.

36. In order to draw the pair of tangents, we follow the following steps.

Steps of construction

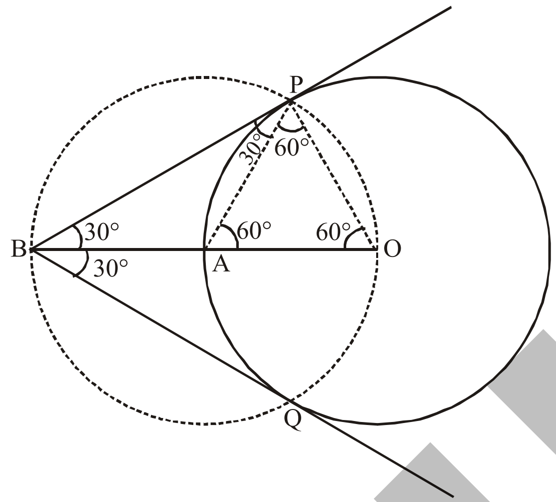
Step 1 : Take a point O on the plane of the paper and draw a circle of radius OA = 5 cm.

Step 2 : Produce OA to B such that OA = AB = 5 cm.

Step 3 : Taking A as the centre draw a circle of radius AO = AB = 5 cm.

Suppose it cuts the circle drawn in step I at P and Q.

Step 4 : Join BP and BQ to get the desired tangents.



OR

Steps of construction

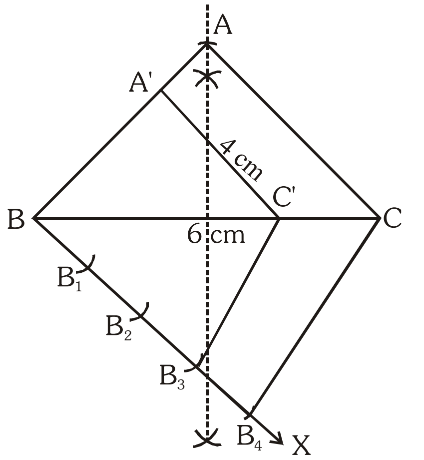
(1) Draw a line segment BC = 6 cm.

(2) Draw a perpendicular bisector of BC which cuts the line BC at Q.

(3) Cut the line OA = 4 cm.

(4) Join A to B and C.

(5) Triangle ABC is the given triangle.



(6) Draw a ray BX making an acute angle.

(7) Mark the four points B_1, B_2, B_3 and B_4 on the ray BX.

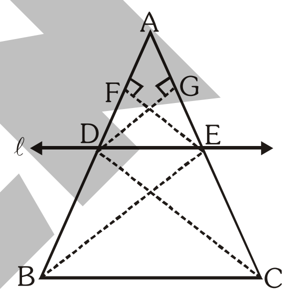
(8) Join B_4C. Draw a line parallel through B_3 to B_4C intersecting extended line segment AB at A'.

Hence ΔA'B'C' is a required triangle.

37. Given : A ΔABC in which a line DE parallel to BC intersects AB at D and AC at E.

To prove : DE divides the two sides in the same ratio.

i.e., $\frac{AD}{DB} = \frac{AE}{EC}$



Construction : Join BE, CD and draw EF $\perp$ AB and DG $\perp$ AC.

Proof : ar(ΔADE) = $\frac{1}{2} \times AD \times EF$

Similarly, ar(ΔBDE) = $\frac{1}{2} \times DB \times EF$

$$\therefore \frac{ar(\Delta ADE)}{ar(\Delta BDE)} = \frac{\frac{1}{2}\times AD \times EF}{\frac{1}{2}\times DB \times EF} = \frac{AD}{DB} \quad \text{.... (i)}$$

$$\text{and } \frac{ar(\Delta ADE)}{ar(\Delta DEC)} = \frac{\frac{1}{2}\times AE \times GD}{\frac{1}{2}\times EC \times GD} = \frac{AE}{EC} \quad \text{.... (ii)}$$

Since, ΔBDE and ΔDEC lie between the same parallel lines DE and BC and on the same base DE.

So, ar(ΔBDE) = ar(ΔDEC) .... (iii)

From (i), (ii) and (iii) we get

$\frac{AD}{DB} = \frac{AE}{EC}$ Hence, proved.

Now, in ΔPQR, we have $AB \parallel PQ$

$\Rightarrow \frac{AR}{PR} = \frac{BR}{QR}$ ..... (i)

In ΔAQR, we have $CB \parallel AQ$

$\Rightarrow \frac{CR}{AR} = \frac{BR}{QR}$ ...... (ii)

From (i) and (ii) we get $\frac{AR}{PR} = \frac{CR}{AR}$

$\Rightarrow AR^2 = PR \cdot CR$

38. Let the usual speed of the train be xkm/h

$\frac{300}{x} - \frac{300}{x+5} = 2$

$\Rightarrow x^2 + 5x - 750 = 0$

$\Rightarrow (x + 30)(x - 25) = 0$

$\Rightarrow x = -30, 25$

$\therefore$ Usual speed of the train = 25 km/h

OR

$\frac{1}{(a+b+x)} = \frac{1}{a} + \frac{1}{b} + \frac{1}{x}$

$\Rightarrow \frac{x-a-b-x}{x(a+b+x)} = \frac{b+a}{ab}$

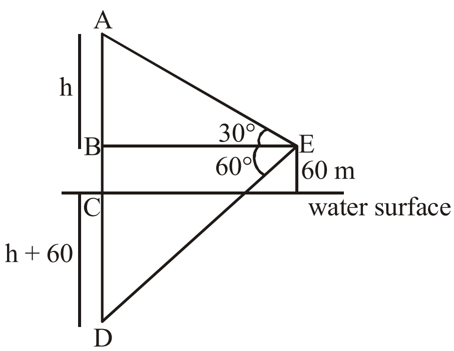
$\Rightarrow -ab = x^2 + (a + b)x$

$\Rightarrow x^2 + ax + bx + ab = 0$

$\Rightarrow (x + a)(x + b) = 0$

$\Rightarrow x = -a, -b$

39.



In ΔABE,

$\frac{h}{x} = \tan 30°$

$\Rightarrow x = h\sqrt{3}$

In ΔBDE,

$\frac{h+60+60}{x} = \tan 60°$

$h + 120 = x\sqrt{3}$

$h + 120 = h\sqrt{3} \times \sqrt{3}$

$2h = 120$

$h = 60$

$\therefore$ height of cloud from surface of water

$= (60 + 60)m = 120$ m

40. Since diagonals of $\|^{gm}$ bisect each other

Coordinates of point O = $\left(\frac{-4+2}{2}, \frac{-3+3}{2}\right)$

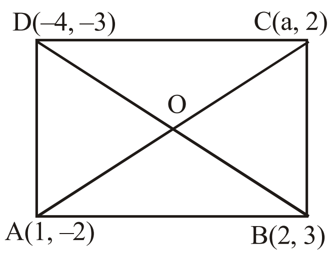
$= \left(\frac{-2}{2}, \frac{0}{2}\right)$

$= (-1, 0)$ ...(1)

Again coordinates of point O

$= \left(\frac{1+a}{2}, \frac{-2+2}{2}\right)$

$= \left(\frac{1+a}{2}, 0\right)$ ...(2)



From (1) and (2) we have

$$\frac{1+a}{2} = -1$$

$\Rightarrow$ $1 + a = -2$

$\Rightarrow$ $a = -2 - 1 = -3$

So, the points of $||^{gm}$ ABCD are

A(1, –2), B(2, 3), C(–3, 2) and D(–4, –3)

Area of ΔABC

$$= \frac{1}{2}|[1(3-2) + 2(2+2) - 3(-2-3)]|$$

$$= \frac{1}{2}|1 + 8 + 15| = 12 \text{ sq units}$$

Area of $||^{gm}$ ABCD = 2 × 12 = 24 sq units

$$AB = \sqrt{(2-1)^2 + (3+2)^2}$$

$$= \sqrt{1^2 + 5^2} = \sqrt{26}$$

Again area of $||^{gm}$ ABCD = AB × height

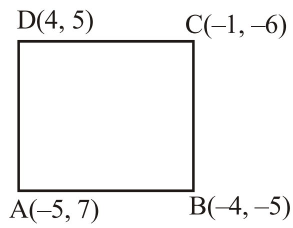
$$24 = \sqrt{26} \times \text{height}$$

$$\text{height} = \frac{24}{\sqrt{26}}$$

$$\text{height} = \frac{24\sqrt{26}}{26}$$

Hence, $a = -3$, height $= \frac{12\sqrt{26}}{13}$.

OR



Area of ΔABC,

$$= \frac{1}{2}|-5(-5+6) - 4(-6-7) - 1(7+5)|$$

$$= \frac{1}{2}|-5 + 52 - 12|$$

$$= \frac{1}{2} \times 35 = \frac{35}{2} \text{ sq. units}$$

Area of ΔACD,

$$= \frac{1}{2}|-5(-6-5) - 1(5-7) + 4(7+6)|$$

$$= \frac{1}{2}|55 + 2 + 52|$$

$$= \frac{109}{2} \text{ sq. units}$$

Area of quadrilateral ABCD $= \frac{35}{2} + \frac{109}{2}$

$$= \frac{144}{2} = 72 \text{ sq. units}$$