CBSE QUESTION PAPER SOLUTION – 2022 (65/3/3)

SUBJECT: MATHEMATICS

TERM-II

SOLUTION

SECTION - A

1. $I = \int \frac{dx}{x^2 - 6x + 13} = \int \frac{1}{(x-3)^2 + 2^2} dx$ **[1]**

$= \frac{1}{2}\tan^{-1}\left(\frac{x-3}{2}\right) + C \quad \left[\because \int \frac{1}{x^2 + a^2} dx = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) + C\right]$ **[1]**

2. Given that

$e^{dy/dx} = x^2$

take log on both sides

$\Rightarrow \log e^{dy/dx} = \log x^2$

$\Rightarrow \frac{dy}{dx} = 2\log x$

$\Rightarrow dy = 2\log x dx$ **[1]**

On integrating both sides

$\Rightarrow \int 1.dy = 2\int 1.\log x \; dx$

$\Rightarrow y = 2[\log x \int 1.dx - \int \frac{1}{x}.\int 1.dx)dx]$

$y = 2x\log x - 2x + c$ **[1]**

3. Given, $\vec{a} = 2\hat{i} - 2\hat{j} + \hat{k}, \vec{b} = \hat{i} + 2\hat{j} - 2\hat{k}$ and $\vec{c} = 2\hat{i} - \hat{j} + 4\hat{k}$

$\Rightarrow \vec{b} + \vec{c} = 3\hat{i} + \hat{j} + 2\hat{k}$

$\therefore$ Projection of $(\vec{b} + \vec{c})$ on $\vec{a}$ $= (\vec{b} + \vec{c}) \cdot \hat{a}$ **[1]**

$= (3\hat{i} + \hat{j} + 2\hat{k}) \cdot \frac{(2\hat{i} - 2\hat{j} + \hat{k})}{3} = \frac{6}{3} = 2$ units **[1]**

4. Given, distance of the point (1,1,1) from the plane $x - y + z + \lambda = 0$ is $\frac{5}{\sqrt{3}}$ i.e. $\left|\frac{1-1+1+\lambda}{\sqrt{3}}\right| = \frac{5}{\sqrt{3}}$

$\Rightarrow \left|\frac{1+\lambda}{\sqrt{3}}\right| = \frac{5}{\sqrt{3}}$ **[1]**

or $\frac{1+\lambda}{\sqrt{3}} = \pm\frac{5}{\sqrt{3}}$

$\Rightarrow 1 + \lambda = \pm 5$

$\therefore \lambda = 4, -6$ **[1]**

5. Let random variable X denotes the number of spade cards ; then the possible values of X are 0, 1 or 2.

$P(X = 0) = P(\text{no spade \& no spade}) = \frac{39}{52}\times\frac{39}{52} = \frac{3}{4}\times\frac{3}{4} = \frac{9}{16}$

$P(X = 1) = P(\text{spade \& no spade or no spade \& spade}) = \left(\frac{13}{52}\times\frac{39}{52}\right)+\left(\frac{39}{52}\times\frac{13}{52}\right)$

$= \left(\frac{1}{4}\times\frac{3}{4}\right)+\left(\frac{3}{4}\times\frac{1}{4}\right) = \frac{3}{8}$

$P(X = 2) = P(\text{spade \& spade}) = \left(\frac{1}{4}\times\frac{1}{4}\right) = \frac{1}{16}$ **[1½]**

Hence, probability distribution of X is

X	0	1	2
P(X)	9/16	6/16	1/16

[½]

6. When a pair of dice is thrown, the sample space S contains 36 outcomes.

Let E : Event that number 5 has appeared on at least one die,

F : Event that sum of the numbers on the dice is 7

E = {(5,1), (5,2), (5,3), (5,4), (5,5), (5,6), (1,5), (2,5), (3,5), (4,5), (6,5)}

F = {(1,6), (2,5), (3,4), (4,3), (5,2), (6,1)}

$E \cap F$ = {(5, 2), (2,5)}

$\Rightarrow P(E\cap F) = \frac{2}{36}, P(F) = \frac{6}{36}$ **[1]**

Required probability = $P(E/F) = \frac{P(E\cap F)}{P(F)} = \frac{(2/36)}{(6/36)} = \frac{1}{3}$ **[1]**

OR

Let

E_1 : Event that A hits the target

E_2 : Event that B hits the target

Given ; $P(E_1) = \frac{1}{3}$, $P(E_2) = \frac{2}{5}$ $\Rightarrow P\left(\overline{E_1}\right)=\frac{2}{3}, P\left(\overline{E_2}\right)=\frac{3}{5}$ **[½]**

Required Probability $= P(\text{target is hit})$

$= 1 - P\left(\bar{E}_1\bar{E}_2\right)$

$= 1 - P\left(\bar{E}_1\right)\cdot P\left(\bar{E}_2\right)$

$= 1-\left(\frac{2}{3}\times\frac{3}{5}\right)=1-\frac{2}{5}=\frac{3}{5}$ **[1½]**

7. Given lines are

$\vec{r} = 3\hat{i}+5\hat{j}+7\hat{k}+\lambda(\hat{i}-2\hat{j}+\hat{k})$

$\vec{r} = (-\hat{i}-\hat{j}-\hat{k})+\mu(7\hat{i}-6\hat{j}+\hat{k})$

$\vec{a}_1 = 3\hat{i}+5\hat{j}+7\hat{k}$ $\quad\vec{b}_1 = \hat{i}-2\hat{j}+\hat{k}$

$\vec{a}_2 = -\hat{i}-\hat{j}-\hat{k}$ $\quad\vec{b}_2 = 7\hat{i}-6\hat{j}+\hat{k}$

Now,

$$\vec{a}_2-\vec{a}_1 = -4\hat{i}-6\hat{j}-8\hat{k}$$

$$\vec{b}_1\times\vec{b}_2 = \begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 1 & -2 & 1\\ 7 & -6 & 1\end{vmatrix} = 4\hat{i}+6\hat{j}+8\hat{k}$$ **[1]**

$\because$ distance b/w lines given by

$$d = \left|\frac{(\vec{a}_2-\vec{a}_1).(\vec{b}_1\times\vec{b}_2)}{|\vec{b}_1\times\vec{b}_2|}\right|$$

$$d = \left|\frac{(-4\hat{i}-6\hat{j}-8\hat{k}).(4\hat{i}+6\hat{j}+8\hat{k})}{|4\hat{i}+6\hat{j}+8\hat{k}|}\right|$$ **[1]**

$$= \left|\frac{-16-36-64}{\sqrt{16+36+64}}\right|$$

$$d = \frac{116}{\sqrt{116}} = \sqrt{116} \text{ units}$$ **[1]**

8. Adjacent sides of a parallelogram are given as $\vec{a} = 2\hat{i}-4\hat{j}+5\hat{k}$ and $\vec{b} = \hat{i}-2\hat{j}-3\hat{k}$.

Then, the diagonal of a parallelogram is given by $\vec{a}+\vec{b}$.

$$\vec{a}+\vec{b} = \left(2\hat{i}-4\hat{j}+5\hat{k}\right)+\left(\hat{i}-2\hat{j}-3\hat{k}\right) = 3\hat{i}-6\hat{j}+2\hat{k}$$

Thus, the unit vector parallel to the diagonal is $\frac{\vec{a}+\vec{b}}{|\vec{a}+\vec{b}|} = \frac{3\hat{i}-6\hat{j}+2\hat{k}}{\sqrt{3^2+(-6)^2+2^2}} = \frac{3}{7}\hat{i}-\frac{6}{7}\hat{j}+\frac{2}{7}\hat{k}$ **[1]**

Also, Area of parallelogram = $|\vec{a}\times\vec{b}|$

$$\because\ \vec{a}\times\vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -4 & 5 \\ 1 & -2 & -3 \end{vmatrix} = \hat{i}(12+10)-\hat{j}(-6-5)+\hat{k}(-4+4)$$ **[1]**

$$\Rightarrow\ \vec{a}\times\vec{b} = 22\hat{i}+11\hat{j}$$

$$\therefore\ \left|\vec{a}\times\vec{b}\right| = \sqrt{22^2+11^2} = 11\sqrt{5}$$

Hence area of the parallelogram is $11\sqrt{5}$ square units. **[1]**

OR

Given, $\vec{a} = 2\hat{i}+2\hat{j}+3\hat{k}$, $\vec{b} = -\hat{i}+2\hat{j}+\hat{k}$ and $\vec{c} = 3\hat{i}+\hat{j}$.

Given, $\vec{a}+\lambda\vec{b}$ is perpendicular to $\vec{c}$.

$$\Rightarrow \left(\vec{a}+\lambda\vec{b}\right)\cdot\vec{c} = 0$$

$$\Rightarrow \vec{a}\cdot\vec{c}+\lambda\left(\vec{b}\cdot\vec{c}\right) = 0$$

$$\Rightarrow \lambda = -\frac{\vec{a}\cdot\vec{c}}{\vec{b}\cdot\vec{c}}$$ **[1]**

$$\Rightarrow \lambda = -\frac{(2\hat{i}+2\hat{j}+3\hat{k})\cdot(3\hat{i}+\hat{j})}{(-\hat{i}+2\hat{j}+\hat{k})\cdot(3\hat{i}+\hat{j})}$$ **[1]**

$$\Rightarrow \lambda = -\frac{6+2+0}{-3+2+0} = 8$$ **[1]**

9. Given differential equation is $x\frac{dy}{dx} - y = x^2 \cdot e^x$

$$\Rightarrow \frac{dy}{dx} - \frac{y}{x} = x \cdot e^x$$

This is linear differential equation of the form $\frac{dy}{dx} + Py = Q$

where P $= -\frac{1}{x}$ and Q = $x \cdot e^x$

$\therefore$ I.F. = $e^{\int -\frac{1}{x}dx} = e^{-\log x} = \frac{1}{x}$ **[1]**

Hence, the general solution is given by $y\left(\frac{1}{x}\right) = \int x \cdot e^x \left(\frac{1}{x}\right)dx$

$$\Rightarrow \frac{y}{x} = \int e^x dx$$

$$\Rightarrow \frac{y}{x} = e^x + c$$ **[1]**

Substituting x = 1 and y = 0, we get:

$0 = e^1 + c \Rightarrow c = -e$

$\therefore$ Required particular solution is $\frac{y}{x} = e^x - e$. **[1]**

OR

Given differential equation is: $x\frac{dy}{dx} = y(\log y - \log x + 1)$

$$\Rightarrow \frac{dy}{dx} = \frac{y}{x}\left(\log\frac{y}{x} + 1\right)$$

Put y = vx $\Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}$ **[1]**

$$\Rightarrow v + x\frac{dv}{dx} = v(\log v + 1)$$

$$\Rightarrow \frac{1}{v\log v}dv = \frac{dx}{x}$$ **[1]**

Integrating both sides $\int \frac{1}{v\log v}dv = \int \frac{1}{x}dx$

$\Rightarrow$ log(log v) = log x + log c = log cx

$\Rightarrow$ log v = cx

$\Rightarrow \log\frac{y}{x} = cx$, is the required general solution. **[1]**

10. Let I = $\int_{\frac{-\pi}{2}}^{\pi/2}\{\sin|x|+\cos|x|\}dx$

$\because$ f(x) = sin|x| + cos|x|

f(–x) = sin|–x| + cos|–x|

f(–x) = sin|x| + cos|x| = f(x)

$\therefore$ f(x) is even function **[1]**

Therefore by prop. $\int_{-a}^{a} f(x)dx = 2\int_0^a f(x)dx$

$I = 2\int_0^{\pi/2}\{\sin|x|+\cos|x|\}dx$

$I = 2\int_0^{\pi/2}\{\sin x+\cos x\}dx$ **[1]**

$I = 2[-\cos x+\sin x]_0^{\pi/2}$

I = 2[0 + 1 – (–1) – 0] = 2 × 2 = 4 **[1]**

11. Given, line $\vec{r} = 4\hat{i}+2\hat{j}+7\hat{k}+\lambda(3\hat{i}+4\hat{j}+2\hat{k})$ and plane $\vec{r}.(\hat{i}-\hat{j}+\hat{k}) = 10 \Rightarrow x-y+z=10$

Equation of line in Cartesion form

$\frac{x-4}{3} = \frac{y-2}{4} = \frac{z-7}{2} = t$ (say)

x = 3t + 4, y = 4t + 2, z = 2t + 7 **[1]**

Satisfy in equation of plane x – y + z = 10

$\Rightarrow$ (3t + 4) – (4t + 2) + (2t + 7) = 10

$\Rightarrow$ t + 9 = 0

$\Rightarrow$ t = 1

So, Point of intersection is (7, 6, 9) **[1]**

Now, distance of point (7, 6, 9) & (1, –2, 9) is

$$d=\sqrt{(7-1)^2+(6+2)^2+(9-9)^2}$$

$$d=\sqrt{36+64+0}=\sqrt{100}$$

d = 10 units **[2]**

12. $I=\int\frac{x^2}{(x^2+1)(3x^2+4)}dx$

Let $x^2 = y$

Let $\frac{x^2}{(x^2+1)(3x^2+4)}=\frac{y}{(y+1)(3y+4)}=\frac{A}{(y+1)}+\frac{B}{(3y+4)}$

$\Rightarrow \quad y=A(3y+4)+B(y+1) \qquad$ ______(1)

from equation (1)

Put y = –1 $\Rightarrow$ A = –1

Put y = –4/3 $\Rightarrow$ B = 4 **[1]**

$\therefore \quad \frac{y}{(y+1)(3y+4)}=\frac{-1}{y+1}+\frac{4}{3y+4}$

i.e. $\frac{x^2}{(x^2+1)(3x^2+4)}=\frac{-1}{x^2+1}+\frac{4}{3x^2+4}$ **[1]**

$$\therefore\ I=\int\left(\frac{-1}{x^2+1}+\frac{4}{3x^2+4}\right)dx$$

$$=-\int\frac{1}{x^2+1}dx+\frac{4}{3}\int\frac{1}{x^2+4/3}dx$$

$$=-\tan^{-1}x+\frac{4}{3}\cdot\frac{\sqrt{3}}{2}\tan^{-1}\left(\frac{x\sqrt{3}}{2}\right)+C$$

$$=\frac{2}{\sqrt{3}}\tan^{-1}\left(\frac{x\sqrt{3}}{2}\right)-\tan^{-1}x+C$$ **[2]**

OR

Let $I=\int_{-2}^{1}\sqrt{5-4x-x^2}\,dx$

$$=\int_{-2}^{1}\sqrt{3^2-(x+2)^2}\,dx$$ **[1]**

$$= \left[\frac{x+2}{2}\sqrt{3^2-(x+2)^2} + \frac{9}{2}\sin^{-1}\left(\frac{x+2}{2}\right)\right]_{-2}^{1}$$ **[2]**

$$= \frac{9}{2}\sin^{-1}1 = \frac{9\pi}{4}$$ **[1]**

13. Given, line $\vec{r} = 4\hat{i}+2\hat{j}+7\hat{k}+\lambda(3\hat{i}+4\hat{j}+2\hat{k})$ and plane $\vec{r}.(\hat{i}-\hat{j}+\hat{k}) = 10 \Rightarrow x-y+z=10$

Equation of line in Cartesion form

$$\frac{x-4}{3} = \frac{y-2}{4} = \frac{z-7}{2} = t \text{ (say)}$$

$x = 3t + 4,\ y = 4t + 2,\ z = 2t + 7$ **[1]**

Satisfy in equation of plane $x - y + z = 10$

$\Rightarrow (3t + 4) - (4t + 2) + (2t + 7) = 10$

$\Rightarrow t + 9 = 0$

$\Rightarrow t = 1$

So, Point of intersection is (7, 6, 9) **[1]**

Now, distance of point (7, 6, 9) & (1, –2, 9) is

$$d = \sqrt{(7-1)^2+(6+2)^2+(9-9)^2}$$

$$d = \sqrt{36+64+0} = \sqrt{100}$$

d = 10 units **[2]**

14. Given, A1 : A2 : A3 = 4 : 4 : 2

$P(A1) = \frac{4}{10},\ P(A2) = \frac{4}{10}$ and $P(A3) = \frac{2}{10}$

Let E be the event that a seed germinates

$\therefore P(E/A1) = \frac{45}{100}, P\left(\frac{E}{A2}\right) = \frac{60}{100}$ and $P(E/A3) = \frac{35}{100}$ **[1]**

(a) $P(E) = P(A1)\cdot P(E/A1) + P(A2)\cdot P(E/A2) + P(A3)\cdot P(E/A3)$

$$= \frac{4}{10}\cdot\frac{45}{100}+\frac{4}{10}\cdot\frac{60}{100}+\frac{2}{10}\cdot\frac{35}{100} = \frac{490}{1000} = \frac{49}{100} = 0.49$$ **[1½]**

(b) $$P(A2/E) = \frac{P(A2)\cdot P(E/A2)}{P(A1)\cdot P(E/A1)+P(A2)\cdot P(E/A2)+P(A3)\cdot P(E/A3)}$$

$= \frac{4}{10} \cdot \frac{60}{100} = \frac{24}{49} = 0.48$ **[1½]**