CBSE QUESTION PAPER SOLUTION – 2022 (65/3/2)

SUBJECT: MATHEMATICS

TERM-II

SOLUTION

SECTION - A

1. Given, $\vec{a} = 2\hat{i} - 2\hat{j} + \hat{k}, \vec{b} = \hat{i} + 2\hat{j} - 2\hat{k}$ and $\vec{c} = 2\hat{i} - \hat{j} + 4\hat{k}$

$\Rightarrow \vec{b} + \vec{c} = 3\hat{i} + \hat{j} + 2\hat{k}$

$\therefore$ Projection of $(\vec{b} + \vec{c})$ on $\vec{a}$ = $(\vec{b} + \vec{c}) \cdot \hat{a}$ **[1]**

$= (3\hat{i} + \hat{j} + 2\hat{k}) \cdot \dfrac{(2\hat{i} - 2\hat{j} + \hat{k})}{3} = \dfrac{6}{3} = 2$ units **[1]**

2. Given that

$$\log\left(\frac{dy}{dx}\right) = ax + by$$

$$\Rightarrow \frac{dy}{dx} = e^{ax+by} = e^{ax}.e^{by}$$

$$\Rightarrow \frac{dy}{e^{by}} = e^{ax}dx$$ **[1]**

On integrating both sides

$$\Rightarrow \int e^{-by}dy = \int e^{ax}dx$$

$$\Rightarrow \frac{e^{-by}}{-b} = \frac{e^{ax}}{a} + c_1$$

$$\Rightarrow e^{-by} = -\frac{b}{a}e^{ax} + C \text{ where } C = -bc_1$$ **[1]**

3. Let random variable X denotes the number of spade cards ; then the possible values of X are 0, 1 or 2.

P(X = 0) = P(no spade & no spade) = $\dfrac{39}{52} \times \dfrac{39}{52} = \dfrac{3}{4} \times \dfrac{3}{4} = \dfrac{9}{16}$

P(X = 1) = P(spade & no spade or no spade & spade) = $\left(\dfrac{13}{52} \times \dfrac{39}{52}\right) + \left(\dfrac{39}{52} \times \dfrac{13}{52}\right)$

$$= \left(\frac{1}{4}\times\frac{3}{4}\right)+\left(\frac{3}{4}\times\frac{1}{4}\right)=\frac{3}{8}$$

P(X = 2) = P(spade & spade) = $\left(\frac{1}{4}\times\frac{1}{4}\right)=\frac{1}{16}$ **[1½]**

Hence, probability distribution of X is

X	0	1	2
P(X)	9/16	6/16	1/16

[½]

4. When a pair of dice is thrown, the sample space S contains 36 outcomes.

Let E : Event that number 5 has appeared on at least one die,

F : Event that sum of the numbers on the dice is 7

E = {(5,1), (5,2), (5,3), (5,4), (5,5), (5,6), (1,5), (2,5), (3,5), (4,5), (6,5)}

F = {(1,6), (2,5), (3,4), (4,3), (5,2), (6,1)}

$E\cap F$ = {(5, 2), (2,5)}

$\Rightarrow\ P(E\cap F)=\frac{2}{36}, P(F)=\frac{6}{36}$ **[1]**

Required probability = P(E/F) = $\frac{P(E\cap F)}{P(F)}=\frac{(2/36)}{(6/36)}=\frac{1}{3}$ **[1]**

OR

Let

E_1 : Event that A hits the target

E_2 : Event that B hits the target

Given ; $P(E_1)=\frac{1}{3}$, $P(E_2)=\frac{2}{5}$ $\Rightarrow P(\overline{E_1})=\frac{2}{3}, P(\overline{E_2})=\frac{3}{5}$ **[½]**

Required Probability = P(target is hit)

$= 1 - P(\overline{E}_1\overline{E}_2)$

$= 1 - P(\overline{E}_1)\cdot P(\overline{E}_2)$

$= 1-\left(\frac{2}{3}\times\frac{3}{5}\right)=1-\frac{2}{5}=\frac{3}{5}$ **[1½]**

5. Given, distance of the point (1,1,1) from the plane $x - y + z + \lambda = 0$ is $\frac{5}{\sqrt{3}}$ i.e. $\left|\frac{1-1+1+\lambda}{\sqrt{3}}\right| = \frac{5}{\sqrt{3}}$

$\Rightarrow \left|\frac{1+\lambda}{\sqrt{3}}\right| = \frac{5}{\sqrt{3}}$ **[1]**

or $\frac{1+\lambda}{\sqrt{3}} = \pm\frac{5}{\sqrt{3}}$

$\Rightarrow 1 + \lambda = \pm 5$

$\therefore \lambda = 4, -6$ **[1]**

6. $I = \int \frac{dx}{x^2 - 6x + 13} = \int \frac{1}{(x-3)^2 + 2^2} dx$ **[1]**

$= \frac{1}{2}\tan^{-1}\left(\frac{x-3}{2}\right) + C \quad \left[\because \int \frac{1}{x^2 + a^2} dx = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) + C\right]$ **[1]**

7. Given ; $\vec{a}.\vec{b} = \vec{a}.\vec{c}$ and $\vec{a} \neq \vec{0}$

$\Rightarrow \quad \vec{a}.\vec{b} - \vec{a}.\vec{c} = 0$ and $\vec{a} \neq \vec{0}$

$\Rightarrow \quad \vec{a}.(\vec{b} - \vec{c}) = 0$ and $\vec{a} \neq \vec{0}$

or $\quad \vec{b} - \vec{c} = \vec{0}$ or $\vec{a} \perp (\vec{b} - \vec{c})$

$\Rightarrow \quad \vec{b} = \vec{c}$ or $\vec{a} \perp (\vec{b} - \vec{c})$ ……..(1) **[1]**

Again ; $\vec{a} \times \vec{b} = \vec{a} \times \vec{c}$ and $\vec{a} \neq \vec{0}$

$\Rightarrow \quad (\vec{a} \times \vec{b}) - (\vec{a} \times \vec{c}) = \vec{0}$ and $\vec{a} \neq \vec{0}$

or $\quad \vec{a} \times (\vec{b} - \vec{c}) = \vec{0}$ and $\vec{a} \neq \vec{0}$

$\Rightarrow \quad (\vec{b} - \vec{c}) = \vec{0}$ or $\vec{a} \parallel (\vec{b} - \vec{c})$ **[1]**

$\Rightarrow \quad \vec{b} = \vec{c}$ or $\vec{a} \parallel (\vec{b} - \vec{c})$ ……..(2)

From (1) and (2) ; we get $\vec{b} = \vec{c}$ [$\because$ $\vec{a}$ cannot be both $\perp$ and $\parallel$ to $(\vec{b} - \vec{c})$ simultaneously] **[1]**

OR

Given that $|\vec{a}| = 3$, $|\vec{b}| = 5$, $|\vec{c}| = 4$ & $\vec{a} + \vec{b} + \vec{c} = \vec{0}$

$\because |\vec{a} + \vec{b} + \vec{c}|^2 = (\vec{a} + \vec{b} + \vec{c}).(\vec{a} + \vec{b} + \vec{c}) \quad (\because |\vec{a}|^2 = \vec{a}.\vec{a})$ **[1]**

$\Rightarrow 0 = |\vec{a}|^2 + \vec{a}.\vec{b} + \vec{a}.\vec{c} + \vec{b}.\vec{a} + |\vec{b}|^2 + \vec{b}.\vec{c} + \vec{c}.\vec{a} + \vec{c}.\vec{b} + |\vec{c}|^2$ **[1]**

$\Rightarrow 0 = 9 + 25 + 16 + 2(\vec{a}.\vec{b} + \vec{b}.\vec{c} + \vec{c}.\vec{a})$

$\Rightarrow \vec{a}.\vec{b} + \vec{b}.\vec{c} + \vec{c}.\vec{a} = -25$ **[1]**

8. We note that $x^3 - x$ ^{3} 0 on [–1, 0] and $x^3 - x \le 0$ on [0, 1] and that $x^3 - x$ ^{3} 0 on [1, 2]. So by property of definite integral we get

$$\int_{-1}^{2} |x^3 - x| dx = \int_{-1}^{0} (x^3 - x) dx + \int_{0}^{1} -(x^3 - x) dx + \int_{1}^{2} (x^3 - x) dx$$ **[1]**

$$= \int_{-1}^{0} (x^3 - x) dx + \int_{0}^{1} (x - x^3) dx + \int_{1}^{2} (x^3 - x) dx = \left[\frac{x^4}{4} - \frac{x^2}{4}\right]_{-1}^{0} + \left[\frac{x^2}{2} - \frac{x^4}{4}\right]_{0}^{1} + \left[\frac{x^4}{4} - \frac{x^2}{2}\right]_{1}^{2}$$ **[1]**

$$= -\left(\frac{1}{4} - \frac{1}{2}\right) + \left(\frac{1}{2} - \frac{1}{4}\right) + (4 - 2) - \left(\frac{1}{4} - \frac{1}{2}\right) = -\frac{1}{4} + \frac{1}{2} + \frac{1}{2} - \frac{1}{4} + 2 - \frac{1}{4} + \frac{1}{2} = \frac{3}{2} - \frac{3}{4} + 2 = \frac{11}{4}$$ **[1]**

9. Given equation of lines are $\frac{1-x}{2} = \frac{y-3}{4} = \frac{z}{-1}$ and $\frac{x-4}{3} = \frac{2y-2}{-4} = z - 1$

In vector form both the lines can be expressed as

$\vec{r} = (\hat{i} + 3\hat{j}) + \lambda(-2\hat{i} + 4\hat{j} - \hat{k})$ and $\vec{r} = (4\hat{i} + \hat{j} + \hat{k}) + \mu(3\hat{i} - 2\hat{j} + \hat{k})$

On comparing it with $\vec{r} = \vec{a}_1 + \lambda\vec{b}_1$ and $\vec{r} = \vec{a}_2 + \mu\vec{b}_2$

$\vec{a}_1 = \hat{i} + 3\hat{j}$, $\vec{b}_1 = -2\hat{i} + 4\hat{j} - \hat{k}$

$\vec{a}_2 = 4\hat{i} + \hat{j} + \hat{k}$ and $\vec{b}_2 = 3\hat{i} - 2\hat{j} + \hat{k}$

The given lines are coplanar if $(\vec{a}_2 - \vec{a}_1) \cdot (\vec{b}_1 \times \vec{b}_2) = 0$

$\vec{a}_2 - \vec{a}_1 = (4\hat{i} + \hat{j} + \hat{k}) - (\hat{i} + 3\hat{j}) = 3\hat{i} - 2\hat{j} + \hat{k}$ **[1]**

$$\vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -2 & 4 & -1 \\ 3 & -2 & 1 \end{vmatrix} = \hat{i}(4-2) - \hat{j}(-2+3) + \hat{k}(4-12)$$ **[1]**

$= 2\hat{i} - \hat{j} - 8\hat{k}$

Now, $(\vec{a}_2 - \vec{a}_1) \cdot (\vec{b}_1 \times \vec{b}_2) = (3\hat{i} - 2\hat{j} + \hat{k}) \cdot (2\hat{i} - \hat{j} - 8\hat{k})$ = 6 + 2 – 8 = 0

Hence, both the lines are coplanar. **[1]**

10. Given differential equation is $x\frac{dy}{dx} - y = x^2 \cdot e^x$

$\Rightarrow \frac{dy}{dx} - \frac{y}{x} = x \cdot e^x$

This is linear differential equation of the form $\frac{dy}{dx} + Py = Q$

where P $= -\frac{1}{x}$ and Q = $x \cdot e^x$

$\therefore$ I.F. $= e^{\int -\frac{1}{x}dx} = e^{-\log x} = \frac{1}{x}$ **[1]**

Hence, the general solution is given by $y\left(\frac{1}{x}\right) = \int x \cdot e^x \left(\frac{1}{x}\right)dx$

$\Rightarrow \quad \frac{y}{x} = \int e^x dx$

$\Rightarrow \quad \frac{y}{x} = e^x + c$ **[1]**

Substituting x = 1 and y = 0, we get:

$0 = e^1 + c \Rightarrow c = -e$

$\therefore$ Required particular solution is $\frac{y}{x} = e^x - e$. **[1]**

OR

Given differential equation is: $x\frac{dy}{dx} = y(\log y - \log x + 1)$

$\Rightarrow \frac{dy}{dx} = \frac{y}{x}\left(\log\frac{y}{x} + 1\right)$

Put y = vx $\Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}$

$\Rightarrow v + x\frac{dv}{dx} = v(\log v + 1)$ **[1]**

$\Rightarrow \frac{1}{v\log v}dv = \frac{dx}{x}$

Integrating both sides $\int \frac{1}{v\log v}dv = \int \frac{1}{x}dx$

$\Rightarrow$ log(log v) = log x + log c = log cx **[1]**

$\Rightarrow$ log v = cx

$\Rightarrow \log\frac{y}{x} = cx$, is the required general solution. **[1]**

11. $I = \int \frac{x^2}{(x^2+1)(3x^2+4)}dx$

Let $x^2 = y$

Let $\frac{x^2}{(x^2+1)(3x^2+4)} = \frac{y}{(y+1)(3y+4)} = \frac{A}{(y+1)} + \frac{B}{(3y+4)}$

$\Rightarrow \quad y = A(3y+4) + B(y+1) \quad$ ______(1)

from equation (1)

Put y = –1 $\Rightarrow$ A = –1

Put y = –4/3 $\Rightarrow$ B = 4 **[1]**

$$\therefore \quad \frac{y}{(y+1)(3y+4)} = \frac{-1}{y+1} + \frac{4}{3y+4}$$ **[1]**

$$\text{i.e. } \frac{x^2}{(x^2+1)(3x^2+4)} = \frac{-1}{x^2+1} + \frac{4}{3x^2+4}$$

$$\therefore\ I = \int\left(\frac{-1}{x^2+1} + \frac{4}{3x^2+4}\right)dx$$

$$= -\int\frac{1}{x^2+1}dx + \frac{4}{3}\int\frac{1}{x^2+4/3}dx$$

$$= -\tan^{-1}x + \frac{4}{3}\cdot\frac{\sqrt{3}}{2}\tan^{-1}\left(\frac{x\sqrt{3}}{2}\right) + C$$

$$= \frac{2}{\sqrt{3}}\tan^{-1}\left(\frac{x\sqrt{3}}{2}\right) - \tan^{-1}x + C$$ **[2]**

OR

$$\text{Let } I = \int_{-2}^{1}\sqrt{5-4x-x^2}\,dx$$

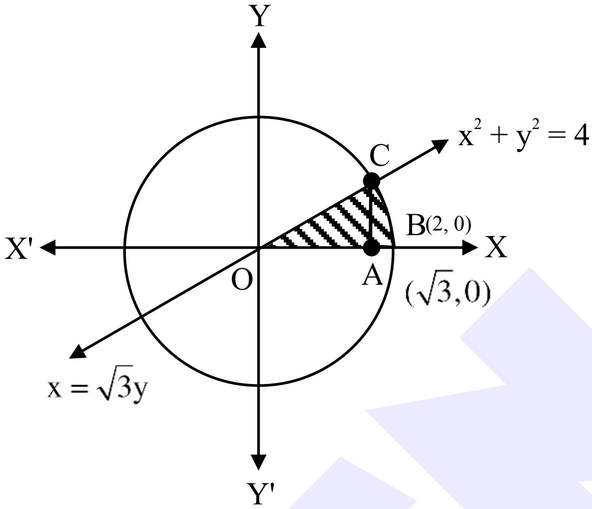
$$= \int_{-2}^{1}\sqrt{3^2-(x+2)^2}\,dx$$ **[1]**

$$= \left[\frac{x+2}{2}\sqrt{3^2-(x+2)^2} + \frac{9}{2}\sin^{-1}\left(\frac{x+2}{2}\right)\right]_{-2}^{1}$$ **[2]**

$$= \frac{9}{2}\sin^{-1}1 = \frac{9\pi}{4}$$ **[1]**

12. Given, curve $x^2 + y^2 = 4$ is a circle $x^2 + y^2 = 4$ is a circle with centre (0, 0) and radius 2. and line is $x = \sqrt{3}y$ Now, point of intersection of line & circle is

$3y^2 + y^2 = 4$



$\Rightarrow 4y^2 = 4$ **[1]**

$\Rightarrow y = \pm 1$

for $y = 1$; $x = \sqrt{3}$

So, point C is $(\sqrt{3},1)$ **[1]**

Required area = area of OACO + Area of ABCA

$$= \int_0^{\sqrt{3}} y_{(\text{line})}dx + \int_{\sqrt{3}}^{2} y_{(\text{circle})}dx$$

$$= \frac{1}{\sqrt{3}}\int_0^{\sqrt{3}} x\ dx + \int_{\sqrt{3}}^{2}\sqrt{4-x^2}\,dx$$

$$= \frac{1}{2\sqrt{3}}[x^2]_0^{\sqrt{3}} + \left[\frac{x}{2}\sqrt{4-x^2} + \frac{4}{2}\sin^{-1}\left(\frac{x}{2}\right)\right]_{\sqrt{3}}^{2}$$ **[1]**

$$= \frac{1}{2\sqrt{3}}[3] + \left\{2\sin^{-1}(1) - \frac{\sqrt{3}}{2} - 2\sin^{-1}\frac{\sqrt{3}}{2}\right\}$$

$$= \frac{\sqrt{3}}{2} + 2\times\frac{\pi}{2} - \frac{\sqrt{3}}{2} - 2\times\frac{\pi}{3}$$

$$= \frac{\pi}{3}\text{Sq.units}$$ **[1]**

13. Given, line $\vec{r} = 4\hat{i} + 2\hat{j} + 7\hat{k} + \lambda(3\hat{i} + 4\hat{j} + 2\hat{k})$ and plane $\vec{r}.(\hat{i} - \hat{j} + \hat{k}) = 10 \Rightarrow x - y + z = 10$

Equation of line in Cartesion form

$$\frac{x-4}{3} = \frac{y-2}{4} = \frac{z-7}{2} = t \text{ (say)}$$

$x = 3t + 4,\ y = 4t + 2,\ z = 2t + 7$ **[1]**

Satisfy in equation of plane $x - y + z = 10$

$\Rightarrow (3t + 4) - (4t + 2) + (2t + 7) = 10$

$\Rightarrow t + 9 = 10$

$\Rightarrow t = 1$

So, Point of intersection is (7, 6, 9) **[1]**

Now, distance of point (7, 6, 9) & (1, –2, 9) is

$$d = \sqrt{(7-1)^2 + (6+2)^2 + (9-9)^2}$$

$$d = \sqrt{36+64+0} = \sqrt{100}$$

d = 10 units **[2]**

14. Given, A1 : A2 : A3 = 4 : 4 : 2

$P(A1) = \frac{4}{10},\ P(A2) = \frac{4}{10}$ and $P(A3) = \frac{2}{10}$

Let E be the event that a seed germinates

$\therefore\ P(E/A1) = \frac{45}{100}, P\left(\frac{E}{A2}\right) = \frac{60}{100}$ and $P(E/A3) = \frac{35}{100}$ **[1]**

(a) P(E) = P(A1) · P(E/A1) + P(A2) ·P(E/A2) + P(A3) · P(E/A3)

$$= \frac{4}{10}\cdot\frac{45}{100} + \frac{4}{10}\cdot\frac{60}{100} + \frac{2}{10}\cdot\frac{35}{100} = \frac{490}{1000} = \frac{49}{100} = 0.49$$ **[1½]**

(b) $$P(A2/E) = \frac{P(A2)\cdot P(E/A2)}{P(A1)\cdot P(E/A1)+P(A2)\cdot P(E/A2)+P(A3)\cdot P(E/A3)}$$

$$= \frac{4}{10}\cdot\frac{60}{100} = \frac{24}{49} = 0.48$$ **[1½]**