# CBSE QUESTION PAPER SOLUTION – 2022 (65/3/1)

## SUBJECT: MATHEMATICS

## TERM-II

## SOLUTION

### SECTION - A

**1.** Given, distance of the point (1,1,1) from the plane $x - y + z + \lambda = 0$ is $\frac{5}{\sqrt{3}}$ i.e. $\left|\frac{1-1+1+\lambda}{\sqrt{3}}\right| = \frac{5}{\sqrt{3}}$ **[1]**

$\Rightarrow \left|\frac{1+\lambda}{\sqrt{3}}\right| = \frac{5}{\sqrt{3}}$

or $\frac{1+\lambda}{\sqrt{3}} = \pm\frac{5}{\sqrt{3}}$

$\Rightarrow 1 + \lambda = \pm 5$

$\therefore \lambda = 4, -6$ **[1]**

**2.** Given, $\vec{a} = 2\hat{i} - 2\hat{j} + \hat{k}, \vec{b} = \hat{i} + 2\hat{j} - 2\hat{k}$ and $\vec{c} = 2\hat{i} - \hat{j} + 4\hat{k}$

$\Rightarrow \vec{b} + \vec{c} = 3\hat{i} + \hat{j} + 2\hat{k}$

$\therefore$ Projection of $(\vec{b} + \vec{c})$ on $\vec{a}$ = $(\vec{b} + \vec{c}) \cdot \hat{a}$ **[1]**

$= (3\hat{i} + \hat{j} + 2\hat{k}) \cdot \frac{(2\hat{i} - 2\hat{j} + \hat{k})}{3} = \frac{6}{3} = 2$ units **[1]**

**3.** Given differential equation is $\frac{dy}{dx} = \frac{3e^{2x} + 3e^{4x}}{e^{x} + e^{-x}}$

$\Rightarrow \frac{dy}{dx} = \frac{3e^{2x}(1+e^{2x})}{e^{-x}(1+e^{2x})}$

$\Rightarrow \frac{dy}{dx} = 3e^{3x}$ **[1]**

$\Rightarrow \int 1 \cdot dy = \int 3e^{3x}\, dx$

$\Rightarrow y = 3\left(\frac{e^{3x}}{3}\right) + C$

$\Rightarrow y = e^{3x} + C$ **[1]**

**4.** Let random variable X denotes the number of spade cards ; then the possible values of X are 0, 1 or 2.

P(X = 0) = P(no spade & no spade) = $\frac{39}{52}\times\frac{39}{52}=\frac{3}{4}\times\frac{3}{4}=\frac{9}{16}$

P(X = 1) = P(spade & no spade or no spade & spade) $=\left(\frac{13}{52}\times\frac{39}{52}\right)+\left(\frac{39}{52}\times\frac{13}{52}\right)$

$=\left(\frac{1}{4}\times\frac{3}{4}\right)+\left(\frac{3}{4}\times\frac{1}{4}\right)=\frac{3}{8}$

P(X = 2) = P(spade & spade) = $\left(\frac{1}{4}\times\frac{1}{4}\right)=\frac{1}{16}$ **[1½]**

Hence, probability distribution of X is

| X | 0 | 1 | 2 |
|---|---|---|---|
| P(X) | 9/16 | 6/16 | 1/16 |

**[½]**

**5.** $I=\int\frac{dx}{x^2-6x+13}=\int\frac{1}{(x-3)^2+2^2}dx$ **[1]**

$=\frac{1}{2}\tan^{-1}\left(\frac{x-3}{2}\right)+C\quad\left[\because\int\frac{1}{x^2+a^2}dx=\frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right)+C\right]$ **[1]**

**6.** When a pair of dice is thrown, the sample space S contains 36 outcomes.

Let E : Event that number 5 has appeared on at least one die,

F : Event that sum of the numbers on the dice is 7

E = {(5,1), (5,2), (5,3), (5,4), (5,5), (5,6), (1,5), (2,5), (3,5), (4,5), (6,5)}

F = {(1,6), (2,5), (3,4), (4,3), (5,2), (6,1)}

$E\cap F$ = {(5, 2), (2,5)}

$\Rightarrow P(E\cap F)=\frac{2}{36}, P(F)=\frac{6}{36}$ **[1]**

Required probability = P(E/F) = $\frac{P(E\cap F)}{P(F)}=\frac{(2/36)}{(6/36)}=\frac{1}{3}$ **[1]**

**OR**

Let

$E_1$ : Event that A hits the target

$E_2$ : Event that B hits the target

Given ; $P(E_1) = \frac{1}{3}$, $P(E_2) = \frac{2}{5}$ $\Rightarrow P\left(\overline{E_1}\right)=\frac{2}{3}, P\left(\overline{E_2}\right)=\frac{3}{5}$ **[½]**

Required Probability $= P(\text{target is hit})$

$= 1 - P\left(\overline{E}_1\overline{E}_2\right)$

$= 1 - P\left(\overline{E}_1\right)\cdot P\left(\overline{E}_2\right)$

$= 1-\left(\frac{2}{3}\times\frac{3}{5}\right)=1-\frac{2}{5}=\frac{3}{5}$ **[1½]**

**7.** Let $I=\int_0^{2\pi}\frac{1}{1+e^{\sin x}}dx$ ....(1)

$\Rightarrow I=\int_0^{2\pi}\frac{1}{1+e^{\sin(2\pi-x)}}dx$ $\left[\because \int_0^a f(x)dx=\int_0^a f(a-x)dx\right]$

$\Rightarrow I=\int_0^{2\pi}\frac{1}{1+e^{-\sin x}}dx$

$\Rightarrow I=\int_0^{2\pi}\frac{e^{\sin x}}{e^{\sin x}+1}dx$ ....(2) **[1]**

From (1) & (2), we get:

$\Rightarrow 2I=\int_0^{2\pi}\frac{1}{1+e^{\sin x}}dx+\int_0^{2\pi}\frac{e^{\sin x}}{e^{\sin x}+1}dx$

$\Rightarrow 2I=\int_0^{2\pi}1\cdot dx$ **[1]**

$\Rightarrow 2I=[x]_0^{2\pi}$

$\Rightarrow 2I = 2\pi$

$\Rightarrow I = \pi$ **[1]**

**8.** Given differential equation is $x\frac{dy}{dx}-y=x^2\cdot e^x$

$\Rightarrow \frac{dy}{dx}-\frac{y}{x}=x\cdot e^x$

This is linear differential equation of the form $\frac{dy}{dx}+Py=Q$

where $P=-\frac{1}{x}$ and $Q = x\cdot e^x$

$\therefore$ I.F. $= e^{\int -\frac{1}{x}dx} = e^{-\log x} = \frac{1}{x}$ **[1]**

Hence, the general solution is given by $y\left(\frac{1}{x}\right) = \int x \cdot e^x \left(\frac{1}{x}\right)dx$

$\Rightarrow \quad \frac{y}{x} = \int e^x dx$

$\Rightarrow \quad \frac{y}{x} = e^x + c$ **[1]**

Substituting x = 1 and y = 0, we get:

$0 = e^1 + c \Rightarrow c = -e$

$\therefore$ Required particular solution is $\frac{y}{x} = e^x - e$. **[1]**

**OR**

Given differential equation is: $x\frac{dy}{dx} = y(\log y - \log x + 1)$

$\Rightarrow \frac{dy}{dx} = \frac{y}{x}\left(\log\frac{y}{x} + 1\right)$

Put y = vx $\Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}$

$\Rightarrow v + x\frac{dv}{dx} = v(\log v + 1)$ **[1]**

$\Rightarrow \frac{1}{v\log v}dv = \frac{dx}{x}$

Integrating both sides $\int \frac{1}{v\log v}dv = \int \frac{1}{x}dx$

$\Rightarrow$ log(log v) = log x + log c = log cx **[1]**

$\Rightarrow$ log v = cx

$\Rightarrow \log\frac{y}{x} = cx$, is the required general solution. **[1]**

**9.** Adjacent sides of a parallelogram are given as $\vec{a} = 2\hat{i} - 4\hat{j} + 5\hat{k}$ and $\vec{b} = \hat{i} - 2\hat{j} - 3\hat{k}$.

Then, the diagonal of a parallelogram is given by $\vec{a} + \vec{b}$.

$\vec{a} + \vec{b} = (2\hat{i} - 4\hat{j} + 5\hat{k}) + (\hat{i} - 2\hat{j} - 3\hat{k}) = 3\hat{i} - 6\hat{j} + 2\hat{k}$

Thus, the unit vector parallel to the diagonal is $\frac{\vec{a} + \vec{b}}{|\vec{a} + \vec{b}|} = \frac{3\hat{i} - 6\hat{j} + 2\hat{k}}{\sqrt{3^2 + (-6)^2 + 2^2}} = \frac{3}{7}\hat{i} - \frac{6}{7}\hat{j} + \frac{2}{7}\hat{k}$ **[1]**

Also, Area of parallelogram = $|\vec{a}\times\vec{b}|$

$$\because\ \vec{a}\times\vec{b}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 2 & -4 & 5\\ 1 & -2 & -3\end{vmatrix}=\hat{i}(12+10)-\hat{j}(-6-5)+\hat{k}(-4+4)$$

$\Rightarrow\ \vec{a}\times\vec{b}=22\hat{i}+11\hat{j}$ **[1]**

$\therefore\ |\vec{a}\times\vec{b}|=\sqrt{22^2+11^2}=11\sqrt{5}$

Hence area of the parallelogram is $11\sqrt{5}$ square units. **[1]**

**OR**

Given, $\vec{a}=2\hat{i}+2\hat{j}+3\hat{k}$, $\vec{b}=-\hat{i}+2\hat{j}+\hat{k}$ and $\vec{c}=3\hat{i}+\hat{j}$.

Given, $\vec{a}+\lambda\vec{b}$ is perpendicular to $\vec{c}$.

$\Rightarrow\ (\vec{a}+\lambda\vec{b})\cdot\vec{c}=0$

$\Rightarrow\ \vec{a}\cdot\vec{c}+\lambda(\vec{b}\cdot\vec{c})=0$

$\Rightarrow\ \lambda=-\dfrac{\vec{a}\cdot\vec{c}}{\vec{b}\cdot\vec{c}}$ **[1]**

$\Rightarrow\ \lambda=-\dfrac{(2\hat{i}+2\hat{j}+3\hat{k})\cdot(3\hat{i}+\hat{j})}{(-\hat{i}+2\hat{j}+\hat{k})\cdot(3\hat{i}+\hat{j})}$ **[1]**

$\Rightarrow\ \lambda=-\dfrac{6+2+0}{-3+2+0}=8$ **[1]**

**10.** Given equation of lines are $\dfrac{1-x}{2}=\dfrac{y-3}{4}=\dfrac{z}{-1}$ and $\dfrac{x-4}{3}=\dfrac{2y-2}{-4}=z-1$

In vector form both the lines can be expressed as

$\vec{r}=(\hat{i}+3\hat{j})+\lambda(-2\hat{i}+4\hat{j}-\hat{k})$ and $\vec{r}=(4\hat{i}+\hat{j}+\hat{k})+\mu(3\hat{i}-2\hat{j}+\hat{k})$

On comparing it with $\vec{r}=\vec{a}_1+\lambda\vec{b}_1$ and $\vec{r}=\vec{a}_2+\mu\vec{b}_2$

$\vec{a}_1=\hat{i}+3\hat{j}$, $\vec{b}_1=-2\hat{i}+4\hat{j}-\hat{k}$

$\vec{a}_2=4\hat{i}+\hat{j}+\hat{k}$ and $\vec{b}_2=3\hat{i}-2\hat{j}+\hat{k}$ **[1]**

The given lines are coplanar if $(\vec{a}_2-\vec{a}_1)\cdot(\vec{b}_1\times\vec{b}_2)=0$

$\vec{a}_2-\vec{a}_1=(4\hat{i}+\hat{j}+\hat{k})-(\hat{i}+3\hat{j})=3\hat{i}-2\hat{j}+\hat{k}$

$$\vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -2 & 4 & -1 \\ 3 & -2 & 1 \end{vmatrix} = \hat{i}(4-2) - \hat{j}(-2+3) + \hat{k}(4-12)$$

$= 2\hat{i} - \hat{j} - 8\hat{k}$ **[1]**

Now, $(\vec{a}_2 - \vec{a}_1)\cdot(\vec{b}_1 \times \vec{b}_2) = (3\hat{i} - 2\hat{j} + \hat{k})\cdot(2\hat{i} - \hat{j} - 8\hat{k})$ = 6 + 2 – 8 = 0

Hence, both the lines are coplanar. **[1]**

**11.** Given curve $4x^2 = y$ _____(1)

and line y = 8x + 12 _____(2)

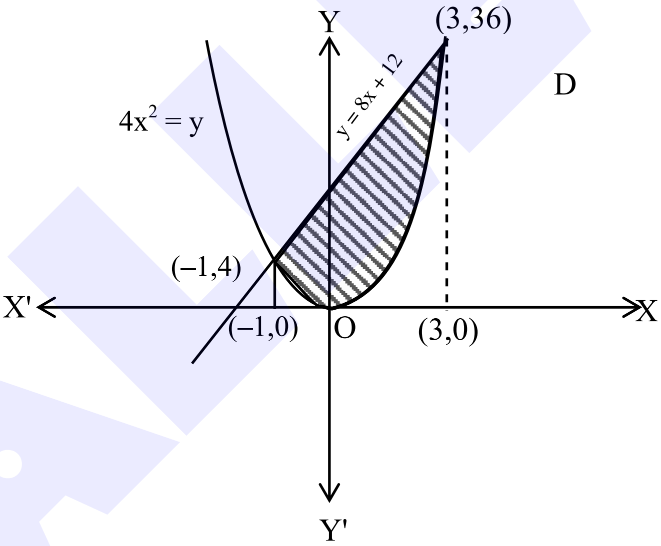
From eq. (1) & (2)

$4x^2 - 8x - 12 = 0$

$\Rightarrow x^2 - 2x - 3 = 0$

$\Rightarrow (x - 3)(x + 1) = 0$

$\Rightarrow x = 3, -1$ **[1]**




**[1]**

From equation (3) when x = 3 ⇒ y = 36 & x = –1 ⇒ y = 4.

So, Point of intersection of curve & line are (3,36) and (–1,4).

Req. Area = $\int_{-1}^{3} \left\{(8x+12) - 4x^2\right\} dx = \left[\frac{8x^2}{2} + 12x - \frac{4x^3}{3}\right]_{-1}^{3}$ **[1]**

$= (36 + 36 - 36) - \left(4 - 12 + \frac{4}{3}\right) = 36 + \frac{20}{3} = \frac{128}{3}$ sq. unit. **[1]**

**12.** $I=\int \frac{x^2}{(x^2+1)(3x^2+4)}dx$

Let $x^2 = y$

Let $\frac{x^2}{(x^2+1)(3x^2+4)}=\frac{y}{(y+1)(3y+4)}=\frac{A}{(y+1)}+\frac{B}{(3y+4)}$

$\Rightarrow \quad y=A(3y+4)+B(y+1) \qquad$ ______(1)

from equation (1)

Put $y = -1 \Rightarrow A = -1$

Put $y = -4/3 \Rightarrow B = 4$ **[1]**

$\therefore \quad \frac{y}{(y+1)(3y+4)}=\frac{-1}{y+1}+\frac{4}{3y+4}$

i.e. $\frac{x^2}{(x^2+1)(3x^2+4)}=\frac{-1}{x^2+1}+\frac{4}{3x^2+4}$ **[1]**

$\therefore \ I=\int\left(\frac{-1}{x^2+1}+\frac{4}{3x^2+4}\right)dx$

$=-\int\frac{1}{x^2+1}dx+\frac{4}{3}\int\frac{1}{x^2+4/3}dx$ **[1]**

$=-\tan^{-1}x+\frac{4}{3}\cdot\frac{\sqrt{3}}{2}\tan^{-1}\left(\frac{x\sqrt{3}}{2}\right)+C$

$=\frac{2}{\sqrt{3}}\tan^{-1}\left(\frac{x\sqrt{3}}{2}\right)-\tan^{-1}x+C$ **[2]**

**OR**

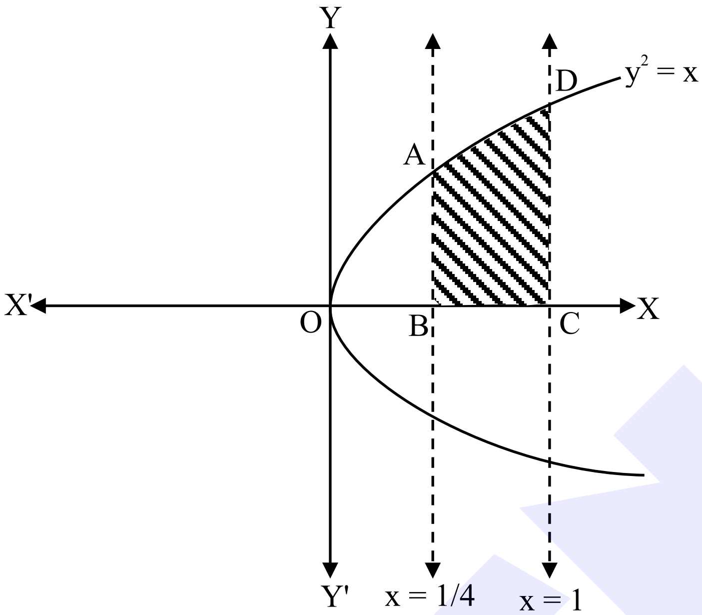
Let $I=\int_{-2}^{1}\sqrt{5-4x-x^2}\,dx$

$=\int_{-2}^{1}\sqrt{3^2-(x+2)^2}\,dx$ [1]

$=\left[\frac{x+2}{2}\sqrt{3^2-(x+2)^2}+\frac{9}{2}\sin^{-1}\left(\frac{x+2}{2}\right)\right]_{-2}^{1}$ **[2]**

$=\frac{9}{2}\sin^{-1}1=\frac{9\pi}{4}$ **[1]**

**13.** The area of the region bounded by the curve $y^2 = x$, $x = \frac{1}{4}, y = 0$ and $x = 1$ is the area ABCD




**[1]**

$\therefore$ Area of ABCD = $\int_{1/4}^{1} y\,dx$ **[1]**

$= \int_{1/4}^{1} \sqrt{x}\,dx = \left[\frac{2}{3}x^{3/2}\right]_{1/4}^{1}$ **[1]**

$= \frac{2}{3}\left[1-\left(\frac{1}{4}\right)^{3/2}\right] = \frac{2}{3}\left[1-\frac{1}{8}\right] = \frac{7}{12}$ sq units. **[1]**

**14.** Given, A1 : A2 : A3 = 4 : 4 : 2

$P(A1) = \frac{4}{10}$, $P(A2) = \frac{4}{10}$ and $P(A3) = \frac{2}{10}$

Let E be the event that a seed germinates

$\therefore$ $P(E/A1) = \frac{45}{100}, P\left(\frac{E}{A2}\right) = \frac{60}{100}$ and $P(E/A3) = \frac{35}{100}$ **[1]**

(a) $P(E) = P(A1) \cdot P(E/A1) + P(A2) \cdot P(E/A2) + P(A3) \cdot P(E/A3)$

$= \frac{4}{10}\cdot\frac{45}{100} + \frac{4}{10}\cdot\frac{60}{100} + \frac{2}{10}\cdot\frac{35}{100} = \frac{490}{1000} = \frac{49}{100} = 0.49$ **[1½]**

(b) $P(A2/E) = \frac{P(A2)\cdot P(E/A2)}{P(A1)\cdot P(E/A1) + P(A2)\cdot P(E/A2) + P(A3)\cdot P(E/A3)}$

$= \frac{4}{10}\cdot\frac{60}{100} = \frac{24}{49} = 0.48$ **[1½]**