CLASS XII - CBSE

MATHEMATICS (SOLUTION)

SECTION – A

1. $\int f(t)dt = \frac{1}{2\sqrt{3}}\log\left|\frac{\sqrt{3}+\tan t}{\sqrt{3}-\tan t}\right|$

Taking derivative on both sides

$$\frac{d}{dt}\int f(t)dt = \frac{d}{dt}\left[\frac{1}{2\sqrt{3}}\log\left|\frac{\sqrt{3}+\tan t}{\sqrt{3}-\tan t}\right|\right]$$

$$\Rightarrow f(t) = \frac{1}{2\sqrt{3}}\frac{d}{dt}\left[\log\left(\sqrt{3}+\tan t\right)-\log\left(\sqrt{3}-\tan t\right)\right]$$

$$\Rightarrow f(t) = \frac{1}{2\sqrt{3}}\left[\frac{\sec^2 t}{\sqrt{3}+\tan t}+\frac{\sec^2 t}{\sqrt{3}-\tan t}\right]$$ **[1]**

$$\Rightarrow f(t) = \frac{\sec^2 t}{2\sqrt{3}}\left[\frac{\sqrt{3}-\tan t+\sqrt{3}+\tan t}{3-\tan^2 t}\right]$$

$$\Rightarrow f(t) = \frac{\sec^2 t}{2\sqrt{3}}\left[\frac{2\sqrt{3}}{3-\tan^2 t}\right]$$

$$\Rightarrow f(t) = \frac{\sec^2 t}{3-\tan^2 t}$$ **[1]**

OR

Let $I = \int \frac{dx}{1+3\sin^2 x}$

Multiply and divide by $\sec^2 x$ in Nr and Dr

$$I = \int \frac{\sec^2 x\,dx}{\sec^2 x+3\tan^2 x} = \int \frac{\sec^2 x\,dx}{1+4\tan^2 x} = \frac{1}{4}\int \frac{\sec^2 x\,dx}{\frac{1}{4}+\tan^2 x}$$ **[1]**

Put $t = \tan x \Rightarrow dt = \sec^2 x\,dx$, then it reduces to

$$\frac{1}{4}\int \frac{dt}{t^2+\left(\frac{1}{2}\right)^2} = \frac{1}{4}\left[2\tan^{-1}(2t)\right]+c$$

$$= \frac{1}{2}\tan^{-1}(2t)+c = \frac{1}{2}\tan^{-1}(2\tan x)+c.$$ **[1]**

2. $\dfrac{d^2y}{dx^2}-\sqrt{\dfrac{dy}{dx}-3}=\sin\left(\dfrac{dy}{dx}\right)\Rightarrow\dfrac{d^2y}{dx^2}-\sin\left(\dfrac{dy}{dx}\right)=\sqrt{\dfrac{dy}{dx}-3}$

Squaring both sides, we get $\left(\dfrac{d^2y}{dx^2}-\sin\left(\dfrac{dy}{dx}\right)\right)^2=\left(\dfrac{dy}{dx}-3\right)$

$\Rightarrow\left(\dfrac{d^2y}{dx^2}\right)^2+\sin^2\left(\dfrac{dy}{dx}\right)-2\sin\left(\dfrac{dy}{dx}\right)\dfrac{d^2y}{dx^2}=\dfrac{dy}{dx}-3$.

The highest order derivative present in equation is $\dfrac{d^2y}{dx^2}$. Order is 2.

Given differential equation is not polynomial in its derivative, So degree is not defined **[2]**

3. $\vec{u}=\hat{i}-2\hat{j}+3\hat{k}$ and $\vec{v}=3\hat{i}-2\hat{j}+\hat{k}$

$\therefore\quad \cos\theta=\dfrac{\vec{u}.\vec{v}}{|\vec{u}||\vec{v}|}$ **[½]**

$=\dfrac{\hat{i}-2\hat{j}+3\hat{k}\ .\ 3\hat{i}-2\hat{j}+\hat{k}}{\sqrt{(1)^2+(-2)^2+(3)^2}\times\sqrt{(3)^2+(-2)^2+(1)^2}}$ **[½]**

$=\dfrac{3+4+3}{\sqrt{14}.\sqrt{14}}=\dfrac{10}{14}=\dfrac{5}{7}$ **[½]**

Hence, $\cos\theta=\dfrac{5}{7}$

$\Rightarrow\quad \sin\theta=\sqrt{1-\cos^2\theta}=\sqrt{1-\dfrac{25}{49}}$

Hence; $\sin\theta=\sqrt{\dfrac{24}{49}}=\dfrac{2}{7}\sqrt{6}$ **[½]**

4. Given points are

A(1, 2, 3), B(–2, 4, 5), C (1, 5, p) and D(–2, 6, 1)

Equation of line joining two points (x_1, y_1, z_1) & (x_2, y_2, z_2) is given by

$\dfrac{x-x_1}{x_2-x_1}=\dfrac{y-y_1}{y_2-y_1}=\dfrac{z-z_1}{z_2-z_1}$

So, equation of line AB is

$\dfrac{x-1}{-3}=\dfrac{y-2}{2}=\dfrac{z-3}{2}$ **[½]**

And, equation of line CD is

$\dfrac{x-1}{-3}=\dfrac{y-5}{1}=\dfrac{z-p}{1-p}$ **[½]**

$\because$ lines AB and CD are perpendicular

$\therefore\ a_1a_2+b_1b_2+c_1c_2=0$

$\Rightarrow 9+2+2(1-p)=0$

$\Rightarrow 13-2p=0$

$\Rightarrow p=\dfrac{13}{2}$ **[1]**

5. The number of aces is a random variable. Let it be denoted by X, clearly, X can take the values 0, 1 or 2.

Now, since the draws are done with replacement, therefore the two draws form independent experiments.

Therefore, P(X = 0) = P(non-ace and non-ace)

= P(non-ace) × P(non-ace)

$= \frac{48}{52} \times \frac{48}{52} = \frac{144}{169}$

P(X = 1) = P(ace and non-ace or non-ace and ace)

= P(ace and non-ace) + P(non-ace and ace)

= P(ace) . P(non-ace) + P(non-ace) . P(ace)

$= \left(\frac{4}{52} \times \frac{48}{52}\right) + \left(\frac{48}{52} \times \frac{4}{52}\right) = \frac{24}{169}$ **[1]**

and P(X = 2) = P(ace and ace)

$= \frac{4}{52} \times \frac{4}{52} = \frac{1}{169}$

Thus, the required probability distribution is : **[1]**

X	0	1	2
P(X)	$\frac{144}{169}$	$\frac{24}{169}$	$\frac{1}{169}$

6. Here ; $P(A) = \frac{2}{5}$, $P(B) = \frac{1}{3}$, $P(C) = \frac{1}{2}$

Also $P(A \cap C) = \frac{1}{5}$ and $P(B \cap C) = \frac{1}{4}$

$\therefore$ $P(C/B) = \frac{P(C \cap B)}{P(B)} = \frac{(1/4)}{(1/3)} = \frac{3}{4}$ **[1]**

and $P(A' \cap C') = P(A \cup C)' = 1 - P(A \cup C)$

$= 1 - [P(A) + P(C) - P(A \cap C)]$

$= 1 - \left[\frac{2}{5} + \frac{1}{2} - \frac{1}{5}\right]$

$= 1 - \frac{7}{10} = \frac{3}{10}$ **[1]**

SECTION – B

7. $f(x)=\int \frac{x^2dx}{(1+x^2)\left(1+\sqrt{1+x^2}\right)}$

Let $x=\tan\theta, dx=\sec^2\theta d\theta = (1+x^2).d\theta$

$f(x)=\int \frac{x^2dx}{(1+x^2)\left(1+\sqrt{1+x^2}\right)}=\int \frac{\tan^2\theta\sec^2\theta d\theta}{\sec^2\theta(1+\sec\theta)}$ **[1]**

$=\int \frac{\tan^2\theta\, d\theta}{1+\sec\theta}=\int \frac{\sin^2\theta d\theta}{\cos\theta(1+\cos\theta)}=\int \frac{\left(1-\cos^2\theta\right)d\theta}{\cos\theta(1+\cos\theta)}$

$=\int \frac{(1-\cos\theta)d\theta}{\cos\theta} = \int \sec\theta d\theta - \int 1 d\theta$

$= \log|\tan\theta + \sec\theta| - \theta + c$

$=\log(x+\sqrt{1+x^2})-\tan^{-1}x+c$ **[1]**

$f(0)=\log(0+\sqrt{1+0})-\tan^{-1}(0)+c$

$0=\log 1-0+c \Rightarrow c=0$

$f(1)=\log(1+\sqrt{1+1^2})-\tan^{-1}(1)=\log(1+\sqrt{2})-\frac{\pi}{4}.$ **[1]**

8. $\frac{dy}{dx}=\frac{x}{2y-x}$. Put $y = vx \Rightarrow v + x\frac{dv}{dx}=\frac{dy}{dx}$

$v+x\frac{dv}{dx}=\frac{x}{2vx-x}=\frac{1}{2v-1}$ **[1]**

$x\frac{dv}{dx}=\frac{1}{2v-1}-v=\frac{1-2v^2+v}{2v-1}=-\frac{(v-1)(2v+1)}{2v-1}$

$\frac{(2v-1)\,dv}{(2v+1)(v-1)}=\frac{-dx}{x}$ ……(i)

Let $\frac{2v-1}{(2v+1)(v-1)}=\frac{A}{2v+1}+\frac{B}{v-1}$ ……(ii)

$2v - 1 = A(v - 1) + B(2v + 1)$

For $v = 1$, $B = \frac{1}{3}$

For $v = -\frac{1}{2}$, $A = \frac{4}{3}$ [put the value of A and B in equation (ii)]

$\frac{2v-1}{(2v+1)(v-1)}=\frac{4}{3(2v+1)}+\frac{1}{3(v-1)}$ [Put in eqution (i)]

$\frac{4}{3(2v+1)}+\frac{1}{3(v-1)}=\frac{-dx}{x}$

$\frac{4}{3}\int\frac{dv}{2v+1}+\frac{1}{3}\int\frac{dv}{v-1}=-\int\frac{dx}{x}$ Integrating both sides :

$\frac{4}{3}.\frac{1}{2}\log(2v+1)+\frac{1}{3}\log(v-1)=\log\frac{1}{x}+\log c$

$\log(2v+1)^{2/3}+\log(v-1)^{1/3}=\log\frac{c}{x}$ **[1]**

$\Rightarrow (v-1)^{1/3}(2v+1)^{2/3}=\frac{c}{x}$

$\left(\frac{y-x}{x}\right)\left(\frac{2y+x}{x}\right)^2=\frac{c^3}{x^3}$

$\Rightarrow (x-y)(x+2y)^2=C$ [Where $C=-c^3$] **[1]**

OR

$\frac{dy}{dx}+\frac{2x}{1+x^2}y=\frac{4x^2}{1+x^2}$

It is linear equation of the form $\frac{dy}{dx}+Py=Q$

Here $P=\frac{2x}{1+x^2}$ and $Q=\frac{4x^2}{1+x^2}$

I.F. $=e^{\int\frac{2x}{1+x^2}dx}=e^{\log(1+x^2)}=(1+x^2)$ **[1]**

Therefore, solution is given by

$y.(1+x^2)=\int\frac{4x^2}{1+x^2}(1+x^2)dx+c=\frac{4x^3}{3}+c$. **[1]**

But it passes through (0,0) therefore $c=0$, hence the equation of curve is $3y(1+x^2)=4x^3$ **[1]**

9. Given, $\vec{r}$ and $\vec{s}$ are two vectors with

$|\vec{r}|=2$; $|\vec{s}|=3$

$|\vec{r}+\vec{s}|=\sqrt{19}$

$\Rightarrow |\vec{r}+\vec{s}|^2=19$ (on squaring both sides)

$(\vec{r}+\vec{s}).(\vec{r}+\vec{s})=19$ $(\because |\vec{a}|^2=\vec{a}.\vec{a})$

$\vec{r}.\vec{r}+\vec{r}.\vec{s}+\vec{s}.\vec{r}+\vec{s}.\vec{s}=19$

$\Rightarrow |\vec{r}|^2+|\vec{s}|^2+2\vec{r}.\vec{s}=19$

$\Rightarrow 4+9+2\vec{r}.\vec{s}=19$

$\Rightarrow \vec{r}.\vec{s}=3$ **[1]**

(i) $(4\vec{r}-\vec{s}).(2\vec{r}+\vec{s})=8|\vec{r}|^2+4\vec{r}.\vec{s}-2\vec{r}.\vec{s}-|\vec{s}|^2$

$=8\times4+2\times3-9=29$ **[1]**

(ii) $\cos\theta=\frac{\vec{r}.\vec{s}}{|\vec{r}||\vec{s}|}=\frac{3}{2\times3}$

$\Rightarrow \cos\theta=\frac{1}{2}\Rightarrow\theta=60°$ **[1]**

10. Equation of plane passing through the point (1,0,–1) is,

$$a(x-1)+b(y-0)+c(z+1)=0 \qquad \ldots\ldots(i)$$

Also, plane (i) is passing through (3, 2, 2) **[1]**

$\therefore \quad a(3-1)+b(2-0)+c(2+1)=0$

or $\quad 2a+2b+3c=0 \qquad \ldots..(ii)$

Plane (i) is also parallel to the line $\frac{x-1}{2}=\frac{y-1}{-2}=\frac{z-2}{3}$

$\therefore \quad 2a-2b+3c=0 \qquad \ldots\ldots(iii)$

From (ii) and (iii), $\frac{a}{-3}=\frac{b}{0}=\frac{c}{2}$ **[1]**

Therefore, the required plane is,

$$-3(x-1)+0(y-0)+2(z+1)=0$$

or $\quad -3x+2z+5=0$. **[1]**

OR

Comparing (ℓ_1) and (ℓ_2) with $\vec{r}=\vec{a}_1+\lambda\vec{b}_1$ and $\vec{r}=\vec{a}_2+\mu\vec{b}_2$ respectively.

We get $\quad \vec{a}_1=\hat{i}+\hat{j},\ \vec{b}_1=2\hat{i}-\hat{j}+\hat{k}$

$\vec{a}_2=2\hat{i}+\hat{j}-\hat{k},\ \vec{b}_2=3\hat{i}-5\hat{j}+2\hat{k}$

Therefore, $\quad \vec{a}_2-\vec{a}_1=\hat{i}-\hat{k}$ **[1]**

and $\quad \vec{b}_1\times\vec{b}_2=(2\hat{i}-\hat{j}+\hat{k})\times(3\hat{i}-5\hat{j}+2\hat{k})$

$$=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 2 & -1 & 1\\ 3 & -5 & 2\end{vmatrix}=3\hat{i}-\hat{j}-7\hat{k}$$ **[1]**

So, $\quad |\vec{b}_1\times\vec{b}_2|=\sqrt{9+1+49}=\sqrt{59}$

Hence, the shortest distance between the given lines is given by

$$d=\left|\frac{(\vec{b}_1\times\vec{b}_2).(\vec{a}_2-\vec{a}_1)}{|\vec{b}_1\times\vec{b}_2|}\right|=\frac{|3-0+7|}{\sqrt{59}}=\frac{10}{\sqrt{59}}\text{ units}$$ **[1]**

SECTION – C

11. $I = \int_0^{\infty} \log\left(x + \frac{1}{x}\right)\frac{1}{1+x^2}\,dx$

Put $x = \tan\theta \Rightarrow dx = \sec^2\theta\, d\theta$

At x = 0 , θ = 0 and **[½]**

At x = ∞ , $\theta = \frac{\pi}{2}$

$\Rightarrow I = \int_0^{\pi/2} \log(\tan\theta + \cot\theta)\frac{\sec^2\theta}{\sec^2\theta}d\theta$

$\Rightarrow\ I = \int_0^{\pi/2} \log(\tan\theta + \cot\theta)d\theta$ **[½]**

$\Rightarrow I = \int_0^{\pi/2} \log\frac{(1+\tan^2\theta)}{\tan\theta}d\theta$

$\Rightarrow I = 2\int_0^{\pi/2} \log\sec\theta\, d\theta - \int_0^{\pi/2}\log\tan\theta\, d\theta$ **[1]**

$\Rightarrow\ I = 2\int_0^{\pi/2}\log\sec\theta\ d\theta \quad \left\{\because \int_0^{\pi/2}\log\tan\theta = 0\right\}$

$\Rightarrow I = -2\int_0^{\pi/2} \log\cos\theta\, d\theta$ **[1]**

$\Rightarrow I = -2\times\frac{-\pi}{2}\log 2 \quad \left\{\because \int_0^{\pi/2}\log\cos\theta = -\frac{\pi}{2}\log 2\right\}$

$\Rightarrow\ I = \pi\log 2$ **[1]**

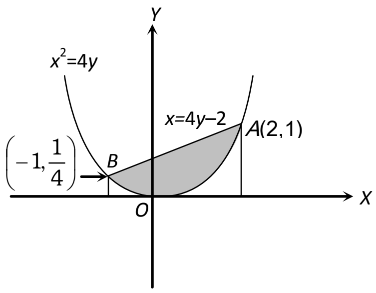
12. Solving the equations $x^2 = 4y$ and $x = 4y - 2$ simultaneously. The points of intersection of the parabola and the line

$x^2 = x + 2 \Rightarrow x^2 - x - 2 = 0$

$\Rightarrow (x + 1)(x - 2) = 0$ or $x = 2, -1$ **[1]**

at x = 2, y = 1 and x = –1, $y = \frac{1}{4}$

Now point A(2, 1) and $B\left(-1, \frac{1}{4}\right)$



∴ The required area = shaded area

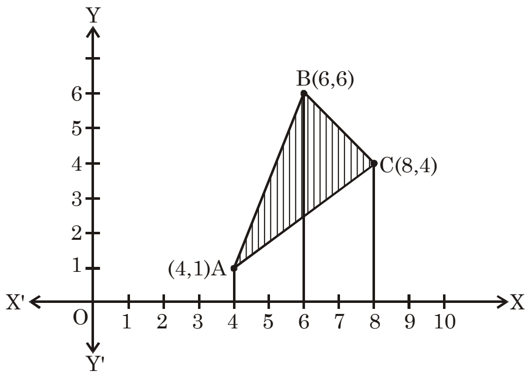
$= \left[\int_{-1}^{2} y\ dx\right] - \left[\int_{-1}^{2} y\ dx\right]$ **[1½]**

$= \int_{-1}^{2}\frac{1}{4}(x+2)dx - \int_{-1}^{2}\frac{1}{4}x^2dx$

$= \frac{1}{4}\left[\frac{x^2}{2} + 2x\right]_{-1}^{2} - \frac{1}{4}\left[\frac{x^3}{3}\right]_{-1}^{2}$

$= \frac{9}{8}$ sq. units **[1½]**

OR



[1]

Equation of line AB $\Rightarrow$ $y - 1 = \dfrac{5}{2}(x-4)$

$\Rightarrow$ $5x - 2y - 18 = 0$

$\Rightarrow$ $y = \dfrac{1}{2}(5x-18)$

Equation of line BC $\Rightarrow$ $y - 6 = (-1)(x - 6)$

$\Rightarrow$ $y = 12 - x$

Equation of line AC $\Rightarrow$ $y - 1 = \dfrac{3}{4}(x-4)$

$\Rightarrow$ $y = \dfrac{1}{4}(3x-8)$

Area of $\Delta = \left[\int_4^6 (\text{line AB})dx + \int_6^8 (\text{line BC})dx\right] - \int_4^8 (\text{line AC})\, dx$ **[1]**

$$= \left[\frac{1}{2}\int_4^6 (5x-18)dx + \int_6^8 (12-x)dx\right] - \frac{1}{4}\int_4^8 (3x-8)dx$$ **[½]**

$$= \left[\frac{1}{2}\left(\frac{5x^2}{2} - 18x\right)_4^6 + \left(12x - \frac{x^2}{2}\right)_6^8\right] - \frac{1}{4}\left(\frac{3x^2}{2} - 8x\right)_4^8$$ **[1]**

$$= \frac{1}{2}\left(5\times\frac{6^2}{2} - 18\times 6 - 5\times\frac{4^2}{2} + 18\times 4\right) + \left(12\times 8 - \frac{8^2}{2} - 12\times 6 + \frac{6^2}{2}\right)$$

$$-\frac{1}{4}\left[3\times\frac{8^2}{2} - 8\times 8 - 3\times\frac{4^2}{2} + 8\times 4\right]$$

$$= \frac{1}{2}[90 - 108 - 40 + 72] + [96 - 32 - 72 + 18] - \frac{1}{4}[96 - 64 - 24 + 32]$$

$$= \left(\frac{1}{2}\times 14\right) + 10 - \left(\frac{1}{4}\times 40\right) = 7 + 10 - 10 = 7 \text{ sq. units}$$ **[½]**

13. Given point (0, 7, –7)

Equation of a line $\vec{r} = (-\hat{i} + 3\hat{j} - 2\hat{k}) + \lambda(-3\hat{i} + 2\hat{j} + \hat{k})$

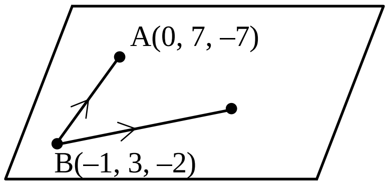
Equation of a plane passing through

the point is $(\vec{r} - \vec{a}) \cdot \vec{n} = 0$

$\left[\vec{r} - (0\hat{i} + 7\hat{j} - 7\hat{k})\right] \cdot \vec{n} = 0$ ....(1) **[½]**

If plane containing the line

$\therefore$ Point B(–1, 3, –2) in a plane



So $\overrightarrow{BA} = \hat{i} + 4\hat{j} - 5\hat{k}$

Parallel vector of given line is $\vec{b} = -3\hat{i} + 2\hat{j} + \hat{k}$

Normal vector of a plane $\vec{n} = \overrightarrow{BA} \times \vec{b}$

$$\vec{n} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 4 & -5 \\ -3 & 2 & 1 \end{vmatrix}$$

$\vec{n} = 14\hat{i} + 14\hat{j} + 14\hat{k}$ **[2½]**

Put in equation (1)

$\left[\vec{r} - (7\hat{j} - 7\hat{k})\right] \cdot (14\hat{i} + 14\hat{j} + 14\hat{k}) = 0$

$\vec{r} \cdot (14\hat{i} + 14\hat{j} + 14\hat{k}) - 7(14 - 14) = 0$

$\vec{r} \cdot (14\hat{i} + 14\hat{j} + 14\hat{k}) = 0$ **[1]**

$\vec{r} \cdot (\hat{i} + \hat{j} + \hat{k}) = 0$

CASE BASED / DATA BASED

14. (i) Let E_1, E_2 and E_3 be the events of selecting flower seeds A_1, A_2 and A_3 respectively and Let E be the event that the seed will germinate.

Then ; $P(E_1) = \frac{4}{10}, P(E_2) = \frac{4}{10}, P(E_3) = \frac{2}{10}$

Also ; $P\left(\frac{E}{E_1}\right) = \frac{45}{100}, P\left(\frac{E}{E_2}\right) = \frac{60}{100}, P\left(\frac{E}{E_3}\right) = \frac{35}{100}$ **[½]**

Now ; $P(E) = P(E_1) \cdot P(E/E_1) + P(E_2) \cdot P(E/E_2) + P(E_3) \cdot P(E/E_3)$ **[½]**

$$= \left(\frac{4}{10}\times\frac{45}{100}\right)+\left(\frac{4}{10}\times\frac{60}{100}\right)+\left(\frac{2}{10}\times\frac{35}{100}\right)$$

$$= \frac{180}{1000}+\frac{240}{1000}+\frac{70}{1000}=\frac{490}{1000}=0.49$$ **[1]**

(ii) Here ; $P\left(\frac{\bar{E}}{E_1}\right)=1-P\left(\frac{E}{E_1}\right)$

$$= 1-\frac{45}{100}=\frac{55}{100}\ ;$$

$$P\left(\frac{\bar{E}}{E_2}\right)=1-P\left(\frac{E}{E_2}\right)$$

$$= 1-\frac{60}{100}=\frac{40}{100}\ ;$$

$$P\left(\frac{\bar{E}}{E_3}\right)=1-P\left(\frac{E}{E_3}\right)$$

$$= 1-\frac{35}{100}=\frac{65}{100}$$ **[½]**

$$\Rightarrow \quad P\left(\frac{E_2}{\bar{E}}\right)=\frac{P(E_2).P\left(\frac{\bar{E}}{E_2}\right)}{P(E_1).P\left(\frac{\bar{E}}{E_1}\right)+P(E_2).P\left(\frac{\bar{E}}{E_2}\right)+P(E_3).P\left(\frac{\bar{E}}{E_3}\right)}$$ **[1]**

$$= \frac{\left(\frac{4}{10}\times\frac{40}{100}\right)}{\left(\frac{4}{10}\times\frac{55}{100}\right)+\left(\frac{4}{10}\times\frac{40}{100}\right)+\left(\frac{2}{10}\times\frac{65}{100}\right)}$$

$$= \frac{80}{110+80+65}=\frac{80}{255}=\frac{16}{51}$$ **[½]**