ALLEN CAREER INSTITUTE KOTA (RAJASTHAN)

CLASSROOM CONTACT PROGRAMME
(Academic Session : 2021 - 2022)

JEE (Main)
SAMPLE TEST
PAPER # 03

JEE(Main) : Leader Course

ANSWER KEY

PART-1 : PHYSICS

SECTION-I	Q.	1	2	3	4	5	6	7	8	9	10
	A.	C	B	A	A	B	C	A	D	B	B
	Q.	11	12	13	14	15	16	17	18	19	20
	A.	A	D	B	C	A	A	D	D	A	B
SECTION-II	Q.	1	2	3	4	5	6	7	8	9	10
	A.	0.40	22.00	9.00	8.00	12.00	0.06	0.56	900.00	8.00	12.53

PART-2 : CHEMISTRY

SECTION-I	Q.	1	2	3	4	5	6	7	8	9	10
	A.	C	C	C	A	B	D	D	C	B	B
	Q.	11	12	13	14	15	16	17	18	19	20
	A.	C	A	C	C	B	B	B	A	C	C
SECTION-II	Q.	1	2	3	4	5	6	7	8	9	10
	A.	7.00	4.00	6.00	2.00	3.75	50.00	272.00	5.00	4.00	4.00

PART-3 : MATHEMATICS

SECTION-I	Q.	1	2	3	4	5	6	7	8	9	10
	A.	C	D	A	A	A	B	A	D	B	C
	Q.	11	12	13	14	15	16	17	18	19	20
	A.	C	A	A	B	B	A	A	B	B	C
SECTION-II	Q.	1	2	3	4	5	6	7	8	9	10
	A.	107.00	8.00	-0.50	2.00	6.00	0.50	67.00	49.00	5.00	3.00

HINT – SHEET

PART-1 : PHYSICS

SECTION-I

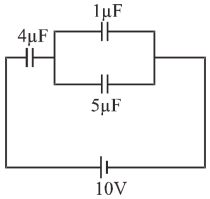
1. Ans (C)

$q = C_{eq}V$

$= \left(\frac{4\times 6}{4+6}\right) \times 10\mu C$

$= 24\mu C$

(In steady state, no current flow through the battery, so p.d. at 2Ω & 5Ω will be zero)



2. Ans (B)

$G = \frac{RS}{R-S}$

3. Ans (A)

$B_z \rightarrow$ always remain parallel to surface

$\varepsilon = \pi r^2 \omega$ By sin ωt

4. Ans (A)

L → Removed, $\tan\frac{\pi}{3} = \frac{X_C}{R}$

$X_C = R\sqrt{3}$

C → Removed,

$\therefore\ X_L = X_C$

$\therefore\ Z = R$

$\therefore\ \cos\phi = \frac{R}{Z} = 1$

5. Ans (B)

Total average energy density of electromagnetic wave is

$\upsilon_{av} = \varepsilon_0 E_{rms}^2$

$= 8.85 \times 10^{-12} \times (720)^2$

$= 4.58 \times 10^{-6}\ Jm^{-3}$

6. Ans (C)

$\sin^2 i = n_1^2 - n_2^2$

$\sin^2 60° = n_1^2 - 1$

$n_1 = \sqrt{\frac{7}{4}} = \frac{2.65}{2} = 1.32$

7. Ans (A)

$\because \frac{1}{f} = ({}_m\mu_\ell - 1)\left[\frac{1}{R_1} - \frac{1}{R_2}\right]$

$= \left[\frac{1.5}{1.75} - 1\right]\left[-\frac{1}{R} - \frac{1}{(+R)}\right]$

$R_1 = -R$ $\quad$ $R_2 = +R$

$\frac{1}{f} = \left(-\frac{1}{7}\right) \times \left(-\frac{2}{R}\right) = \frac{2}{7R} \Rightarrow F = 3.5\ R$

f positive so converging lens

8. Ans (D)

Width of central bright fringe

$= \frac{2\lambda D}{a} = \frac{2 \times 600 \times 10^{-9} \times 2}{1 \times 10^{-3}} m$

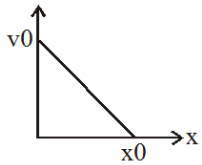
$= 2.4 \times 10^{-3}$

$= 2.4$ mm

10. Ans (B)

$v\frac{dv}{dx}$ is negative

$\frac{dv}{dx}$ is constant



But, v is decreasing with x

$\therefore \left| v\frac{dv}{dx} \right|$ is decreasing

17. Ans (D)

Using Einstein photoelectric equation,

$E - \phi = \frac{1}{2}mv^2$

$E_1 = \phi + \frac{1}{2}(2V)^2;\ E_2 = \phi + \frac{1}{2}(V)^2$

$\Rightarrow \frac{E_1 - \phi}{E_2 - \phi} = 4$

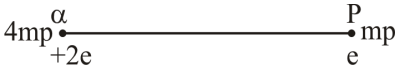
$\Rightarrow \frac{\left(\frac{hc}{\lambda_1} - \phi\right)}{\left(\frac{hc}{\lambda_2} - \phi\right)} = 4$

$\Rightarrow \phi = \left(\frac{hc}{3}\right)\left(\frac{4}{\lambda_2} - \frac{1}{\lambda_1}\right)$

$\Rightarrow \phi = \left(\frac{1240}{3}\right)\left(\frac{4}{540} - \frac{1}{350}\right)$

$\approx 1.8eV$

18. Ans (D)



$K_i + U_i = K_b + V_i$

$0 + \frac{K(2e)e}{r} = K_\alpha + K_p + 0$

$K_\alpha = \frac{2}{5}\frac{Ke^2}{r}$

$= \frac{2}{5} \times \frac{1}{4\pi\varepsilon_0}\frac{e^2}{r}$

$K_\alpha = \frac{e^2}{10\pi\varepsilon_0 r}$

19. Ans (A)

Work function of 'Y' is greater than 'X' and slope of V versus f is h/e

PART-1 : PHYSICS

SECTION-II

1. Ans (0.40)

$\Delta x_1 = \Delta x_2$

$t_1(\mu_1 - 1) = t_2(\mu_2 - 1)$

$1.2 \times (1.5 - 1) = t(2.5 - 1)$

$t = 0.4\ \mu m$

2. Ans (22.00)

The apparent frequency of the horn of police van as heard by Motor-cyclist is

$v' = \left(\frac{\upsilon - u_m}{\upsilon - 22}\right) 176\ Hz \quad \left(\because\ v' = \left(\frac{\upsilon - \upsilon_0}{\upsilon - \upsilon_s}\right) v\right)$

$\because \upsilon = 330$ m/s

$$v' = \frac{(330 - u_m)}{308} \times 176\ Hz \quad .....(i)$$

The apparent frequency of siren as heard by the cyclist must also be 'v' because the monocyclist does not observe any beats. Thus,

$$v' = \left(\frac{330 + u_m}{330}\right) \times 165 \quad .....(ii)$$

$\left(\because\ v' = \frac{\upsilon + \upsilon_0}{\upsilon} v\right)$

From eq, (i) and (ii), we get

$$\frac{330 - u_m}{308} \times 176 = \frac{330 + u_m}{330} \times 165$$

Solving it for u_m we get

$u_m = 22$ m/s

3. Ans (9.00)

Fill the empty space with $+\rho$ and $-\rho$ charge density.

$$|E_A| = 0 + \frac{k\rho . \frac{4}{3}\pi\left(\frac{R}{2}\right)^3}{\left(\frac{R}{2}\right)^2} = k\rho\frac{4}{3}\pi\left(\frac{R}{2}\right)$$

$$|E_B| = \frac{k\rho . \frac{4}{3}\pi R^3}{R^2} - \frac{k\rho . \frac{4}{3}\pi\left(\frac{R}{2}\right)^3}{\left(\frac{3R}{2}\right)^2}$$

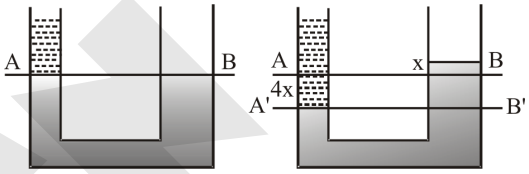
$$= k\rho\frac{4}{3}\pi R - k\rho\frac{4}{3}\pi\frac{R}{18} = k\rho . \frac{4}{3}\pi\left(\frac{17R}{18}\right)$$

$$\frac{E_A}{E_B} = \frac{9}{17} = \frac{18}{34}$$

7. Ans (0.56)

If the rise of level in the right limb be x cm. the fall of level of mercury in left limb be 4x cm because the area of cross section of right limb is 4 times as that of left limb.

$\therefore$ Level of water in left limb is (36 + 4x) cm.



Now equating pressure at interface of Hg and water (at A'B')

$(36 + 4x) \times 1 \times g = 5x \times 13.6 \times g$

By solving we get x = 0.56 cm.

9. Ans (8.00)

$$v_1 = \frac{(m_1 - m_2)}{(m_1 + m_2)} . u_1$$

$$= \frac{(m - 2m)}{m + 2m} . u_1 = -\frac{41}{3}$$

$$1 - \frac{k_1}{k_2} = 1 - \frac{\frac{1}{2}m\left(\frac{41}{3}\right)^2}{\frac{1}{2}mu_1^2} = \frac{8}{9}$$

PART-2 : CHEMISTRY

SECTION-I

3. Ans (C)

$$\underset{\text{impurity}}{FeO} + \underset{\text{flux}}{SiO_2} \rightarrow \underset{\text{slag}}{FeSiO_3}$$

8. Ans (C)

$BaSO_4 = BaCl_2 + H_2SO_4 - 2HCl$

$\therefore \quad \Lambda^{\circ}_{eq}(BaSO_4)$

$= \Lambda^{\circ}_{eq}(BaCl_2) + \Lambda^{\circ}_{eq}(H_2SO_4) - \Lambda^{\circ}_{eq}(HCl)$

$= x_1 + x_2 - x_3$

Now, $\Lambda^{\circ}_{eq} = \Lambda_{eq}$ for saturated $BaSO_4$ solution

$$= \frac{1000 \times \kappa}{N}$$

$$\therefore \quad x_1 + x_2 - x_3 = \frac{1000 \times y}{N}$$

$$\Rightarrow \quad N = \frac{10^3 \times y}{x_1 + x_2 - x_3}$$

$\therefore$

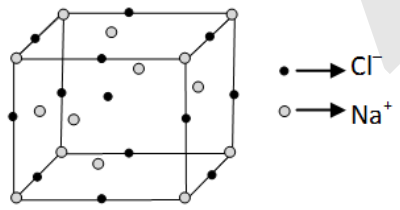
$$M = \frac{N}{v.f.} = \frac{10^3 \times y}{2(x_1 + x_2 - x_3)} = \text{solubility(s)}$$

$$\therefore \quad K_{SP} = S^2 = \frac{10^6 y^2}{4(x_1 + x_2 - x_3)^2}$$

9. Ans (B)

$Fe(OH)_3$ is positively charged sol, hence if direct current is passed coagulation will take place at cathode (negative electrode)

10. Ans (B)



Rock salt structure

If all the atoms touching one face plane are removed then

$$\text{No. of } Na^+ \text{ left} = 8 \times \frac{1}{4} + 1 = 3$$

$$\text{No. of } Cl^- \text{ left} = 5 \times \frac{1}{2} + 4 \times \frac{1}{8} = 3$$

Hence formula is NaCl

Also since equal number of cations and anions are missing defect is schottky defect.

11. Ans (C)

$$\int dH = n \int C_{p,m}\, dT$$

$$\Delta H = 10 \int_{300K}^{400K} (C_{v,m} + R)dT$$

$$\Delta H = 10 \int_{300K}^{400K} (20 + 10^{-2}\, T + 8.314)dT$$

$\Delta H = 31814$ J

12. Ans (A)

NCERT XII part-I Exercise 2.35

From Henry's law,

$P_{gas} = KH \times X_{gas}$

$760 = 4.5 \times 10^5 \times X_{gas}$

$$X_{gas} = \frac{760}{4.5 \times 10^5} = 168.8 \times 10^{-5} = 1.688 \times 10^{-3}$$

$$X_{gas} = \frac{760}{4.5 \times 10^5} = 168.8 \times 10^{-5}$$

$= 1.688 \times 10^{-3}$

13. Ans (C)

0.05 mol XY_2 = 5 gm

$$1 \text{ mol } XY_2 = \frac{5}{0.05} = 100$$

$X + 2Y = 100$ ....(i)

3.01×10^{23} molecule X_2Y_3 = 85 gm

1 mol X_2Y_3 = NA molecule = 170 gm

$2X + 3Y = 170$ ....(ii)

On solving,

$X = 40, Y = 30$

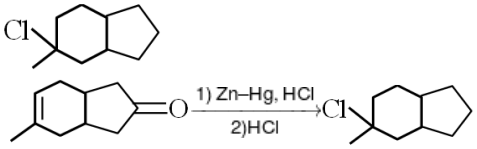
14. Ans (C)

$$C=C-C \xrightarrow[CCl_4]{NBS} C=C-\underset{\underset{Br}{|}}{C} \xrightarrow[\text{ether}]{Na} C=C-C-C\diagdown\!\!\diagup C=C$$

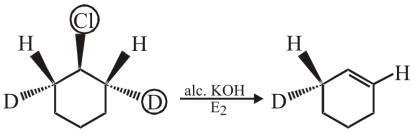
15. Ans (B)

In DNA four bases are present adenine(A), guanine(G), cytosine (C) and thymine (T). Out of which pyrimidine bases are cytosine and thymine

16. Ans (B)

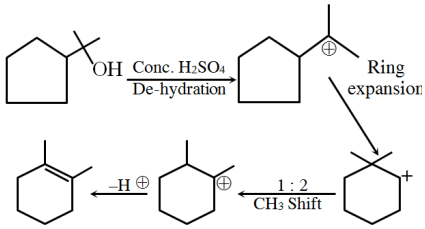


19. **Ans (C)**



Antielimination takes place so that removal of –Cl & H or D possible only from opposite site.

20. **Ans (C)**



PART-2 : CHEMISTRY

SECTION-II

1. **Ans (7.00)**

XeF_2, I_3^- ⟶ linear (planar)

BCl_3 ⟶ Plane triangle (planar)

XeF_4 ⟶ Square planar (planar)

$[XeF_5]^-$ ⟶ Pentagonal planar (planar)

ClF_3 ⟶ Bent 'T' - shape (planar)

H_2O ⟶ 'V' - shape (planar)

2. **Ans (4.00)**

Cl_2 → Greenish yellow

NO_2 → Brown gas

CdS → Yellow

$Cu_2[Fe(CN)_6]$ → Chocolate brown

3. **Ans (6.00)**

F and O are more electronegative than Cl.

4. **Ans (2.00)**

$$\text{Slope} = \frac{-E_a}{2.303\,R} = -\frac{1}{2.303} \Rightarrow E_a = R = 2 \text{ calorie}$$

5. **Ans (3.75)**

$$HCOONa + HCl \rightarrow HCOOH + NaCl$$

t=0	0.25 × 20ml =5 milli-mole	0.1 × 30ml =3 milli-mole	0	0
End	2 milli-mole	0	3 milli-mole	3milli-mole

$$pH = pK_a + \log \frac{[HCOONa]}{[HCOOH]}$$

$$= 3.75 + \log \frac{(2/50ml)}{(3/50ml)} = 3.57$$

6. **Ans (50.00)**

$\pi_1 = \pi_2$

$i_1c_1 = i_2c_2$

$$(1 + 2\alpha_1) \times \frac{17.4 \times 1000}{174 \times 100}$$

$$= (1 + \alpha_1) \times \frac{5.85 \times 1000}{58.5 \times 100}$$

$\alpha_1 = 0.5$ $\quad \therefore \alpha$ in % = 50%

7. **Ans (272.00)**

$nN_2(g) + nH_2(g) \rightarrow -(-NH-NH-)_n-$

$\Delta H = [n \times BE(N \equiv N) + n \times BE(H-H)$

$- BE(N-N) \times 2n + 2n \times BE(N-H)]$

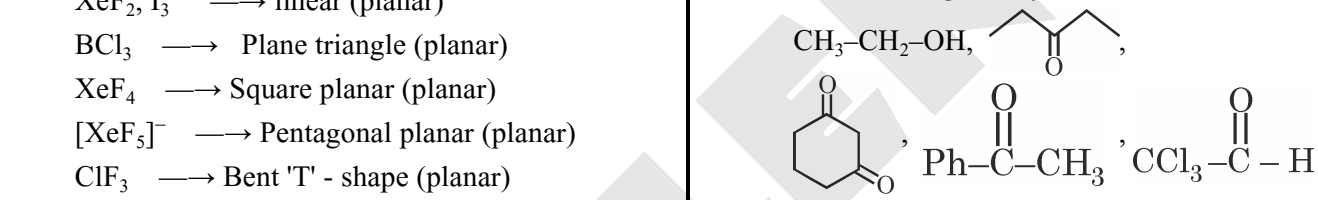
$\because n = 1$

$\Rightarrow \Delta H = [BE(N \equiv N) + BE(H-H) -$

$[BE(N-N) + BE(N-H)] \times 2$

8. **Ans (5.00)**

Chloroform given by



PART-3 : MATHEMATICS

SECTION-I

1. **Ans (C)**

$$\lim_{x \to 1^-} \frac{\sqrt{\pi} - \sqrt{2\sin^{-1}x}}{\sqrt{1-x}} \times \frac{\sqrt{\pi} + \sqrt{2\sin^{-1}x}}{\sqrt{\pi} + \sqrt{2\sin^{-1}x}}$$

$$\lim_{x \to 1^-} \frac{2\left(\frac{\pi}{2} - \sin^{-1}x\right)}{\sqrt{1-x} \cdot \left(\sqrt{\pi} + \sqrt{2\sin^{-1}x}\right)}$$

$$\lim_{x \to 1^-} \frac{2\cos^{-1}x}{\sqrt{1-x}} \cdot \frac{1}{2\sqrt{\pi}}$$

Put $x = \cos\theta$

$$\lim_{\theta \to 0^+} \frac{2\theta}{\sqrt{2}\sin\left(\frac{\theta}{2}\right)} \cdot \frac{1}{2\sqrt{\pi}} = \sqrt{\frac{2}{\pi}}$$

2. **Ans (D)**

$\frac{x}{1+|x|}$ is always differentiable (also at x = 0).

Also $(x+2)(x-2)|(x-1)(x-2)(x-3)|$

$(x+2)(x-2)|x-2||x-1||x-3|$

is Not differentiable at x = 1, 3.

3. **Ans (A)**

Since f(x) is decreasing in the interval (–2, –1), therefore,

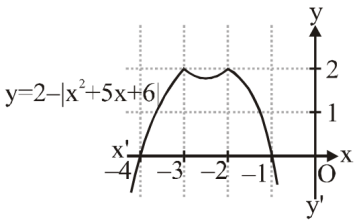
$f'(x) < 0 \Rightarrow 6x^2 + 18x + \lambda < 0$

The value of l should be such that the equation $6x^2 + 18x + \lambda = 0$ has roots –2 & –1.

Therefore, $(-2)(-1) = \frac{\lambda}{6} \Rightarrow \lambda = 12$

4. **Ans (A)**

f(x) will have maxima at x = –2 only if $a^2 + 1 \geq 2$ or or $|a| \geq 1$.



5. **Ans (A)**

Let y + 1 = Y

$\therefore \quad \frac{dY}{dx} = Y^2 e^{\frac{x^2}{2}} - xY$

Put $-\frac{1}{Y} = k$

$\Rightarrow \frac{dk}{dx} + k(-x) = e^{\frac{x^2}{2}}$

$I.F. = e^{-\frac{x^2}{2}}$

$\therefore \quad k = (x + c)e^{x^2/2}$

Put $k = -\frac{1}{y+1}$

$\therefore \quad y + 1 = -\frac{1}{(x + c)\, e^{x^2/2}} \quad ...(i)$

when x = 2, y = 0, then $c = -2 - \frac{1}{e^2}$

differentiate equation (i) & put x = 1

we get $\left(\frac{dy}{dx}\right)_{x=1} = -\frac{e^{3/2}}{(1 + e^2)^2}$

6. **Ans (B)**

$I = 2\int_0^4 f\left(x^2\right) dx$ {Even function}

$= 2\int_0^4 \left(4x^3 - g(4 - x)\right) dx$

$= 2\left(\frac{4x^4}{4}\Big|_0^4 - \int_0^4 g(4 - x)\, dx\right)$

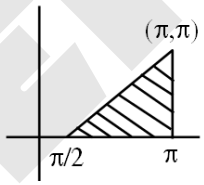
$= 2(256 - 0) = 512$

7. **Ans (A)**

$y = \left|\frac{\pi}{2} - \sin^{-1}(\sin x)\right| + \left|\frac{\pi}{2} - \cos^{-1}(\cos x)\right|$

$y = \left|\frac{\pi}{2} - (\pi - x)\right| + \left|\frac{\pi}{2} - x\right|$

$y = \left|x - \frac{\pi}{2}\right| + \left|\frac{\pi}{2} - x\right| = 2x - \pi \ \forall x \in \left[\frac{\pi}{2}, \pi\right]$



$\text{Area} = \frac{1}{2}\left(\frac{\pi}{2}\right)\pi = \frac{\pi^2}{4}$

8. **Ans (D)**

We know that asymptotes of rectangular hyperbola is mutually perpendicular, thus other asymptote should be

$4x + 3y + \lambda = 0$.

Also, intersection point of asymptotes is also the centre of the hyperbola.Hence, intersection point of $4x + 3y + \lambda = 0$ and $3x - 4y - 6 = 0$ is $\left(\frac{18 - 4\lambda}{25}, \frac{-12\lambda - 96}{100}\right)$ and it should lie on the line x – y – 1 = 0.

$\therefore \quad \frac{18 - 4\lambda}{25} - \frac{-12\lambda - 96}{100} - 1 = 0$

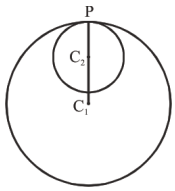
$\Rightarrow \quad \lambda = 17$

Hence, equation of other asymptote is 4x + 3y + 17 = 0

9. **Ans (B)**

Points of intersection of the lines $x^2 + 2xy + 3x + 6y = 0$ is $\left(-3, \frac{3}{2}\right)$

$\therefore$ center of the required circle is $c_1 \equiv \left(-3, \frac{3}{2}\right)$



consider the circle $x(x-4) + y(y-3) = 0$

$C_2 \equiv (2, \frac{3}{2})$ and $r_2 = \frac{5}{2}$

Radius of required circle is $r_1 = |C_1P|$

$= |C_1C_2| + |C_2P|$

$= 5 + \frac{5}{2}$

$= \frac{15}{2}$

Equation of circle is

$$(x+3)^2 + \left(y - \frac{3}{2}\right)^2 = \frac{225}{4}$$

or $x^2 + y^2 + 6x - 3y - 45 = 0$.

10. **Ans (C)**

The given plane passes through $\vec{a}$ and is parallel to the vectors $(\vec{b} - \vec{a})$ and $\vec{c}$. So it is normal to $(\vec{b} - \vec{a}) \times \vec{c}$.

Hence, its equation is

$$(\vec{r} - \vec{a}).\left((\vec{b} - \vec{a}) \times \vec{c}\right) = 0$$

$$\Rightarrow \vec{r} \cdot (\vec{b} \times \vec{c} + \vec{c} \times \vec{a}) = [\vec{a}\ \ \vec{b}\ \ \vec{c}]$$

The length of the perpendicular form the origin to this plane is $\dfrac{[\vec{a}\ \ \vec{b}\ \ \vec{c}]}{|\vec{b} \times \vec{c} + \vec{c} \times \vec{a}|}$

11. **Ans (C)**

For $[\vec{a}\ \ \vec{b}\ \ \vec{c}]$ to be greatest $\vec{a}$ must be perpendicular to both $\vec{b}$ and $\vec{c}$ i.e. collinear with $\vec{b} \times \vec{c}$.

$$\vec{b} \times \vec{c} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -1 & 4 & 6 \\ 2 & -7 & -10 \end{vmatrix} = 2\hat{i} + 2\hat{j} - \hat{k}$$

$$\therefore\ \vec{a} = \frac{2\hat{i} + 2\hat{j} - \hat{k}}{3}$$

12. **Ans (A)**

Let m be the slope of line which passing, through (h, k)

$\therefore$ equation of line $y - k = m(x - h)$ ...(1)

eq. of line perpendicular to (1) & passing through (0, 0) is

$$y - 0 = -\frac{1}{m}(x - 0) \quad \ldots(2)$$

from (1) & (2) eliminate m, we get

$x^2 + y^2 - hx - ky = 0$, which is the required locus of the point

13. **Ans (A)**

Let no. of terms = 2n

$$\therefore\ K = \frac{\frac{n}{2}[2a + (n-1)d]}{\frac{n}{2}[2(a + nd) + (n-1)d]}$$

$$= \frac{2 + (n+1)d}{2 + (3n-1)d}$$

$$= 1 - \frac{2d}{\left(\frac{2-d}{n}\right) + 3d}$$

For 'K' to be constant, d = 0 or d = 2

$\therefore$ K = 1, $\frac{1}{3}$

Sum = $1 + \frac{1}{3} = \frac{4}{3}$

14. **Ans (B)**

$x^2 - 2x + 5 + 4\cos(a + bx) = 0$

$(x-1)^2 + 4(1 + \cos(a + bx)) = 0$

$\Rightarrow x = 1$ and $\cos(a + bx) = -1$

$\cos(a + b) = -1$

$a + b = \pi, 3\pi \in (0, 5)$

max. value of $a + b = 3\pi$

16. **Ans (A)**

$(4k_1) + (4k_2 + 1) + (4k_2 + 2) = 35$

$k_1 + k_2 + k_3 = 8$

Ans. $= {}^{10}C_2 = 45$

17. **Ans (A)**

$${}^9C_3\left(\frac{1}{6}\right)^3\left(\frac{5}{6}\right)^6 \times \left(\frac{1}{6}\right) = \frac{84 \times 5^6}{6^{10}}$$

18. **Ans (B)**

$\cos\theta + i\sin\theta = \frac{3}{z} - 2$

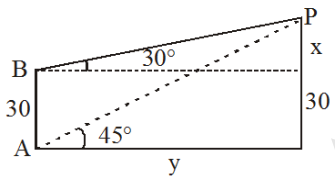
$\cos\theta + i\sin\theta = \frac{3 - 2z}{z}$

$1 = \frac{|3 - 2z|}{|z|}$

$|z|^2 = |3 - 2z|^2 \Rightarrow x^2 + y^2 - 4x + 3 = 0$

put $z = x + iy$

19. **Ans (B)**



$\tan 45° = 1 = \frac{x + 30}{y} \Rightarrow x + 30 = y$ (i)

$\tan 30° = \frac{1}{\sqrt{3}} = \frac{x}{y} \Rightarrow x = \frac{y}{\sqrt{3}}$ (ii)

from (i) and (ii) $y = 15\,(3 + \sqrt{3})$

20. **Ans (C)**

p	q	$p \to q$	-p	$q \to -p$	$(p \to q) \wedge (q \to -p)$
T	T	T	F	F	F
T	F	F	F	T	F
F	T	T	T	T	T
F	F	T	T	T	T

Clearly, $(p \to q) \wedge (q \to -p)$ is equivalent to -p

PART-3 : MATHEMATICS

SECTION-II

1. **Ans (107.00)**

$3[x] + 1 = 4[x] - 4 - 10$

$[x] = +15$

$\therefore y = 46$

$\therefore [x + 2y] = [x] + 92 = 107$

3. **Ans (-0.50)**

$$\frac{dy}{dx} - \frac{t}{1+t}\,y = \frac{1}{1+t}$$

which is linear form

$$I.F. = e^{\int \frac{-t}{1+t}dt} = e^{-t+\log(1+t)} = (1+t)e^{-t}$$

$$\therefore\ y(1+t)e^{-t} = \int \frac{1}{(1+t)}(1+t)\cdot e^{-t}\,dt$$

$$= \int e^{-t}\,dt = -r^{-t} + c$$

when $t = 0,\ y = -1$ so $-1 = -1 + c$

$\therefore\quad c = 0$

$\therefore\quad y(1+t)e^{-t} = -e^{-t}$

so, $t = 1,\ y \cdot 2 \cdot e^{-1} = -e^{-1}$

$\therefore\ y = \frac{-1}{2}$

4. **Ans (2.00)**

$$\int \frac{xe^x + e^x}{(1 + xe^x)^2}\,dx$$

put $xe^x = t$

$\Rightarrow e^x\,(x+1)\,dx = dt$

$$\therefore I = \int \frac{dt}{(1+t)^2} = -\frac{1}{(1+t)} + \lambda$$

$$= \frac{1}{1 + xe^x} + \lambda = f(x)$$

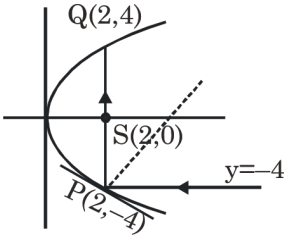
Where λ is constant of integration

$\because f(0) = 1 \Rightarrow \lambda = 2$

$$\therefore f(x) = \frac{2xe^x + 1}{xe^x + 1}$$

$$\therefore \lim_{x\to\infty} f(x) = 2$$

5. Ans (6.00)



Equation of parabola is

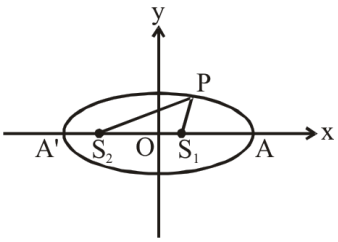
$y^2 = 8x$ $\quad \therefore P = (2,-4)$

By reflection property.

reflected say passes through focus

$\therefore Q = (2,4)$

6. Ans (0.50)



$2a + 2ae = 15$

$\Rightarrow 2a(1 + e) = 15$

$\Rightarrow 10(1 + e) = 15$

$\Rightarrow 1 + e = \frac{3}{2} \Rightarrow e = \frac{1}{2}$

7. Ans (67.00)

$3 \log_b a = 3 + d$

$3 \log_c b = 3 + 2d$

$7 \log_a c = 3 + 3d$

$\Rightarrow 3^2 \times 7 = 3(3 + d)(3 + 2d)(3 + 2d)$

$(1 + d)(3 + d)(3 + 2d) = 21$

$\Rightarrow d = \frac{1}{2}$

$\therefore 6 \log_b a = 7 \Rightarrow a^6 = b^7$

$3 \log_c b = 4 \Rightarrow b^3 = c^4$

$\Rightarrow a^{18} = b^{21} = c^{28}$

$\therefore p + q + r = 18 + 21 + 28 = 67$

8. Ans (49.00)

Let T_{1+1} is max.

$$T_{1+1} = \frac{{}^{100}c_r}{(r+1)(r+2)(r+3)(r+4)}$$

$$= \frac{{}^{104}c_{r+4}}{101 \cdot 102 \cdot 103 \cdot 104} \text{ is max, when}$$

$r + 4 = 52$

$r = 48$

so term is 49^{th}

9. Ans (5.00)

If, $A + B = 45°$

$\tan(A + B) = 1$

$\because \tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$

$\Rightarrow \tan A + \tan B = 1 - \tan A \tan B$

$\Rightarrow (1 + \tan A)(1 + \tan B) = 2$

LHS

$= [(1 + \tan 1°)(1 + \tan 44°)][(1 + \tan 43°)] ..... [(1 + \tan 45°)]$

$\left[\text{for each } (1 + \tan\theta)\left[1 + \tan\left(\frac{\pi}{4} - \theta\right) = 2\right]\right]$

$= 2^{22}(1 + 1)$

$= 2^{23}$

$= 2^{\lambda}$

then, $\lambda = 23$.

Hence the sum of digits of λ is $2 + 3$

$= 5$

10. Ans (3.00)

The given data is :

6, 8, 10, 12, 14, 16, 18, 20, 22, 24

$\therefore$

$$\bar{x} = \frac{6 + 8 + 10 + 12 + 14 + 16 + 18 + 20 + 22 + 24}{10}$$

$$= \frac{150}{10} = 15$$

The table is as below :

x_i	$x_i - \bar{x}$	$(x_i - \bar{x})^2$
6	–9	81
8	–7	49
10	–5	25
12	–3	9
14	–1	1
16	1	1
18	3	9
20	5	25
22	7	49
24	9	81
Total		**330**

Hence, Variance $(\sigma^2) = \frac{\sum (x_i - \bar{x})^2}{n} = \frac{330}{10} = 33$