(1001CJM203121003)

Test Pattern

ALLEN CAREER INSTITUTE
KOTA (RAJASTHAN)
path to success

CLASSROOM CONTACT PROGRAMME
(Academic Session : 2021 - 2022)

JEE (Main)
SAMPLE TEST
PAPER # 02

JEE(Main) : Leader Course

ANSWER KEY

PART-1 : PHYSICS

		1	2	3	4	5	6	7	8	9	10
SECTION-I	Q.	1	2	3	4	5	6	7	8	9	10
	A.	C	B	B	B	A	C	D	C	C	C
	Q.	11	12	13	14	15	16	17	18	19	20
	A.	D	D	D	C	C	D	B	D	C	C
SECTION-II	Q.	1	2	3	4	5	6	7	8	9	10
	A.	3.00	5.00	75.00	2.25	90.00	3.50	5.00	18.31	448.00	0.03

PART-2 : CHEMISTRY

		1	2	3	4	5	6	7	8	9	10
SECTION-I	Q.	1	2	3	4	5	6	7	8	9	10
	A.	B	D	C	C	C	C	C	D	C	A
	Q.	11	12	13	14	15	16	17	18	19	20
	A.	B	C	B	D	A	C	B	B	A	C
SECTION-II	Q.	1	2	3	4	5	6	7	8	9	10
	A.	19.00	10.00	2.00	0.10	18.00	8.00	3.00	13.00	3.00	8.00

PART-3 : MATHEMATICS

		1	2	3	4	5	6	7	8	9	10
SECTION-I	Q.	1	2	3	4	5	6	7	8	9	10
	A.	C	D	B	B	B	D	A	B	D	D
	Q.	11	12	13	14	15	16	17	18	19	20
	A.	A	C	A	B	A	A	B	A	C	B
SECTION-II	Q.	1	2	3	4	5	6	7	8	9	10
	A.	10.10	2.00	2.00	2.00	5.00	3500.00	11.00	18.00	3.50	13.00

HINT – SHEET

PART-1 : PHYSICS

SECTION-I

1. **Ans (C)**

$I = \Delta P = m(v-u) = 2\left(\sqrt{2gh} - 0\right)$

$= 2\sqrt{2 \times 10 \times 5}$

$= 20$ N–S

2. **Ans (B)**

$$v_1 = \frac{(m_1 - em_2)u_1}{m_1 + m_2} + \frac{(1+e)m_2u_2}{m_1 + m_2}$$

$$v_2 = \frac{(m_2 - em_1)u_2}{m_1 + m_2} + \frac{(1+e)m_1u_1}{m_1 + m_2}$$

here $u_2 = 0$, $m_1 = m_2$

$v_2 = 3v_1$

3. **Ans (B)**

As liq. 1 is above liq. 2

$\therefore\ \rho_1 < \rho_2$

As the body is floating between the layer of liq. 1 & liq.2

$\therefore\ \rho_3 > \rho_1$ & $\rho_3 < \rho_2$

Hence, $\rho_1 < \rho_3 < \rho_2$

4. **Ans (B)**

max. permissible acceleratic for block to not to leave plat form $= a_{max} = g$

$\omega^2 d = g$

$(2\pi f)^2\ d = g$

$$f = \frac{1}{2\pi}\sqrt{\frac{g}{d}}$$

5. **Ans (A)**

$dQ = dU + dW$

$5 = Q + W_{AB} + W_{BC} + W_{CA}$

$5 = (2 - 1) + 0 + W_{CA}$

$W_{CA} = -5$ Joule

6. **Ans (C)**

$-\dfrac{dT}{dt} = K(T - T_0)$

$\dfrac{50 - 45}{5} = K(47.5 - T_0)$

$\dfrac{45 - 40}{8} = K\,(42.5 - T_0)$

On dividing

$\dfrac{8}{5} = \dfrac{47.5 - T_0}{42.5 - T_0}$

$340 - 8T_0 = 237.5 - 5T_0$

$\Rightarrow T_0 = 34°C;\ 3T_0 = 102.5$

8. **Ans (C)**

In γ-decay A & Z remains same.

9. **Ans (C)**

${}_{90}Th^{220} \longrightarrow {}_{2}He^{4} + {}_{88}Ra^{216}$

$m_\alpha E_\alpha = m_R E_R$

$4E_\alpha = 216 E_R$

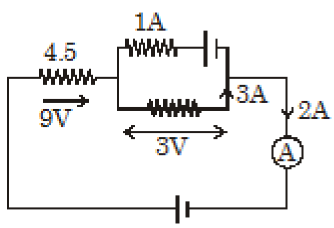
$E_R = \dfrac{E_\alpha}{54}$

12. **Ans (D)**

$3 = 6 - i \times 3$

$i = 1A$

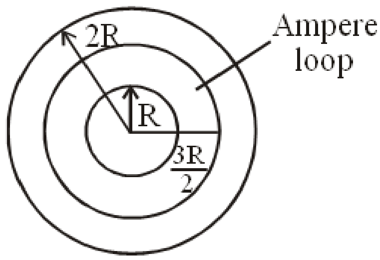
$3V = 3 \times R$



$R = 1\Omega$

$E = 12V$

14. **Ans (C)**



$\oint \vec{B}\cdot\vec{dl} = \mu_0 i$

$$B2\pi\left(\frac{3R}{2}\right) = \frac{\mu_0 I\left(\pi\left(\frac{3R}{2}\right)^2 - \pi R^2\right)}{\pi(2R)^2 - \pi R^2}$$

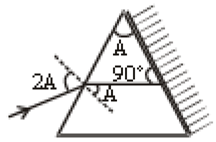
15. **Ans (C)**

$$P = \frac{v_0^2 R}{2\,(\omega^2 L^2 + R^2)}$$

$$\frac{dP}{dR} = 0 = \frac{2\,(\omega^2 L^2 + R^2)\,v_0^2 - 2v_0^2 R \times 2R}{4(\omega^2 L^2 + R^2)^2} = 0$$

$$R = \omega L = \frac{5}{100\pi} \times 100\pi = 5\Omega$$

17. **Ans (B)**



$$\mu = \frac{\sin 2A}{\sin A} = \frac{2\sin A\cos A}{\sin A},\ \mu = 2\cos A.$$

18. **Ans (D)**

$$v = \sqrt{\frac{T}{m}} = \sqrt{\frac{0.5g}{1/20}} = \sqrt{100} = 10\ m/s$$

20. **Ans (C)**

$$f = \frac{4v}{2\ell} = \frac{2\sqrt{\frac{T}{\mu}}}{\ell} = \frac{2\sqrt{\frac{52}{5.2\times10^{-3}}}}{2}$$

$f = 100$ Hz

PART-1 : PHYSICS

SECTION-II

1. **Ans (3.00)**

Orbital velocity of satellite, $v = \sqrt{\frac{GM}{R}} = v_0$

$\therefore v' = \sqrt{\frac{GM}{R+2R}} = \sqrt{\frac{GM}{3R}} = \frac{v_0}{\sqrt{3}}$

2. **Ans (5.00)**

Case (1)

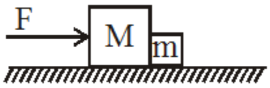


figure (i)

$a = \frac{F}{M+m}$

$N_1 = \frac{mF}{m+M}$

Case (2)

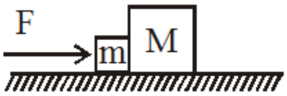


figure (ii)

$F - N_2 = \frac{mF}{M+m}$

$N_2 = F - \frac{mF}{m+M} = \frac{MF}{m+M}$

$\frac{N_2}{N_1} = \frac{M}{m} = \frac{10}{2} = 5$

3. **Ans (75.00)**

$a = \frac{100-(A+Bv)}{15}$

initially $a_0 = \frac{100-25}{15} = 5$

$a = \frac{5}{2} = \frac{75-0.5v}{15}$

$v = 75$ m/s

5. **Ans (90.00)**

$\frac{N_1}{N_2} = \frac{(N_0)e^{-\lambda_1 t}}{(N_0)_2 e^{-\lambda_2 t}} = \frac{1}{e^2}$

$\Rightarrow \; e^{(\lambda_1-\lambda_2)t} = e^2$

$\Rightarrow \; \lambda_1 - \lambda_2 = \frac{2}{t}$

$\Rightarrow \; 10\lambda - \lambda_2 = \frac{2}{\left(\frac{2}{a\lambda}\right)} = a\lambda$

$\Rightarrow \; \lambda_2 = \lambda$

$\Rightarrow \; \lambda_2$ is 90% lesser than λ_1

$\Rightarrow \; p = 90$

6. **Ans (3.50)**

Radiant pressure by perfectly absorbing surface is

$RP = \frac{I}{C} = \frac{P}{4\pi r^2}\left(\frac{1}{c}\right)$

$= \frac{60}{100} \cdot \frac{22W}{4\left(\frac{22}{7}\right)(1m)^2} \cdot \frac{1}{3 \times 10^8 m/s}$

$RP = 3.5 \times 10^{-9}\ Nm^{-2} \Rightarrow y = 3.5$

7. **Ans (5.00)**

Magnetic force on rod = $BI\ell$

Weight of the rod = mg

For no tension in wire, $BI\ell = mg$

or $I = \frac{mg}{Bl} = \frac{1 \times 10}{2 \times 1} = 5A$

8. **Ans (18.31)**

If mirror rotated by q, then reflected ray rotates by

$2\theta = 2 \times 3.5 = 7°$

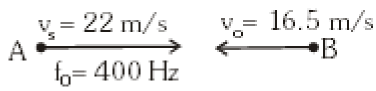
for θ small $\theta \approx \tan\theta$

$\frac{d}{1.5} = \frac{7\pi}{180}$

$d = \frac{7 \times 3.14 \times 1.5}{180}$

$= 0.1831$ m $= 18.31$ cm

9. **Ans (448.00)**



As we know for given condition

$f_{app} = f_0\left(\frac{v + v_{observer}}{v - v_{source}}\right)$

$= 400\left(\frac{340 + 16.5}{340 - 22}\right)$

$f_{app} = 448$ Hz

10. **Ans (0.03)**

$\frac{B}{D} = \frac{\lambda}{d}$ = angular fringe width

$\frac{6000 \times 10^{-10}}{d} = \frac{\pi}{180}$ radian

$\left(\begin{array}{l} 180° = \pi \text{ radian} \\ \therefore 1° = \frac{\pi}{180} \text{ radian} \end{array}\right)$

$d = \frac{6 \times 10^{-7} \times 180}{3.14}$ m

$d = \frac{6 \times 18 \times 10^{-6}}{3.14} \times 10^3$ mm

$d \approx 0.03$ mm

PART-2 : CHEMISTRY

SECTION-I

2. Ans (D)

$\Delta T_b = i.\ k_b.\ m$

$= [1 + 0.6\ (5 - 1)] \times 0.52 \times 1$

$= 1.768$

$T_b = 1.768 + 373 = 374.76$ K

5. Ans (C)

$A \rightarrow$ Products

$t = 0 \quad 1$ M

$t = 1$hr $\quad 0.125$ M

We know

$$K_t = 2.303 \log \frac{a_0}{a_t}$$

$$K = \frac{2.303}{60 \times 60} \log \frac{1}{0.125} \quad [t = 1\text{hr} = 60 \times 60 \text{ sec}]$$

$$= \frac{2.303}{3600} \log 8$$

$$= \frac{2.303}{3600} \times 3\log 2 \ [\log 8 = \log 2^3 = 3\log 2]$$

$K \ = 0.00058$ sec-1

$$t_{1/2} = \frac{0.693}{0.00058} \text{ sec} \quad [t_{1/2} = \frac{0.693}{K}]$$

$= 1200$ sec.

6. Ans (C)

$$\underset{(\text{Red})}{\text{HIn}} \rightleftharpoons H^+ + \underset{\text{yellow}}{In^-}$$

$$pH = pKIn + \log 10 \frac{(In^-)}{(HIn)}$$

at 1st condition;

$$(pH)_1 = pKIn + \log_{10} \frac{10}{90} = pKIn - \log_{10} 9 \quad ..(1)$$

at 2nd condition;

$$(pH)_2 = pKIn + \log_{10} \frac{90}{10} = pKIn + \log_{10} 9 \quad ..(2)$$

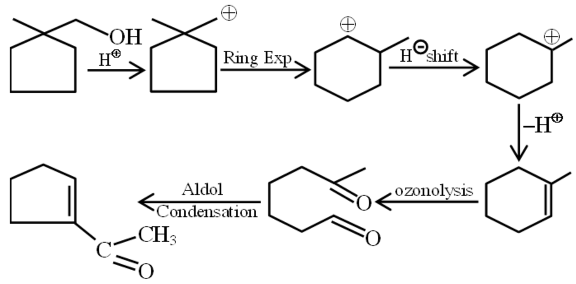
change in $pH = 2 \log_{10} 9 = 2 \log_{10} 3^2 = 4 \log_{10} 3$

$= 4 \times 0.48 = 1.92$

11. Ans (B)

Due to presence of EWG $\left[\begin{array}{c} O \\ \| \\ R-C- \end{array}\right]$ Rate of SN^2 reaction is higher than SN^1 reaction.

12. Ans (C)



13. Ans (B)

Morphine

14. Ans (D)

Cu,Ag, Au

15. Ans (A)

According to $(n + \ell)$ rule

$n = 5$

$5s \rightarrow 1$ (orbital)

$5p \rightarrow 3$

$5d \rightarrow 5$

$5f \rightarrow 7$

$5g \rightarrow 9$

$\Rightarrow$ 5H not possible

19. Ans (A)

I.P. $B < C < O < N$

PART-2 : CHEMISTRY

SECTION-II

3. Ans (2.00)

$$\frac{(t_{1/2})_1}{(t_{1/2})_2} = \left(\frac{P_2}{P_1}\right)^{n-1}$$

$$\frac{440}{880} = \left(\frac{182}{364}\right)^{n-1}$$

$$\frac{1}{2} = \left(\frac{1}{2}\right)^{n-1}$$

$n - 1 = 1$

$n = 2$

4. Ans (0.10)

$$Y_A = \frac{P_A^o x_A}{P_A^o x_A + P_B^o x_B}$$

$$= \frac{1 \times \frac{1}{4}}{1 \times \frac{1}{4} + 3 \times \frac{3}{4}} = \frac{\frac{1}{4}}{\frac{10}{4}} = 0.1$$

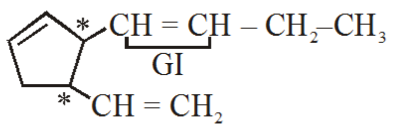
5. **Ans (18.00)**

(A) = CrO_2Cl_2

(B) = Na_2CrO_4

(C) = CrO_5

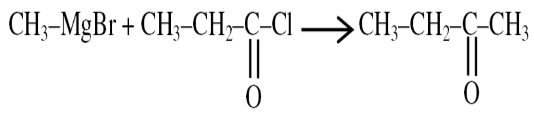
6. **Ans (8.00)**

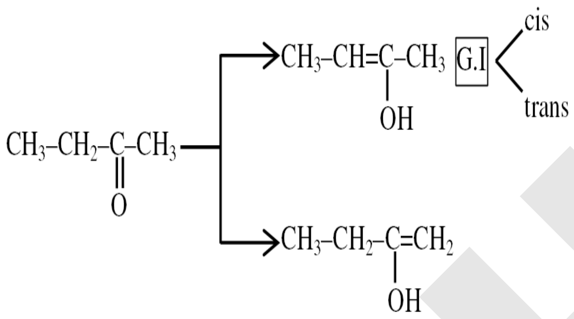


n = 3

no of S.I. = $2^3 = 8$

7. **Ans (3.00)**





8. **Ans (13.00)**

EAN = atomic number – O.S. + 2(CN)

$Ni(CO)_x \Rightarrow 36 = 28 - 0 + 2(x)$

$8 = 2x \quad \Rightarrow \quad x = 4$

$Fe(CO)_y \Rightarrow 36 = 26 - 0 + 2y$

$2y = 10 \Rightarrow y = 5$

$K_z[Fe(CN)_6] \Rightarrow 36 = 26 - \text{O.S.} + 2 \times 6$

$\Rightarrow$ O.S. = +2

$K^{+1}[Fe(CN)_6]^{-z}$

$+2 + 6(-1) = -z$

$-4 = -z \quad \Rightarrow \quad z = 4$

$x + y + z \Rightarrow 4 + 5 + 4 = 13$

PART-3 : MATHEMATICS

SECTION-I

1. **Ans (C)**

$$\bar{x} = \frac{\sum x_i}{n}$$

$$\Rightarrow 4 = \frac{1+2+6+x_1+x_2}{5}$$

$$\Rightarrow x_1 + x_2 = 11 \quad ....(1)$$

$$\Rightarrow \sigma^2 = \frac{1}{n}\sum x_i^2 - \bar{x}^2$$

$$\Rightarrow 5.2 = \frac{1+4+36+x_1^2+x_2^2}{5} - 16$$

$$\Rightarrow 106 = x_1^2 + x_2^2 + 41$$

$$\Rightarrow x_1^2 + x_2^2 = 65 \quad ....(2)$$

Solving (1) and (2)

$x_1 = 4 \quad ; \; x_2 = 7$

2. **Ans (D)**

$2\sin^2 x + 3\sin x - 2 > 0$

$\Rightarrow (2\sin x - 1)(\sin x + 2) > 0$

$\Rightarrow \sin x > \frac{1}{2} \quad \Rightarrow \quad x \in \left(\frac{\pi}{6}, \frac{5\pi}{6}\right)$

Also, $x^2 - x - 2 < 0$

$(x - 2)(x + 1) < 0$

$-1 < x < 2$

Also; $2 < \frac{5\pi}{6}$, we obtain that x must lie in $\left(\frac{\pi}{6}, 2\right)$

3. **Ans (B)**

$D_1 = p^2 - 12q$; $D_2 = r^2 + 4q$; $D_3 = s^2 - 8q$

Case I - If q<0 then $D_1>0, D_3>0$ and D_2 may or may not be +ve

Case II- If q > 0 then $D_2>0$ and D_1, D_3 may or may not be +ve

Case III- If q= 0 then D_1, D_2, D_3

so, given equation has at least two real roots

4. Ans (B)

Coefficient of x^6 in

$(1+{}^6C_1x^6)\ (1+{}^5C_1x^5)(1+{}^4C_1x^4)$

$(1+{}^3C_1x^3+{}^3C_2x^6)$

$(1+{}^2C_1x^2+{}^2C_2x^4)(1+x)$

$\Rightarrow$ coefficient of x^6 in

$\Rightarrow (1+6x^6+5x^5+4x^4)(1+2x^2+3x^3+x^4+6x^5+3x^6)(1+x)$

$\Rightarrow$ coefficient of x^6 in $(11x^5+17x^6)(1+x) = 28$

5. Ans (B)

$$\text{Let}\left(\frac{A}{3}\right) = B$$

$$|\text{adj}B^{-1}| = |B^{-1}|^2 = \frac{1}{|B|^2} = \frac{1}{\left|\frac{A}{3}\right|^2}$$

$$= \frac{3^6}{|A|^2} = \frac{3^6}{9^2} = 3^2 = 9$$

6. Ans (D)

Let $S_1 = 2^2 + 4^2 + 6^2 + \ldots\ldots + 100^2$ ..... (1)

and $S = 1^2 + 3^2 + 5^2 + \ldots\ldots + 99^2$ .... (2)

eq^n (1) – (2) $S_1 - S = [(2^2-1^2)+(4^2-3^2)] +$ .....$+ (100^2-99^2)]$

$\Rightarrow S_1 = S + (1 + 2 + 3 + 4 + \ldots + 100)$

$\Rightarrow S_1 = S + 5050$

7. Ans (A)

$D = x^k.y^k.z^k =$

$$\begin{vmatrix} 1 & ar & a^2r^2 \\ 1 & ar^2 & a^2r^4 \\ 1 & ar^3 & a^2r^6 \end{vmatrix} = a^{3k}.r^{6k}.a^3r^3 \begin{vmatrix} 1 & 1 & 1 \\ 1 & r & r^2 \\ 1 & r^2 & r^4 \end{vmatrix}$$

$= a^{3(k+1)}.r^{6k+3}(1-r)(r-r^2)(r^2-1)$

Clearly, $k = -1$

$$\therefore \Delta = r^{-2}(1-r)^2(r^2-1) = (r-1)^2\left(1-\frac{1}{r^2}\right)$$

8. Ans (B)

Given $\Rightarrow T_9 = 5\,T_2\ \&\ T_{13} = 2T_6 + 5$

So $a + 8d = 5\,(a + d)\ \&\ a + 12d = 2\,(a + 5d) + 5$

$\Rightarrow a = 3$

9. Ans (D)

$$\lim_{x\to a}\frac{(a+2x)^{1/3}-(3x)^{1/3}}{(3a+x)^{1/3}-(4x)^{1/3}}(a\neq 0)\quad\left[\frac{0}{0}\text{form}\right]$$

Put $x = a + h$

$$\text{So, }\lim_{h\to 0}\frac{(a+2a+2h)^{1/3}-(3a+3h)^{1/3}}{(3a+a+h)^{1/3}-(4a+4h)^{1/3}}$$

$$=\lim_{h\to 0}\frac{(3a)^{1/3}\left[\left(1+\frac{2h}{3a}\right)^{1/3}-\left(1+\frac{3h}{3a}\right)^{1/3}\right]}{(4a)^{1/3}\left[\left(1+\frac{h}{4a}\right)^{1/3}-\left(1+\frac{4h}{4a}\right)^{1/3}\right]}$$

$$=\lim_{h\to 0}\left(\frac{3}{4}\right)^{1/3}\left[\frac{1+\frac{2h}{9a}-1-\frac{3h}{9a}+\text{higher degree terms}}{1+\frac{h}{12a}-1-\frac{4h}{12a}+\text{higher degree terms}}\right]$$

$$=\left(\frac{3}{4}\right)^{1/3}\left(\frac{\frac{2}{9}-\frac{3}{9}}{\frac{1}{12}-\frac{4}{12}}\right)=\left(\frac{3}{4}\right)^{1/3}\left(\frac{\frac{-1}{9}}{\frac{-3}{12}}\right)$$

$$=\left(\frac{3}{4}\right)^{1/3}\frac{4}{(3)^2}=\frac{4^{1-\frac{1}{3}}}{3^{2-\frac{1}{3}}}=\frac{4^{2/3}}{3^{5/3}}=\frac{2^{4/3}}{3^{5/3}}=\frac{2}{3}\left(\frac{2}{9}\right)^{1/3}$$

Hence, option (D) is correct.

10. Ans (D)

If function $f:[-7,0] \to R$ be continuous on $[-7,0]$ and differentiable on $(-7, 0)$, then according to LMVT, we have

$$\frac{f(-1)-f(-7)}{(-1)-(-7)} = f'(x) \leqslant 2, \forall x \in (-7,-1)$$

$$\Rightarrow \frac{f(-1)-(-3)}{6} \leqslant 2$$

$\Rightarrow f(-1) \le 9$ ...(i)

$$\text{Similarly, } \frac{f(0)-f(-7)}{0-(-7)} = f'(x) \le 2,\ \forall x\in(-7,0)$$

$$\Rightarrow \frac{f(0)-(-3)}{7} \leqslant 2$$

$\Rightarrow f(0) \le 11$ ...(ii)

From Eqs. (i) and (ii), we get

$f(-1) + f(0) \le 20$

$\therefore f(-1) + f(0)$ lies in the interval $(-\infty, 20]$

11. Ans (A)

$$\frac{dy}{dx} = \frac{y}{x} + \sec\frac{y}{x}$$

Let $y = vx$

$$\frac{dy}{dx} = v + x\frac{dv}{dx}$$

$$v + \frac{xdv}{dx} = v + \sec v$$

$$\cos v\, dv = \frac{dx}{x}$$

$$\sin v = \ell nx + c$$

$$\sin\left(\frac{y}{x}\right) = \ell nx + c$$

$\because$ passing through

$$\left(1, \frac{\pi}{6}\right) \Rightarrow \sin\frac{\pi}{6} = c \Rightarrow c = \frac{1}{2}$$

12. Ans (C)

$$\int_0^{\pi} \frac{x \tan x}{\operatorname{sex} x + \tan x} dx$$

$$= \frac{\pi}{2}\int_0^{\pi} \frac{\tan x}{\operatorname{sex} x + \tan x} dx$$

$$= \frac{\pi}{2}\int_0^{\pi} \tan x(\sec x - \tan x)dx$$

$$= \frac{\pi}{2}\int_0^{\pi} (\tan x \sec x - \tan^2 x)dx$$

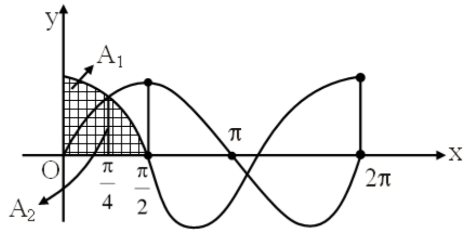
$$= \frac{\pi}{2}\int_0^{\pi} (\sec x \tan x - \sec^2 x + 1)dx$$

$$= \frac{\pi}{2}[\sec x - \tan x + x]_0^{\pi}$$

$$= \frac{\pi}{2}[(-1 - 0 + \pi) - (1 - 0 + 0)]$$

$$= \frac{\pi}{2}(\pi - 2)$$

13. Ans (A)



$$A_1 + A_2 = \int_0^{\pi/2} \cos x\, dx = (\sin x)\Big|_0^{\pi/2} = 1$$

$$A_1 = \int_0^{\pi/4} (\cos x - \sin x)\, dx = (\sin x + \cos x)_0^{\pi/4} = \sqrt{2} - 1$$

$$\because A_2 = 1 - (\sqrt{2} - 1) = 2 - \sqrt{2}$$

$$\therefore \frac{A_1}{A_2} = \frac{\sqrt{2} - 1}{2 - \sqrt{2}} = \frac{\sqrt{2} - 1}{\sqrt{2}\left(\sqrt{2} - 1\right)} = \frac{1}{\sqrt{2}}$$

14. Ans (B)

Point of intersection of L_1 & L_2 is (1, 0, 1)

Line L passes through (1, 0, 1)

$$\frac{1-\alpha}{\ell} = -\frac{1}{m} = \frac{1-\gamma}{-2} \qquad ...(1)$$

acute angle bisector of L_1 & L_2

$$\vec{r} = \hat{i} + \hat{k} + \lambda\left(\frac{\hat{i} - \hat{j} + 3\hat{k} - 3\hat{i} - \hat{j} + \hat{k}}{\sqrt{11}}\right)$$

$$\vec{r} = \hat{i} + \hat{k} + t\left(\hat{i} + \hat{j} - 2\hat{k}\right)$$

$$\Rightarrow \frac{\ell}{1} = \frac{m}{1} = \frac{-2}{-2} \quad \Rightarrow \quad \ell = m = 1$$

$$\text{From (1)} \quad \frac{1-\alpha}{1} = -1 \quad \Rightarrow \quad \alpha = 2$$

$$\& \quad \frac{1-\gamma}{-2} = -1 \quad \Rightarrow \quad \gamma = -1$$

16. Ans (A)

$y^2 = 4\lambda x$, $P(\lambda, 2\lambda)$

Slope of the tangent to the parabola at point P

$$\frac{dy}{dx} = \frac{4\lambda}{2y} = \frac{4\lambda}{2x2\lambda} = 1$$

Slope of the tangent to the ellipse at P

$$\frac{2x}{a^2} + \frac{2yy'}{b^2} = 0$$

As tangents are perpendicular $y' = -1$

$$\Rightarrow \frac{2\lambda}{a^2} - \frac{4\lambda}{b^2} = 0 \Rightarrow \frac{a^2}{b^2} = \frac{1}{2}$$

$$\Rightarrow e = \sqrt{1 - \frac{1}{2}} = \frac{1}{\sqrt{2}}$$

17. Ans (B)

$r = \sqrt{4+1+20} = 5$

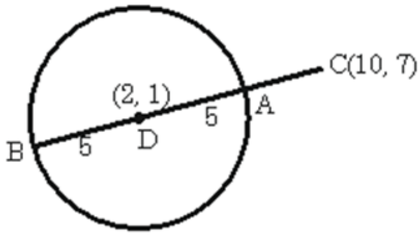
Slope of line

$DP = \frac{7-1}{10-2} = \frac{3}{4}$

$\tan\theta = \frac{3}{4} \Rightarrow \cos\theta = \frac{4}{5}$ and $\sin\theta = \frac{3}{5}$

For point A & B

$(2 \pm 5\cos\theta, \quad 1 \pm 5\sin\theta)$



$\left(2 \pm 5\left(\frac{4}{5}\right), \quad 1 \pm 5\left(\frac{3}{5}\right)\right)$

$(2 \pm 4, \quad 1 \pm 3)$

A(6, 4), B(–2, –2)

Now AB = 10

$\Rightarrow PA + PB = 15 > AB$

$\Rightarrow$ locus of P be an ellipse

$\Rightarrow 2a = 15, 2ae = 10$

$\Rightarrow e = \frac{10}{15} = \frac{2}{3}$

18. Ans (A)

perpendicular vector to the plane

$$\vec{n} = \begin{vmatrix} i & j & k \\ 1 & 2 & -1 \\ -1 & 1 & -2 \end{vmatrix} = -3\hat{i} + 3\hat{j} + 3\hat{k}$$

Eq. of plane

$-3(x-1) + 3(y-1) + 3z = 0$

$\Rightarrow x - y - z = 0$

$$d_{(2,1,4)} = \frac{|2-1-4|}{\sqrt{1^2+1^2+1^2}} = \sqrt{3}$$

20. Ans (B)

Let the line is $y = mx + c$

$\frac{y - mx}{c} = 1$

$$3x^2 - y^2 - 2x\left(\frac{y-mx}{c}\right) + 4y\frac{(y-mx)}{c} = 0$$

x^2 coeff + y^2 coeff = 0

$3 - 1 + \frac{2m}{c} + \frac{4}{c} = 0$

$2c + 2m + 4 = 0$

$m + c = -2$

$m = -2 - c$

$y = (-2 - c)x + c$

$(y + 2x) + c(x - 1) = 0$

$L_1 + \lambda L_2 = 0 \Rightarrow x = 1$ and $y + 2x = 0$

$y = -2$

(1,–2)

PART-3 : MATHEMATICS

SECTION-II

1. Ans (10.10)

$\bar{x} = \frac{1}{101}[1 + (1+d) + (1+2d) + \ldots + (1+100d)]$

$= \frac{1}{101} \times \frac{101}{2}[1 + (1+100d)] = 1 + 50d.$

$\therefore$ Mean deviation from mean

$= \frac{1}{101}[|1-(1+50d)| + |1+d-(1+50d)| + \ldots + |1+100d-(1+50d)|]$

$= \frac{2|d|}{101}[1 + 2 + \ldots + 50]$

$= \frac{2|d|}{101}\frac{50(51)}{2} = \frac{2550}{101}|d|$

Now, $\frac{2550}{101}|d| = 255 \Rightarrow |d| = 10.1$

Thus, we may take d = 10.1

2. **Ans (2.00)**

$\sin^4x + \cos^4x = \sin x \cos x$

or $(\sin^2x + \cos^2x)^2 - 2\sin^2x\cos^2x = \sin x \cos x$

or $1 - \dfrac{\sin^2 2x}{2} = \dfrac{\sin 2x}{2}$

or $\sin^2 2x + \sin 2x - 2 = 0$

or $(\sin^2x + 2)(\sin 2x - 1) = 0$

or $\sin 2x = 1$

or $2x = (4n+1)\dfrac{\pi}{2}, n \in Z$

or $x = (4n+1)\dfrac{\pi}{4}, n \in Z$

$= \dfrac{\pi}{4}, \dfrac{5\pi}{4} \quad (x \in [0, 2\pi])$

Thus, there are two solutions.

3. **Ans (2.00)**

3,1,1 & 2,2,1 $\rightarrow$ two Methods only

4. **Ans (2.00)**

$2x^3 + 5x^2 + 2x - 1 = (x+1)(2x^2 + 3x - 1)$

$\Rightarrow$ either $x = -1$ is root of equation or

$x^2 + ax + b = 0$

& $2x^2 + 3x - 1 = 0$ have both roots in common.

5. **Ans (5.00)**

It is given that the function $f: R \rightarrow R$ satisfies,

$f(x+y) = f(x).f(y), \forall x, y \in R \Rightarrow f(x) = a^x$

$\because f(1) = 3$

$\Rightarrow a = 3$

$\therefore f(x) = 3^x$

Now, it is given that $\sum_{i=1}^{n} f(i) = 363$

$\Rightarrow 3 + 3^2 + 3^3 + ... + 3^n = 363$

$\Rightarrow \dfrac{3(3^n - 1)}{2} = 363 \Rightarrow 3^n = 243 = 3^5 \Rightarrow n = 5$

7. **Ans (11.00)**

$$I = \int \frac{2\sin^2\theta.\cos\theta(\sin^6\theta + \sin^4\theta + \sin^2\theta)\sqrt{2\sin^4\theta + 3\sin^2\theta + 6}}{2\sin^2\theta} d\theta$$

$$I = \int (t^6 + t^4 + t^2)\sqrt{2t^4 + 3t^2 + 6}\, dt$$

(Let $\sin\theta = t$, $\cos\theta\, d\theta x = dt$)

$$= \int (t^5 + t^3 + t)\sqrt{2t^6 + 3t^4 + 6t^2}\, dt$$

Let $2t^6 + 3t^4 + 6t^2 = z$

$(12t^5 + 12t^3 + 12t)dt = dz$

$\because I = \int \sqrt{z} \cdot \dfrac{dx}{12} = \dfrac{1}{12}\dfrac{z^{3/2}}{\frac{3}{2}} + C$

$$= \frac{1}{18}\left(2\sin^6\theta + 3\sin^4\theta + 6\sin^2\theta\right)^{3/2} + C$$

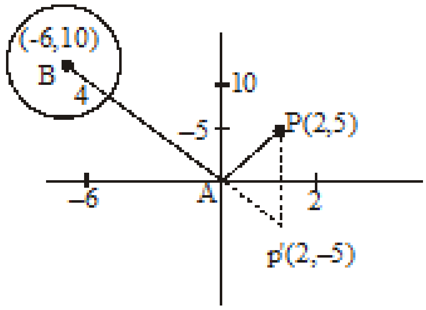
$$= \frac{1}{18}\left((1-\cos^2\theta)(2(1-\cos^2\theta)^2 + 3 - 3\cos^2\theta + 6)\right)^{3/2} + C$$

$$= \frac{1}{18}\left((1-\cos^2\theta)(2\cos^4\theta - 7\cos^2\theta + 11)\right)^{3/2} + C$$

$$= \frac{1}{18}\left(-2\cos^6\theta + 9\cos^4\theta - 18\cos^2\theta + 11\right)^{3/2} + C$$

$\because K_1 + K_2 + K_3 = 18 + (-18) + 11 = 11$

10. **Ans (13.00)**



BP'=17