ALLEN CAREER INSTITUTE KOTA (RAJASTHAN) path to success

# CLASSROOM CONTACT PROGRAMME
(Academic Session : 2021 - 2022)

JEE (Main)
SAMPLE TEST PAPER # 01

## JEE(Main) : Leader Course

## ANSWER KEY

### PART-1 : PHYSICS

| | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| SECTION-A | Q. | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| | A. | B | A | B | D | A | D | A | B | C | D |
| | Q. | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 |
| | A. | D | A | B | B | B | D | C | A | B | A |
| SECTION-B | Q. | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| | A. | 49.00 | 6.00 | 2.00 | 887.00 | 2.00 | 16.00 | 1.00 | 650.00 | 517.00 | 0.66 |

### PART-2 : CHEMISTRY

| | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| SECTION-A | Q. | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| | A. | C | D | D | B | B | C | C | D | D | D |
| | Q. | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 |
| | A. | A | B | C | C | A | C | D | B | C | C |
| SECTION-B | Q. | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| | A. | 70.02 | 75.00 | 0.16 | 0.30 | 8.00 | 45.00 | 43.00 | 13.00 | 7.00 | 5.00 |

### PART-3 : MATHEMATICS

| | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| SECTION-A | Q. | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| | A. | D | C | A | B | D | A | A | A | B | B |
| | Q. | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 |
| | A. | B | A | A | A | D | B | C | C | A | B |
| SECTION-B | Q. | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| | A. | 15.00 | 48.00 | 4.00 | 25.00 | 36.00 | 4.00 | 20.00 | 0.00 | 2.00 | 1.00 |

## HINT – SHEET

### PHYSICS

#### SECTION-I

**1. Ans ( B )**

For Disc $\tau = R \times F = I\alpha$ Here $\left(I = \frac{MR^2}{2}\right)$

$1 \times 10 = \frac{2 \times 1}{2} \times \alpha$

$\alpha = 10\ r/s^2$

then $\omega = \omega_0 + \alpha t$

$= 0 + 10 \times 2$

$= 20\ r/s$

**2. Ans ( A )**

$x\ cm = \frac{\int x.\,dx}{\int dm}$

Mass per unit length $\frac{dm}{dx} = \lambda = \frac{Kx^2}{L}$

$dm = \frac{Kx^2 dx}{L}$

$x_{cm} = \frac{\int_0^L x\left(\frac{Kx^2}{L}\right)dx}{\int_0^L \frac{Kx^2}{L}dx} \equiv \frac{3L}{4}$

**3.** **Ans ( B )**

$$K\varepsilon_{Rot} = \frac{1}{2}\frac{mR^2}{2}\cdot\frac{v^2}{R^2} = \frac{1}{4}mv^2$$

$$K\varepsilon_{trans} = \frac{1}{2}mv^2$$

$$K\varepsilon_{Total} = \frac{3}{4}mv^2$$

$$\frac{K\varepsilon_{Rot}}{K\varepsilon_{Total}} = \frac{1}{3}$$

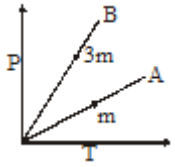
**5.** **Ans ( A )**

$PV = nRT$

$$PV = \frac{m}{M}.RT$$

$$A\ \frac{P}{T} = \frac{m}{M}.\frac{R}{V}$$

$$B\ \frac{P}{T} = \frac{3m}{M}.\frac{R}{V}$$




$$\frac{\text{Slope of B}}{\text{Slope of A}} = \frac{3}{1}$$

**6.** **Ans ( D )**

$P \propto T^C \Rightarrow P^{T-C} = K \quad PT^{r/1-r} = K$

$$-C = \frac{7/5}{1-(7/5)} = \frac{7/5}{-2/5} = \frac{7}{2}$$

**8.** **Ans ( B )**

$$i = \frac{Q}{t} = \frac{ne}{t} = 1.8\times10^{14}\times1.6\times10^{-19}$$

$$= 28.8\times10^{-6}A = 29\mu A$$

**9.** **Ans ( C )**

In γ-decay A & Z remains same.

**12.** **Ans ( A )**

Electrical equivalent

$$i = \frac{E_1+E_2}{4+8} = \frac{E}{12} = 4$$

$E = 48$ V

$V_{PR} = E_1 - 4i$

$= 24 - 4\times4 = 8$V

**14.** **Ans ( B )**

Potential energy of the system $U = \frac{kQ_1Q_2}{r}$

where $K = \frac{1}{4\pi\epsilon_0}$

Potential energy of the configuration

$$U = \frac{kQq}{a} + \frac{kq^2}{a} + \frac{kqQ}{a\sqrt{2}} = 0$$

$$Q = \frac{-\sqrt{2}q}{\sqrt{2}+1}$$

Hence Q is equal to $\frac{-\sqrt{2}q}{\sqrt{2}+1}$

**15.** **Ans ( B )**

$$R_1 = \frac{200^2}{40} = 1000\Omega$$

$$R_2 = \frac{200^2}{100} = 400\Omega$$

$$i = \frac{200}{900} = \frac{2}{9}$$

$$P = \frac{4}{81}\times900 = \frac{400}{9}W = 44.4W$$

**16.** **Ans ( D )**

$$\lambda = \frac{h}{mv} = \frac{h}{qrB}$$

$$= \frac{\lambda_\alpha}{\lambda_{pr}} = \frac{q_{pr}\times r_{pr}}{q_\alpha\times r_\alpha} = \frac{1}{2}$$

**17.** **Ans ( C )**

$$x = \frac{n\lambda D}{d} = \text{same}$$

$n_1\lambda_1 = n_2\lambda_2$

$n_1\times12000 = n_2\times10000$

$$\frac{n_1}{n_2} = \frac{5}{6}$$

12000 Å की 5th Bright 10000Å की 6th Bright fringe से coincide करेगी।

$$x = \frac{5\times12000\times10^{-10}\times2}{2\times10^{-3}} = 6\text{ mm}$$

**18.** **Ans ( A )**

wave no. $= \frac{1}{\lambda}$ for 1 cm thickness

$$\frac{4}{\lambda_g} = \frac{5}{\lambda_w}$$

$$\frac{4}{5} = \frac{\lambda_g}{\lambda_w} = \frac{\mu_m}{\mu_g} = \frac{4/3}{\mu_g}$$

$$\mu_g = \frac{5}{3}$$

**19. Ans ( B )**

$\delta = i + e - A \Rightarrow A = 45°$ ($i = 15°$; $e = 60°$)

**20. Ans ( A )**

$y = 3 \sin 6.28 (0.5\, x - 50\, t)$

$\omega = 6.28 \times 50 \quad k = 6.28 \times 0.5$

$v = \frac{\omega}{k} = 100 \text{ cm/ sec}$

$k = \frac{2\pi}{\lambda} \quad \Rightarrow \quad \lambda = \frac{2\pi}{k} = \frac{2\pi}{6.28 \times 0.5}$

$\lambda = 2$ cm

## PHYSICS

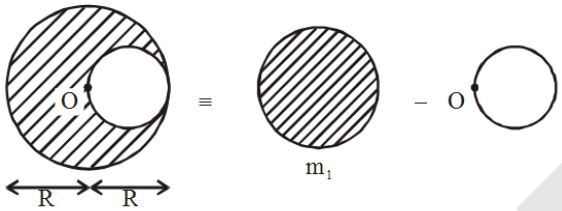
### SECTION-II

**1. Ans ( 49.00 )**

time of flight = (5 sec) × 2 = 10 sec

$\mu = gt = 9.8 \times 5 = 49$ m/s

**2. Ans ( 6.00 )**




Mass of disc α are a

$m_1 \alpha \pi(R)^2$ ; $m_2 \propto \frac{(1+e)m_1 u_1}{m_1 + m_2}$

$m_1 = m \quad ; \quad m_2 = \frac{m}{4}$

CM of remaining part

$x_{cm} = \frac{m_1 x_1 - m_2 x_2}{m_1 + m_2} = \frac{m(0) - \frac{m}{4}\left(\frac{R}{2}\right)}{m - \frac{m}{4}} = -\frac{R}{6}$

**3. Ans ( 2.00 )**

$x = 2$, $U_f = 2^2 - 3 \times 2 = -2$

$x = 0$, $U_i = 0$

$x = 0$, $KE_i = 0$

$U_i + KE_i = U_f + KE_f$

$0 - 0 = -2 + KE_f$

$\boxed{KE_f = 2J}$

**4. Ans ( 887.00 )**

$\frac{E}{t} = s\, AT^4$

$= 6 \times 10^{-8} \times 1.6 \times 310^4$

$= 9.6 \times 10^{-8} \times 81$

$= 887$

**5. Ans ( 2.00 )**

$\mu = \frac{A_{max} - A_{min}}{A_{max} + A_{min}} \Rightarrow \frac{60}{100} = \frac{8 - X}{8 + X}$

$\Rightarrow 48 + 6X = 80 - 10X \Rightarrow 16X = 80 - 48$

$X = \frac{32}{16} = 2$

**6. Ans ( 16.00 )**

$P^1 = 10 \times \frac{66}{100} = 6.6 \text{ Watt}$

$P^1 = \frac{nE}{t} = \frac{n}{t} \times \frac{hc}{\lambda} \Rightarrow \frac{n}{t} = \frac{P^1 \times \lambda}{hc} = \frac{6.6 \times 5896 \times 10^{-10}}{20 \times 10^{-26}}/s$

$\frac{n}{t} = 1965.6 \times 10^{16}/s$

**7. Ans ( 1.00 )**

This is linear variation of resistance.

$\frac{i \times 4}{i \times R_0} = \frac{AD}{DC} = \frac{40}{60} = \frac{2}{3}$

$\frac{4}{R_0} = \frac{2}{3} R_0 = 6 \frac{R \times 12}{R + 12} = 6 \Rightarrow R = 12$

Now, $R = R_0 (1 + \alpha \Delta T)$

$12 = 6 (1 + \alpha\, 100)$

$\alpha = 1 \times 10^{-2}$

$x = 1$

**8. Ans ( 650.00 )**

$\frac{P}{Q} = \frac{S}{625}$

$\frac{Q}{P} = \frac{S}{676}$

$\therefore \frac{S}{625} = \frac{676}{S}$

$S = 25 \times 26$

$= 650\ \Omega$

**9. Ans ( 517.00 )**

After filling frequency increases, so $n_A$ increases(↑). Also it is given that beat frequency increases (i.e., x↑)

Hence $n_A\uparrow - n_B = x\uparrow$ ...(i) → Correct

$n_B - n_A\uparrow = x\uparrow$ ...(ii) → Wrong

$\Rightarrow n_A = n_B + x = 512 + 5 = 517$ Hz.

**10. Ans ( 0.66 )**

$\frac{1}{-0.5} + \frac{1}{0.2} = \frac{2}{r}$

$\Rightarrow \frac{2}{r} = 3$ ; $r = 0.667$ m

## CHEMISTRY

### SECTION-I

**3. Ans ( D )**

$r = k[N_2O_5]^1$

$[N_2O_5] = \dfrac{2.4 \times 10^{-5}}{3 \times 10^{-5}} \Rightarrow 0.8M$

**5. Ans ( B )**

Combination of weak base (NH4OH) and its salt with strong acid (NH4Cl) act as basic buffer solution

$\underset{\text{(weak base)}}{NH_4OH} + \underset{\left(\text{Strong acid}\right)}{HCl} \rightleftharpoons NH_4Cl + H_2O$

| | | | | |
|---|---|---|---|---|
| initial | 2 mole | 1 mole | 0 | 0 |
| After reaction | $(2-1)$ $=1$mole | $(1-1)$ $=0$ | 1 mole | 1 mole |

1 mole $NH_4OH$ & 1 mole $NH_4Cl$ are present in mixture after reaction. Hence it will also act as basic buffer.

**6. Ans ( C )**

$C(s) + 2H_2(g) + \dfrac{1}{2}O_2(g) \rightarrow CH_3OH(\ell)$

$H = (\Delta H_f)CH_3OH = 2$

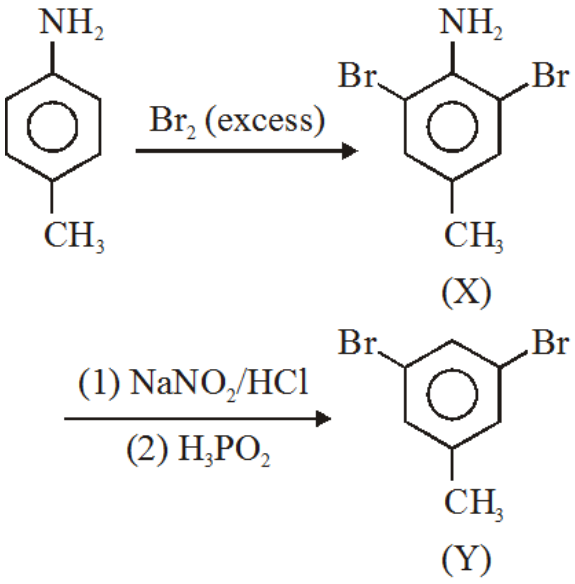
This required reaction $+ (2)^{nd}$ reaction $+ 2 \times (3)^{rd}$ reaction $- (1)^{st}$ reaction

$= -393 + 2\times(-286) - (-726)$

$= -393 - 572 + 726$

$= -239$ KJ $mol^{-1}$

**12. Ans ( B )**




**14. Ans ( C )**

(A) $AuCl_4^-$ – $dsp^2$

(B) $[Co(OX)_3]^{3-}$ – $d^2sp^3$

(C) $RhCl(PPh_3)_3$ – $dsp^2$

(D) $[Fe(NH_3)_6]^{2+}$ – $sp^3d^2$

**15. Ans ( A )**

Oxidation state of o in superoxide is (–1/2) & in peroxide is (–1).

**19. Ans ( C )**

$Zn^{+2}(d^{10})$ unpaired $e^- = 0$

$Sc^{+3}(d^0)$ unpaired $e^- = 0$

**20. Ans ( C )**

$N_2^+$ $1s^2(s)$

$1s^2(\sigma^*)2s^2(\sigma)2s^2\sigma^*\pi\, 2p_x^2\ 2p_y^2\ \sigma 2p_z^1$

$4e^-$ in antibonding M.O.

## CHEMISTRY

### SECTION-II

**2. Ans ( 75.00 )**

$FeSO_4(NH_4)_2SO_4 . 6H_2O + Fe_2(SO_4)_3$

Mohr salt

x mol (4–x) mol

$\because$ (Eq) $K_2Cr_2O_7$ = (Eq) Mohr solt

$(Cr^{+6} \rightarrow Cr^{+3})$ $(Fe^{+2} \rightarrow Fe^{+3})$

$N \times V_{(L)}$ = mol × n factor

$1 \times 6 \times 0.5$ = x × 1

x = 3 mol

$\because$ % of Mohr's salt $\underset{\text{in mixlure}}{} = \dfrac{3}{4} \times 100 = 75\%$

**3. Ans ( 0.16 )**

$2SO_2 + O_2 \longrightarrow 2SO_3$

$+\frac{1}{2}\frac{d[SO_3]}{dt} = \frac{-d[O_2]}{dt} = R\cdot O\cdot R$

$\dfrac{1}{2}\left(\dfrac{0.8}{80}\text{mol. lit}^{-1}\text{ sec}^{-1}\right) = \dfrac{d[O_2]}{dt}$

$\dfrac{1}{200}\text{mol . lit}^{-1}\text{ sec}^{-1} = \dfrac{d[O_2]}{dt}$

Now $\dfrac{d[O_2]}{dt} = \dfrac{1}{200} \times 32\text{g.lit}^{-1}\text{ sec}^{-1}$

$\Rightarrow 0.16$ g.lit$^{-1}$ sec$^{-1}$

**4. Ans ( 0.30 )**

$K_C = 10^{30}$

$E_{cell} = E°_{cell} - \frac{0.059}{2}\log K_C$

$E°_{cell} = 0.01\log \times 10^{30}$

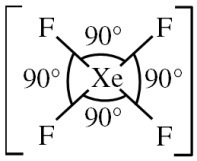
$E°_{cell} = +0.30$ V

**7. Ans ( 43.00 )**

SBH can't reduced ester group.

**8. Ans ( 13.00 )**

$[Cu(H_2O)_4]\ SO_4 \times H_2O \Rightarrow x = 4$




$\Rightarrow$ 90° angles = y = 4

$[Pt(NH_3)_6]^{+y}, 4Cl^-$

(Number of ions = z = 5)

x + y + z

4 + 4 + 5 = 13

## MATHEMATICS

### SECTION-I

**1. Ans ( D )**

Consider $\sim [p \rightarrow (\sim p \vee q)] \equiv p \wedge \sim (\sim p \vee q)]$

$\equiv p \wedge (p \wedge \sim q) \equiv p \wedge p \wedge \sim q \equiv p \wedge \sim q$

**2. Ans ( C )**

$\sin\frac{\pi}{14}\sin\frac{3\pi}{14}\sin\frac{5\pi}{14}\sin\frac{9\pi}{14}\sin\frac{11\pi}{14}\sin\frac{13\pi}{14}$

$\left(\sin\frac{13\pi}{14} = \sin\left(\pi - \frac{\pi}{14}\right)\right) = \sin\frac{\pi}{14}$

$= \sin\frac{11\pi}{14} = \sin\left(\pi - \frac{3\pi}{14}\right) = \sin\frac{3\pi}{14}$

$= \sin\frac{9\pi}{14} = \sin\left(\pi - \frac{5}{14}\right) = \left(\sin\frac{5\pi}{14}\right)$

$= \left(\sin\frac{\pi}{14}\sin\frac{3\pi}{14}\sin\frac{5\pi}{14}\right)^2$

$= \left[\cos\left(\frac{\pi}{2} - \frac{\pi}{14}\right)\cos\left(\frac{\pi}{2} - \frac{3\pi}{14}\right)\cos\left(\frac{\pi}{2} - \frac{5\pi}{14}\right)\right]^2$

$= \left(\cos\frac{\pi}{7}\cos\frac{2\pi}{7}\cos\frac{3\pi}{7}\right)^2$

$= \left(\cos\frac{\pi}{7}\cos\frac{2\pi}{7}\cos\left(\pi - \frac{4\pi}{7}\right)\right)^2$

$= \left(-\cos\frac{\pi}{7}\cos\frac{2\pi}{7}\cos\frac{4\pi}{7}\right)^2 = \left(\frac{\sin 2^3 \cdot \frac{\pi}{7}}{2^3 \sin\frac{\pi}{7}}\right)^2$

$= \left(\frac{1}{8}\right)^2 = \frac{1}{64}$

**4. Ans ( B )**

*→ Any Head or tail; P(H) = P(T) = $\frac{1}{2}$, P (*)=1

for at least four consecutive Heads, we have following patterns

(i) H H H H ∗ ∗∗

(ii) T H H H H ∗ ∗

(iii) ∗ T H H H H∗

(iv) ∗ ∗T H H H H

$\Rightarrow \left(\frac{1}{2}\right)^4 1 + 3\left(\frac{1}{2}\right)^5$

$\Rightarrow \frac{5}{32}$

**5. Ans ( D )**

$A^T = A$

$B^T = -B$

$A - B = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ ...(1)

$A^T - B^T = \begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix}$

$A + B = \begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix}$ ...(2)

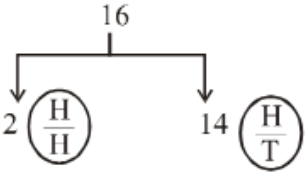
on adding eq. (1) and (2)

$2A = \begin{bmatrix} 2 & 5 \\ 5 & 8 \end{bmatrix}$

$A = \begin{bmatrix} 1 & 5/2 \\ 5/2 & 4 \end{bmatrix}$

$|A| = \frac{4 - 25}{4} = \frac{-9}{4}$

**6. Ans ( A )**




$\frac{2}{16} \times 1 + \frac{14}{16} \times \frac{1}{2} = \frac{9}{16}$

**8. Ans ( A )**

$\sum_{r=0}^{50} {}^{100}C_r \cdot {}^{200}C_{150+r} = \sum_{r=0}^{50} {}^{100}C_r \ {}^{200}C_{50-r} = {}^{300}C_{50}$

**9. Ans ( B )**

Equation of given curve, $y = f(x) = x\log_e x$, $(x>0)$

$\therefore \left.\frac{dy}{dx}\right|_{x=c} = f'(c) = 1 + \log_e c$

$\because$ The tangent to the given curve $y = f(x)$ at point $x = c$ is parallel to line segment joining points $(1,0)$ and $(e, e)$

So, $1 + \log_e c = \frac{e}{e-1} \Rightarrow \log_e c = \frac{e}{e-1} - 1 = \frac{1}{e-1}$

$\Rightarrow c = e^{\left(\frac{1}{e-1}\right)}$ is positive.

**10. Ans ( B )**

Given expression is $2y =$

$$\left(\cot^{-1}\left(\frac{\sqrt{3}\cos x + \sin x}{\cos x - \sqrt{3}\sin x}\right)\right)^2$$

$$= \left(\cot^{-1}\left(\frac{\sqrt{3}\cot x + 1}{\cos x - \sqrt{3}}\right)\right)^2$$

[dividing each term of numerator and denominator by sin x]

$$= \left(\cot^{-1}\left(\frac{\cot\frac{\pi}{6}\cot x + 1}{\cos x - \cot\frac{\pi}{6}}\right)\right)^2 \left[\because \cot\frac{\pi}{6} = \sqrt{3}\right]$$

$$= \left(\cot^{-1}\left(\cot\left(\frac{\pi}{6} - x\right)\right)\right)^2$$

$$\left[\because \cot(A-B) = \frac{\cot A \cot B + 1}{\cot B - \cot A}\right]$$

$$= \begin{cases} \left(\frac{\pi}{6} - x\right)^2, & 0 < x < \frac{\pi}{6} \\ \left(\pi + \left(\frac{\pi}{6} - x\right)\right)^2 & \frac{\pi}{6} < x < \frac{\pi}{2} \end{cases}$$

$$\left[\because \cot^{-1}(\cot\theta) = \begin{cases} \pi + \theta, & -\pi < \theta < 0 \\ \theta, & 0 < \theta < \pi \\ \theta - \pi, & \pi < \theta < 2\pi \end{cases}\right]$$

$$\Rightarrow 2y = \begin{cases} \left(\frac{\pi}{6} - x\right)^2, & 0 < x < \frac{\pi}{6} \\ \left(\frac{7\pi}{6} - x\right)^2, & \frac{\pi}{6} < x < \frac{\pi}{2} \end{cases}$$

$$\Rightarrow 2\frac{dy}{dx} = \begin{cases} 2\left(\frac{\pi}{6} - x\right)(-1), & 0 < x < \frac{\pi}{6} \\ 2\left(\frac{7\pi}{6} - x\right)(-1), & \frac{\pi}{6} < x < \frac{\pi}{2} \end{cases}$$

$$\Rightarrow \frac{dy}{dx} = \begin{cases} x - \frac{\pi}{6}, & 0 < x < \frac{\pi}{6} \\ x - \frac{7\pi}{6}, & \frac{\pi}{6} < x < \frac{\pi}{2} \end{cases}$$

**12. Ans ( A )**

$$S = \lim_{n\to\infty}\left\{\frac{1}{(n+1)(n+2)} + \frac{1}{(n+2)(n+4)} + \ldots + \frac{1}{(n+n)(n+2n)}\right\}$$

$$= \lim_{n\to\infty}\sum_{r=1}^{n} \frac{n^2}{(n+r)(n+2r)}\cdot\frac{1}{n}$$

$$= \lim_{n\to\infty}\sum_{r=1}^{n} \frac{1}{\left(1+\frac{r}{n}\right)\left(1+\frac{2r}{n}\right)}\cdot\frac{1}{n}$$

$\frac{r}{n} \to x, \ \frac{1}{n} \to dx$

$a = \lim_{n\to\infty}(r/n)_{r=1} = 0, \ b = \lim\left(\frac{r}{n}\right)_{r=n} = 1$

$$\Rightarrow S = \int_0^1 \frac{dx}{(1+x)(1+2x)} = \int_0^1 \left(\frac{2}{1+2x} - \frac{1}{1+x}\right)dx$$

$$= [\log(1+2x) - \log(1+x)]_0^1 = \log(3/2)$$

**13. Ans ( A )**

$$4\alpha\int_{-1}^{0} e^{-\alpha|x|}\,dx + 4\alpha\int_{0}^{2} e^{-\alpha|x|}\,dx = 5$$

$$4\alpha\int_{-1}^{0} e^{\alpha x}\,dx + 4\alpha\int_{0}^{2} e^{-\alpha x}\,dx = 5$$

$\Rightarrow 4e^{-2\alpha} + 4e^{-\alpha} - 3 = 0$

Let $e^{-\alpha} = t$

$4t^2 + 4t - 3 = 0$

$(2t+3)(2t-1) = 0$

$t = \frac{-3}{2}; t = \frac{1}{2}$

$e^{-\alpha} = -\frac{3}{2}; e^{-\alpha} = \frac{1}{2}$ (possible)

(not possible)

$-\alpha = \ell n\left(\frac{1}{2}\right) \Rightarrow \alpha = \ell n 2$

**14. Ans ( A )**

$$I = \int \frac{3e^x - 5e^{-x}}{4e^x + 5e^{-x}}dx$$

Let $3e^x - 5e^{-x} = A(4e^x + 5e^{-x}) + B(4e^x - 5e^{-x}) + c$

Then $4(A + B) = 3$, $5(A - B) = -5$, $C = 0$

$\therefore A = -1/8$, $b = 7/8$

$$\therefore I = -\frac{1}{8}\int dx\frac{7}{8}\int \frac{(4e^x - 5e^{-x})\,dx}{4e^x + 5e^{-x}}$$

$$= -\frac{1}{8}x + \frac{7}{8}\ln\left(4e^x + 5e^{-x}\right)$$

$\Rightarrow a = -1/8$, $b = 7/8$

**15. Ans ( D )**

$$\frac{xdy - ydx}{x^2} = \sqrt{1 - \left(\frac{y}{x}\right)^2}\cdot\frac{dx}{x}$$

$$\frac{d\left(\frac{1}{x}\right)}{\sqrt{1 - \left(\frac{y}{x}\right)^2}} = \frac{dx}{x} \Rightarrow \sin^{-1}\left(\frac{y}{x}\right) = \log x + C$$

$x = 1$, $y = 0$

$\Rightarrow C = 0$

$y = x\sin(\log x)$

**16. Ans ( B )**

Use the property : Focus divides the focal chords into 2 line segments such that their harmonic mean is equal to the semi latus rectum of the parabola.

## MATHEMATICS

### SECTION-II

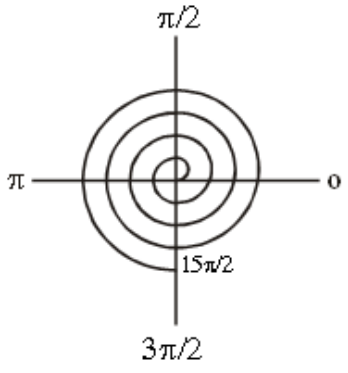
**1. Ans ( 15.00 )**

$2\tan^2 x - 5\sec x = 1$

$2(\sec^2 x - 1) - 5\sec x = 1$

$2\sec^2 x - 5\sec - 3 = 1$

$\therefore \cos x = \frac{1}{3}$




**2. Ans ( 48.00 )**

Let 7 observations be $x_1, x_2, x_3, x_4, x, x_6, x_7$

$$\bar{x} = 8 \Rightarrow \sum_{i=1}^{7} x_i = 56 \quad ...(1)$$

Also $\sigma^2 = 16$

$$\Rightarrow 16 = \frac{1}{7}\left(\sum_{i=1}^{7} x_i^2\right) - (\bar{x})^2$$

$$\Rightarrow 16 = \frac{1}{7}\left(\sum_{i=1}^{7} x_i^2\right) - 64$$

$$\Rightarrow 16 = \frac{1}{7}\left(\sum_{i=1}^{7} x_i^2\right) - 64 \quad ...(2)$$

Now, $x_1 = 2$, $x_2 = 4$, $x_3 = 10$, $x_4 = 12$, $x_5 = 14$

$\Rightarrow x_6 + x_7 = 14$ (from (1)) & $x_6^2 + x_7^2 = 100$ (from (2))

$\Rightarrow$

$$x_6^2 + x_7^2 = (x_6 + x_7)^2 - 2x_6.x_7 \Rightarrow x_6.x_7 = 48$$

**3. Ans ( 4.00 )**

P(hitting the target at least once) > 0.99

$$\Rightarrow 1 - \underbrace{\frac{1}{4}\cdot\frac{1}{4}\cdot\frac{1}{4}\cdots\cdots\frac{1}{4}}_{n\text{ times}} > 0.99$$

$$\Rightarrow 1 - \left(\frac{1}{4}\right)^n > 0.99 \Rightarrow 4^n > \frac{1}{0.01} \Rightarrow 4^n > 100$$

So minimum value of n to satisfy the inequality is 4.

**4. Ans ( 25.00 )**

Sum of 9 term = $9 \times 15 = 135$

New sum when Mean is 16

$= 16 \times 10 = 160$

New term = $160 - 135 \Rightarrow 25$

**6. Ans ( 4.00 )**

Apply LMVT in $x \in [1, 2]$

$$-2 \le \frac{f(2) - f(1)}{2 - 1} \le 2$$

$$\Rightarrow -2 \le f(2) - 2 \le 2$$

$$0 \le f(2) \le 4 \quad ...(1)$$

Apply LMVT in $x \in [2, 4]$

$$-2 \leqslant \frac{f(4) - f(2)}{4 - 2} \leqslant 2 \Rightarrow -4 \le 8 - f(2) \le 4$$

$4 \leqslant f(2) \leqslant 12$ from (1) & (2) we get $f(2) = 4$

**7. Ans ( 20.00 )**

$\int \frac{x+\cos 2x+1}{x\cos^2 x}dx = f(x) + K\log|x| + C,$

and $f(\pi/4) = 1$

$\int \frac{x+2\cos^2 x}{x\cos^2 x}dx = \int \frac{x}{x\cos^2 x}dx + \int \frac{2}{x}dx$

$= \tan x + 2\log|x| + C$

$f(x) = \tan x,\ K = 2$ (by comparing)

Now, $f(0) + 10K = \tan 0 + 10\times 2 = 0 + 20 = 20$

**8. Ans ( 0.00 )**

$\int_0^1 \tan^{-1}\left(\frac{2x-1}{1+x-x^2}\right)dx$

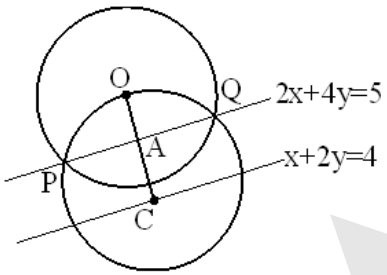
$= \int_0^1 \tan^{-1}\left(\frac{x-(1-x)}{1+x(1-x)}\right)dx$

$= \int_0^1 (\tan^{-1}(x) - \tan^{-1}(1-x))dx$

$= \int_0^1 \tan^{-1}(x) - \int_0^1 \tan^{-1}(1-x)dx$

$= 0$

**9. Ans ( 2.00 )**




M-I

$OA = \frac{\sqrt{5}}{2}$ $\qquad$ $OC = \frac{4}{\sqrt{5}}$

$CQ = OC = \frac{4}{\sqrt{5}}$ and $CA = \frac{3}{2\sqrt{5}}$

$\therefore OQ = \sqrt{OA^2 + AQ^2} = \sqrt{OA^2 + (CQ^2 - CA^2)}$

$\therefore \sqrt{\frac{5}{4} + \frac{16}{5} - \frac{9}{20}} = \sqrt{4}$

$\Rightarrow 2 = r$

**10. Ans ( 1.00 )**

$\vec{p}, \vec{q}, \vec{r}$ being coplanar

$[\vec{p}\ \vec{q}\ \vec{r}] = 0$

$$\begin{vmatrix} a+1 & a & a \\ a & a+1 & a \\ a & a & a+1 \end{vmatrix} = 0$$

$\Rightarrow a = -\frac{1}{3}$

$3(\vec{p}\ .\ \vec{q})^2 = \lambda|\vec{r}\times\vec{q}|^2$

$3\left(\frac{1}{9}\right) = \lambda\left(\frac{1}{3}\right)$

$\lambda = 1.$