(1001CJA102120048)

Test Pattern

ALLEN CAREER INSTITUTE KOTA (RAJASTHAN)

CLASSROOM CONTACT PROGRAMME

JEE(Advanced)
FULL SYLLABUS

SAMPLE PAPER-5

ANSWER KEY — PAPER-2

PART-1 : PHYSICS

SECTION-I	Q.	1	2	3	4	5	6
	A.	A,C	B,C,D	B,C	A,C	A,B	A,C
SECTION-II	Q.	1	2	3	4	5	6
	A.	0.76 to 0.77	57.82	13.12	5.64 to 5.66	61.20	5.80 to 6.00
SECTION-III	Q.	1	2	3	4	5	6
	A.	3	3	8	0	1	5

PART-2 : CHEMISTRY

SECTION-I	Q.	1	2	3	4	5	6
	A.	B,C	B,C	A,B,C,D	A,B,D	A,B,C,D	A,C
SECTION-II	Q.	1	2	3	4	5	6
	A.	–3.00	3.00 OR 7.00	–105.50	4.00	4.00	94.00
SECTION-III	Q.	1	2	3	4	5	6
	A.	2	4	3	9	4	3

PART-3 : MATHEMATICS

SECTION-I	Q.	1	2	3	4	5	6
	A.	A,C	B,D	A,C	B,C	A,D	A,B
SECTION-II	Q.	1	2	3	4	5	6
	A.	4.80	0.50	0.84	0.50	0.24 or 0.25	1.50
SECTION-III	Q.	1	2	3	4	5	6
	A.	1	2	1	4	2	3

SAMPLE PAPER-5

PAPER-2

PART-1 : PHYSICS SOLUTION

SECTION-I

1. Ans. (A,C)

Sol. $F = M\frac{\partial B}{\partial x} = ia^2 \times \frac{\mu_0 I}{2\pi x^2}$

Circular path $\frac{\mu_0 I i a^2}{2\pi b^2} = \frac{mv^2}{b}$

$k = \frac{\mu_0 I i a^2}{4\pi b}$

for escaping U + k = U' + k' = 0

$U = -\vec{M}.\vec{B}$

$= -\frac{\mu_0 I}{2\pi b} ia^2$

$k = \frac{\mu_0 I i a^2}{2\pi b}$

2. Ans. (B,C,D)

Sol. At equator $g'' = g - \omega^2 R$

Time period :

$T'' = 2\pi\sqrt{\frac{L}{g''}}$ = 2 sec

where g = 9.8 m/s²

(A) g' = g > g'' at the poles

$T = 2\pi\sqrt{\frac{L}{g'}} = 2\pi\sqrt{\frac{L}{g}} < 2\,\text{sec}$

(B) L' = L (1 + α ΔT) > L; α is coefficient of linear expansion

$T' = 2\pi\sqrt{\frac{L'}{g}} > 2\,\text{sec}$

(C) $g''' = g_0 - \omega^2 r$: inside statellite

also $mg_0 = m\omega^2 r$

$g''' = 0$

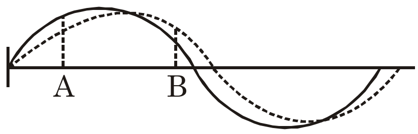
$T''' = 2\pi\sqrt{\frac{L}{g'''}}$ = infinite > 2 sec

(D) $g_1 = \frac{g''}{\left[1+\frac{h}{R}\right]^2} < g''$

$T_1 = 2\pi\sqrt{\frac{L}{g_1}} > 2\,\text{sec}$

3. Ans. (B,C)

Sol.



$\frac{\sqrt{3}}{2} = \sin\phi_A$

$\phi_A = \frac{\pi}{3}$

$\frac{1}{2} = \sin\phi_B$

$\phi_B = \frac{5\pi}{6}$

$\frac{5\pi}{6} - \frac{\pi}{3} = \frac{2\pi}{\lambda} \times 13$

$\Rightarrow \frac{3}{6} = \frac{13}{\lambda} \times 2$

λ = 52 cm

AC = λ = 52 cm

BC = 52 – 13 = 39 cm

y = 0 ⇒ kx + π/3 = π & 2π

$\frac{2\pi x}{52} + \frac{\pi}{3} = \pi$

$x = \frac{52}{3}\text{cm} \ \& \ \frac{130}{3}\text{cm}$

4. Ans. (A,C)

Sol. $\dfrac{\frac{dH}{dt}}{\text{mass}} = \dfrac{4eAT_0^3(T-T_0)\sigma}{\rho V}$

$\propto \dfrac{1}{r}$

$\dfrac{dH}{Adt} = 4eT_0^3(T-T_0)$

5. Ans. (A,B)

Sol. $v^2 = 2as = 2 \times 2 \times 200$

$v = 20\sqrt{2}$ m/s

$k = \dfrac{1}{2} \times 1200 \times 800$

$= 480$ kJ

$f = ma = 2400$ N

6. Ans. (A,C)

Sol. (1) $f = -2.5$ cm

$\dfrac{1}{v} + \dfrac{1}{-25} = -\dfrac{1}{2.5}$

$\dfrac{1}{v} = \dfrac{1}{25} - \dfrac{10}{2.5} = \dfrac{-9}{25}$

$v = -\dfrac{25}{9}$cm, $m = \dfrac{-v}{u} = -\dfrac{1}{9}$

(2) $\dfrac{1}{v'} - \dfrac{1}{25} = \dfrac{1}{2.5}$

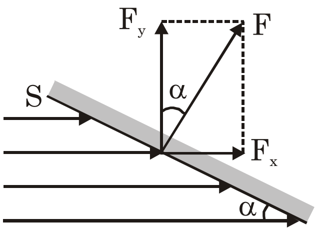
$\dfrac{1}{v'} = \dfrac{10}{25} + \dfrac{1}{25} \Rightarrow v' = \dfrac{25}{11}$cm

$m' = -\dfrac{v'}{u'} = +\dfrac{1}{11}$cm

SECTION-II

1. Ans. 0.76 to 0.77

Sol.



$\Delta p_m = 2mv \sin\alpha$

In a unit of time the area S (figure) will be struck by all the molecules that are located at a distance v from it—that is, by molecules located in a volume $Sv\sin\alpha$. If n is the concentration of the molecules, $N = nSv\sin\alpha$ molecules will strike the area S per unit time.

Thus the net momentum imparted to the object per unit time is directed perpendicular to the surface and is equal to

$\Delta p_m.N = 2mv\sin\alpha.nSv\sin\alpha$

$= 2nmv^2S\sin^2\alpha,$

$F = 2v^2S\sin^2\alpha.nm$

$F_y = F\cos\alpha = 2v^2S\sin^2\alpha\cos\alpha,nm$

which is maximum at $\tan\alpha = \sqrt{2}$

$F_{y\max} = \dfrac{4}{3\sqrt{3}}\rho v^2S = 0.77\rho v^2S$

2. Ans. 57.82

Sol. 10 VSd = 9 CSd

2 VSd = 0.9 CSd × 2

Reading

$= 5.5 \text{ mm} + \dfrac{(30-2\times0.9)\text{CSd}}{50\text{CSd}} \times 0.5\text{mm}$

$= 5.5 + 0.282$ mm

$= 5.782$ mm

3. Ans. 13.12

Sol. $\dfrac{1}{f} = (1.4-1)\left(\dfrac{1}{25} - \dfrac{1}{-25}\right) = \dfrac{0.8}{25} = \dfrac{4}{125}$

$f = 31.25$

$NS = 35 - 31.25 = 3.75 = t\left(1 - \dfrac{1}{\mu}\right)$

$= t\left(1 - \dfrac{1}{1.4}\right) = t \times \dfrac{0.4}{1.4}$

$t = 3.75 \times 3.5 = 13.125$ cm

≈ 13.12

4. Ans. 5.64 to 5.66

Sol. $T = Cr^x\rho^y\sigma^z$

$\Rightarrow M^0L^0T^1 = L^x(ML^{-3})^y(MT^{-2})^z$

$z = -\dfrac{1}{2}$

$y = \dfrac{1}{2}$

$x = \dfrac{3}{2}$

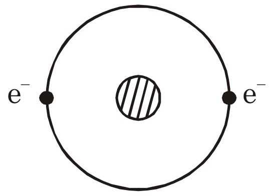
$T = C\sqrt{\dfrac{\rho r^3}{\sigma}}$

$\dfrac{T}{2} = (2^3)^{1/2}$

$T = 4\sqrt{2} = 5.64$ to 5.66 sec.

5. Ans. 61.20

Sol.



$$\frac{e^2}{4\pi\varepsilon_0 r^2}-\frac{e^2}{4\pi\varepsilon_0\times 4r^2}=\frac{mv^2}{r}$$

$$\frac{3e^2}{16\pi\varepsilon_0}=mv^2r$$

$$\frac{h}{2\pi}=mvr\times 2$$

$$v=\frac{3e^2}{4\varepsilon_0 h}$$

$$r=\frac{h}{4\pi\times m\times 3e^2}4\varepsilon_0 h=\frac{\varepsilon_0 h^2}{3\pi me^2}$$

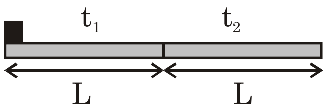
$$E=\frac{3e^2}{16\pi\varepsilon_0 r}-\frac{e^2}{4\pi\varepsilon_0 r}\times 2+\frac{e^2}{4\pi\varepsilon_0\times 2r}$$

$$=-\frac{3e^2\times 3\pi me^2}{16\pi\varepsilon_0\times\varepsilon_0 h^2}$$

$= 9/2 \times 13.6$

6. Ans. 5.80 to 6.00

Sol.



$a_b = -\mu g$

$$2L=\frac{1}{2}(\mu g)(t_1+t_2)^2$$

$$L=0\times t_2+\frac{1}{2}\mu g t_2^2$$

$$t_2=\sqrt{\frac{2L}{\mu g}}$$

$$t_1=\left(2-\sqrt{2}\right)\sqrt{\frac{L}{\mu g}}$$

$$\frac{\mu mg}{2M}=a_1;\ a_2=\frac{\mu mg}{M}$$

$$v_1=\frac{\mu mg}{2M}\times t_1$$

$$v_2=v_1+\frac{\mu mg}{M}t_2$$

$$=\frac{\mu mg}{2M}t_1+\frac{\mu mg}{M}t_2$$

$$\frac{v_2}{v_1}=1+\frac{2t_2}{t_1}=\frac{\sqrt{2}+1}{\sqrt{2}-1}$$

SECTION-III

1. Ans. 3

Sol. Time constant : $\tau = RC$

$$C=\frac{2\pi\varepsilon_0(1+K)ab}{(b-a)}$$

$K = 2$

$$C=\frac{6\pi\varepsilon_0 ab}{(b-a)}$$

$$R=\frac{1}{2\pi\sigma}\frac{(b-a)}{(ab)}$$

$$\tau=\frac{3\varepsilon_0}{\sigma}=\frac{x\varepsilon_0}{\sigma}$$

$x = 3$

2. Ans. 3

Sol. 40 gm → 1 mole

2000 gm → 50 mole

$$1\times C\int_T^8\left(\frac{T}{T_0}\right)^3 dT=50C\int_1^T\left(\frac{T}{T_0}\right)^3 dT$$

$$\frac{8^4}{4}-\frac{T^4}{4}=\frac{50}{4}\left(T^4-1^4\right)$$

$8^4 + 50 = 51T^4$

$$T=\left(\frac{8^4+50}{51}\right)^{1/4}\approx 3k$$

3. Ans. 8

Sol. $kx\cos37° = T\cos53°$

$kx\cos53 + T\cos37 = mg$

$$\Rightarrow\ \frac{T(\cos^2 53°+\cos^2 37)}{\cos 37}=mg\ \Rightarrow\ T=\frac{4mg}{5}$$

$$k\vec{x}+\vec{T}+m\vec{g}=0$$

$k\vec{x}$ does not change suddenly

$$\Rightarrow\ k\vec{x}+m\vec{g}=m\vec{a}=-\vec{T}$$

$$ma=\frac{4mg}{5}$$

$\Rightarrow\ a = 8\ m/s^2$

4. Ans. 0

Sol. When slider is at A, the applied potential difference across the photoelectric cell is a retarding potential difference of 10 V.

Energy of incident photon

$$E = \frac{hc}{\lambda} = \frac{1240\ \text{eVnm}}{350\ \text{nm}} = 3.5\ \text{eV}$$

Maximum KE of emitted photoelectrons is

$$KE_{max} = \frac{hc}{\lambda} - \phi = 3.5 - 2.2 = 1.3\ \text{eV}$$

∴ Stopping potential difference

$V_S = 1.3$ V

(a) The applied potential difference is $V(= 10\ V) > V_s\ (= 1.3\ V)$

Hence there is no photocurrent and reading of Ammeter is zero.

5. Ans. 1

Sol. $\frac{dm}{dt} \times S \times \Delta T = 0.7\ \rho J^2 V$

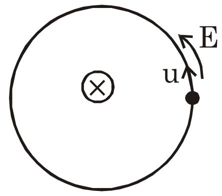
$$\frac{dm}{dt} = \frac{0.7}{10} \times 10^{-6} \times \left(\frac{3}{10^{-6}}\right)^2 \times \frac{10 \times 10^{-6}}{4200 \times 90}$$

= 0.01 kg/min = 1 ×10 gm/min

6. Ans. 5

Sol. $E = \frac{R}{2}\frac{dB}{dt} = \frac{R}{2}\alpha$

$\frac{mdV}{dt} = qE \qquad \Rightarrow v = \frac{qE_t}{m} = \frac{qR\alpha t}{2m}$



$$-N + qvB = \frac{mv^2}{R}$$

$$-N = \frac{mv^2}{R} - qvB$$

$$N = \frac{-q^2\alpha^2Rt^2}{4m} + q(B_0 + \alpha t)\frac{qR\alpha t}{2m}$$

$$= \frac{q^2\alpha^2Rt^2}{4m} + \frac{q^2B_0R\alpha t}{2m}$$

$$= \frac{q^2\alpha tR}{4m}[2B_0 + \alpha t]$$

$$= \frac{10^{-6} \times 10^{-3} \times 5 \times 1}{4 \times \frac{10^{-6}}{16}}[0.02 + 0.005]$$

$= 5 \times 10^{-4}$ N

PART–2 : CHEMISTRY SOLUTION

SECTION-I

1. Ans.(B,C)

2. Ans.(B,C)

Sol. p-chloroaniline and anilinium hydrochloride distinguised by Sandmeyer reaction $NaHCO_3$ and $AgNO_3$.

Therefore option A, B and C are correct.

3. Ans.(A,B,C,D)

Informations based on definitions

4. Ans.(A,B,D)

5. Ans.(A,B,C,D)

Sol. Main reaction of carbonyl compounds is Nucleophilic addition.

Reaction with ammonia derivatives are nucleophile addition Reaction.

$>C = O + HO - NH_2 \rightarrow\ >C=N - OH$

Hydroxylamine — Oxime

$>C = O + NH_2 - NH_2 \rightarrow\ >C= N - NH_2$

Hydrazine — Hydrozone

$>C= O + NH_2–NH–C_6H_5 \rightarrow\ >C= N–NH–C_6H_5$

Phenyldrazine

Phenylhydrazone

$>C= O + NH_2–NH–C(=O)–NH_2 \rightarrow\ >C=N–NH–C(=O)–NH_2$

Semicarbazide

Semicarbazone

Therefore option (A), (B), (C) and (D) are correct.

6. Ans.(A,C)

(A) conductivity will increase on adding electrolyte.

(C) Macromolecular colloids are normally lyophilic.

SECTION-II

1. Ans.(–3.00)

Sol. $2XeF_6 + SiO_2 \longrightarrow \underset{\substack{\text{Non polar} \\ (P)}}{SiF_4} + \underset{\substack{\text{Polar} \\ (Q)}}{2XeOF_4}$

x = 6

y = 3

⇒ 3 – 6 = (–3)

2. Ans. (3.00 OR 7.00)

3. Ans. (–105.50)

$$-13.7=\left[(\Delta_f H^0)_{\underset{(aq.)}{KCl}}+(-68.4)\right]-[(-116.5)+(-39.3)]$$

$$(\Delta_f H^0)_{\underset{(aq.)}{KCl}}=-101.1\text{kcal}$$

$4.4 = (-101.1) - (\Delta_f H^0)_{KCl(s)}$

$(\Delta_f H^0)_{KCl(s)} = -105.5$ kcal

4. Ans.(4.00)

5. Ans.(4.00)

6. Ans.(94.00)

For ppt of $SrCrO_4$

$[Sr^{+2}]\ [CrO_4^{--}] = 4 \times 10^{-9}$

$[CrO_4^{--}] = 2 \times 10^{-8}$ M

$[Ba^{+2}] \times (2 \times 10^{-8}) = 1.2 \times 10^{-10}$

$[Ba^{+2}] = 0.6 \times 10^{-2}$ M

$$\%\text{ of } Ba^{2+}\text{ ppt}=\frac{(0.1-0.6\times10^{-2})}{0.1}\times100=94\%$$

SECTION-III

1. Ans.(2)

2. Ans.(4)

3. Ans.(3)

$$V=\frac{58.5}{2.167}\approx 27\text{cm}^3$$

$a^3 = 27\ cm^3$

a = 3 cm

4. Ans.(9)

5. Ans.(4)

6. Ans.(3)

CH_4	G
V	V
n	n
T	T
σ	σ

$$\frac{(u_{avg})_{CH_4}}{(u_{rms})_G}=\frac{\sqrt{\frac{8}{\pi\times16}}}{\sqrt{\frac{3}{M_G}}}$$

$$\frac{48}{\pi}=\frac{8}{16\pi}\times\frac{M_G}{3}$$

$M_G = 288$

$$\frac{(Z_1)_{CH_4}}{(Z_1)_G}=\frac{\sqrt{\frac{1}{16}}}{\sqrt{2}\times\sqrt{\frac{1}{288}}}=\sqrt{\frac{288}{32}}=\sqrt{\frac{9}{1}}=\frac{3}{1}$$

PART-3 : MATHEMATICS SOLUTION

SECTION–I

1. Ans. (A,C)

$(1 + \lambda) > 0$, graph of

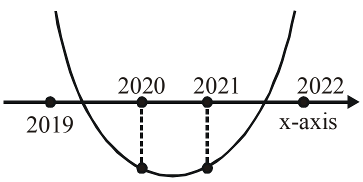
$y = (x - 2019)(x - 2022) + \lambda(x - 2020)(x - 2021)$ is

$f(2019) = 2\lambda$

$f(2020) = -2$

$f(2021) = -2$

$f(2020) = 2\lambda$



2. Ans. (B,D)

$y = m(x - a)$ is a focal chord so, angle between normals drawn at A and B = 90°
length of focal chord = 4a $\text{cosec}^2\alpha$

for m = 1, $\alpha=\frac{\pi}{4}\Rightarrow \text{cosec}^2\frac{\pi}{4}=2$

AB = 4a × 2 = 8a

3. Ans. (A,C)

n(A) = number of elements in A = m
n(B) = number of elements in B = 12 – m
$m \neq 0,6,12$

For A, we have 10 elements to choose from

$$N=\sum_{m=1}^{11}{}^{10}C_{m-1}-{}^{10}C_5=272$$

4. Ans. (B,C)

$$(Z_1-Z_0)(\bar{Z}_1-\bar{Z}_0)=16R^2$$

$$\Rightarrow Z_1\bar{Z}_1-Z_0\bar{Z}_1-\bar{Z}_0Z_1+|Z_0|^2=16R^2 \quad ..(1)$$

$$\left(\frac{1}{\bar{Z}_1}-Z_0\right)\left(\frac{1}{Z_1}-\bar{Z}_0\right)=256R^2$$

$$\frac{1}{Z_1\bar{Z}_1}-\frac{Z_0}{Z_1}-\frac{\bar{Z}_0}{\bar{Z}_1}+|Z_0|^2=256R^2$$

put $|Z_1|^2 = k$

$$1-Z_0\bar{Z}_1-\bar{Z}_0Z_1+K|Z_0|^2=256R^2K$$

(1) – (2)

$\Rightarrow (K - 1) + (1 - K)\ |Z_0|^2 = 16R^2(1 - 16K)$

$\Rightarrow (1 - K) + (|Z_0|^2 - 1) = 16R^2(1 - 16K)$

$\Rightarrow (1-K)=(1-16K)$

$$\Rightarrow K=\frac{31}{511}$$

5. Ans. (A,D)

A(0,0,0), B(1,2,3)

one two points on L_1

foot of perpendicular of B in P_1

$$\frac{x-1}{1}=\frac{y-2}{1}=\frac{z-3}{1}=-\frac{(1+2+3)}{3}=-2$$

$\Rightarrow$ x= –1, y = 0, z = 1

Q(–1,0,1)

D.R^5 of AQ < –1, 0,1 >

a : b : c :: –1, 0,1

6. Ans. (A,B)

P"(x) = 6(x – 1)

$$\Rightarrow P'(x)=\frac{6(x-1)^2}{2}+C_1$$

$O = 3 + C_1 \Rightarrow C_1 = -3$

$P'(x) = 3(x-1)^2 - 3$

$P(x) = (x-1)^3 - 3x + C$

O = –3 + C

$P(x) = (x-1)^3 - 3x + 3$

$= (x-1)^3 - 3(x-1)$

$= (x-1)[x^2 - 2x - 2]$

SECTION–II

1. Ans. 4.80

We have $A^3 - 6A^2 + 7A + 2I = 0$

we have $A^3 - 6A^2 + 7A + 2I = 0$

$\Rightarrow$ B = 2I

$|B| = 8 \Rightarrow |B|^{3-2} = 8$

$\Rightarrow$ C = 8B = 16I

2. Ans. 0.50

$$N=\int_0^1 x\left\{\frac{d}{dx}\left(4\times(1-x^2)^3\times(-2x)\right)\right\}dx$$

$$=-8x^2(1-x^2)^3\Big|_0^1+8\int_0^1(1-x^2)^3\times xdx$$

$$=0+4\int_0^1(1-x^2)^3(2x)dx$$

$$=-4\int_1^0 t^3dt=1$$

3. Ans. 0.84

$\vec{r}\times\vec{b}=\vec{c}\times\vec{b}$

$\Rightarrow(\vec{r}-\vec{c})\times\vec{b}=\vec{0}$

$\vec{r}=\vec{c}+\lambda\vec{b}$

$0=\vec{c}\,.\,\vec{a}+\lambda\vec{b}\,.\,\vec{a}$

= 1 + 2 + 3 + (3 + 4 + 3)λ

$$\lambda=-\frac{6}{10}=-\frac{3}{5}$$

$$\vec{r}=\hat{i}+\hat{j}+\hat{k}-\frac{9}{5}\hat{i}-\frac{6}{5}\hat{j}-\frac{3}{5}\hat{k}$$

$$\vec{r}=-\frac{4}{5}\hat{i}+\frac{-1}{5}\hat{j}+\frac{2}{5}\hat{k}$$

$$|\vec{r}|^2=\frac{21}{25}=0.84$$

4. Ans. 0.50

point of intersection of

$y=(2-\sqrt{3})x$ and $y-1=x(2+\sqrt{3})+2+\sqrt{3}$

$$\Rightarrow(2-\sqrt{3})x-1=x(2+\sqrt{3})+2+\sqrt{3}$$

$$\Rightarrow-2\sqrt{3}x=3+\sqrt{3}\Rightarrow x=\frac{\sqrt{3}+1}{-2}$$

$$y=(2-\sqrt{3})\frac{(1+\sqrt{3})}{(-2)}=\frac{2+\sqrt{3}-3}{-2}=\frac{-1+\sqrt{3}}{-2}$$

$$\Rightarrow\alpha=\frac{\sqrt{3}-1}{2},\beta=\frac{\sqrt{3}+1}{2}$$

$$\Rightarrow\alpha\beta=\frac{2}{4}=\frac{1}{2}=0.50$$

5. Ans. 0.24 or 0.25

$$P\left(\frac{W}{L}\right)=\frac{2}{3},P\left(\frac{L}{W}\right)=\frac{1}{3}$$

$$P\left(\frac{W}{L}\right)=\frac{1}{3},P\left(\frac{L}{L}\right)=\frac{2}{3}$$

Required probability

= P(WWWL) + P(WWLW)

+ P(WLWW) + P(LWWW)

$$=\left(\frac{2}{3}\right)^3\times\frac{1}{3}+\left(\frac{2}{3}\right)^2\times\left(\frac{1}{3}\right)^2+\left(\frac{2}{3}\right)\left(\frac{1}{3}\right)^2\times\left(\frac{2}{3}\right)$$

$$+\left(\frac{1}{3}\right)^2\times\left(\frac{2}{3}\right)^2$$

$$=\frac{8+4+4+4}{3^4}=\frac{20}{81}=.24,.25$$

6. Ans. 1.50

$\frac{T_{n+1}}{2^{n+1}} = \frac{T_n}{2^n} + \frac{1}{2} \times 2^n$

$\Rightarrow \frac{T_2}{2^2} - \frac{T_1}{2} = \frac{1}{2} \times 2$

$\frac{T_3}{2^3} - \frac{T_2}{2^2} = \frac{1}{2} \times 2^2$

$\frac{T_n}{2^n} - \frac{T_{n-1}}{2^{n-1}} = \frac{1}{2} \times 2^{n-1}$

$\frac{T_n}{2^n} - \frac{T_1}{2} = \frac{1}{2}\frac{\left(2^{n-1}-1\right)\times 2}{2^{-1}} = 2^{n-1} - 1$

$\Rightarrow T_n = 2^{2n-1} - 2^{n-1}$

$\Rightarrow S_n = \frac{2.4^n - 2 - 3.2^n + 3}{3}$

$= \frac{2.4^n - 3.2^n + 1}{3}$

SECTION–III

1. Ans. 1

$D = \begin{vmatrix} 1 & 1 & -2 \\ 2 & -3 & 1 \\ 1 & -5 & 4 \end{vmatrix} = 0$

and system has solution

$\Rightarrow \begin{vmatrix} 0 & 1 & -2 \\ 0 & -3 & 1 \\ k & -5 & 4 \end{vmatrix} = 0 \Rightarrow k = 0$

on solving given equations

$\Rightarrow x = y = z = \lambda$(suppose)

As $x_0, y_0, z_0 \in R - \{0\}$, $\lambda \neq 0$

$\frac{x_0y_0 + y_0z_0 + z_0x_0}{x_0^2 + y_0^2 + z_0^2} = \frac{\lambda^2 + \lambda^2 + \lambda^2}{\lambda^2 + \lambda^2 + \lambda^2} = 1$

2. Ans. 2

$f(x) = ax^n + bx^{n-1} + c\,x^{n-2} + \ldots$, $n \geq 2$

$2 + 2(ax^n + bx^{n-1} + cx^{n-2} + \ldots)$

$= a\{(x+1)^n + (x-1)^n\} + b\{(x+1)^{n-1} + (x-1)^{n-1}\}$

$+ c\{(x+1)^{n-2} + (x-1)^{n-2}\}$

$+ \ldots$

compare co-eff - of x^{n-2}

$\Rightarrow 2c = a \times {}^nC_2 \times 2 + 2c$

$\Rightarrow 0 = an(n-1)$ not possible

similarly not possible for $n < 2$

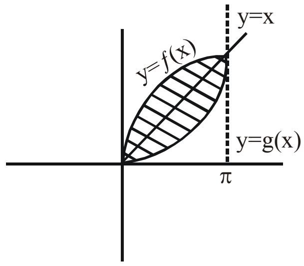
for $n = 2$ $f(x) = x^2 + bx + c$ satisfies

3. Ans. 1

$(2x + 3y - 8)(x + 2y - 5) = 40$

$\Rightarrow a = 2, b = 6, h = \frac{7}{2}, g = -9, f = -\frac{31}{2}$

4. Ans. 4



$A = \int_0^{\pi} \left(f(x) - g(x)\right)dx = 2\int_0^{\pi}\left(f(x) - x\right)dx = 4$

5. Ans. 2

Use $\tan\theta = \cot\theta - 2\cot 2\theta$

$\Rightarrow \tan x + \frac{1}{2}\tan\frac{x}{2} + \frac{1}{2^2}\tan\left(\frac{x}{2^2}\right) + \ldots$

$\ldots + \frac{1}{2^{n-1}}\tan\left(\frac{x}{2^{n-1}}\right) = \frac{1}{2^{n-1}}\cot\frac{x}{2^{n-1}} - 2\cot 2x$

Now differentiate w.r.t x

$\Rightarrow \sec^2 x + \frac{1}{2^2}\sec^2\frac{x}{2} + \frac{1}{2^4}\sec^2\frac{x}{2^2} + \ldots$

$+ \frac{1}{2^{n-2}}\sec^2\frac{x}{2^{n-1}}$

$= 4\,\mathrm{cosec}^2 2x - \frac{1}{2^{nx-2}}\mathrm{cosec}^2\frac{x}{2^{n-1}}$

$\Rightarrow a = 2$

6. Ans. 3

Replace x by $\frac{x-1}{x}$, we get

$f\left(\frac{x-1}{x}\right) + f\left(\frac{1}{1-x}\right) = \tan^{-1}\left(\frac{x-1}{x}\right)$ ...(2)

Replace x by $\frac{1}{1-x}$ in (1) we get

$f\left(\frac{1}{1-x}\right) + f(x) = \tan^{-1}\left(\frac{1}{1-x}\right)$ ...(3)

(1) – (2) + (3), we get

$2f(x) = \tan^{-1}x + \tan^{-1}\left(\frac{1}{1-x}\right) - \tan^{-1}\left(\frac{x-1}{x}\right)$

...(4)

Also,

$2f(1-x)$

$= \tan^{-1}(1-x) + \tan^{-1}\left(\frac{1}{x}\right) - \tan^{-1}\left(\frac{x}{x-1}\right)$ ..(5)

put $x = \frac{1}{2}$ and add (4) and (5)

$\Rightarrow 4f\left(\frac{1}{2}\right) = \frac{3\pi}{2}$

$\Rightarrow N = f\left(\frac{1}{2}\right) = \frac{3\pi}{8}$