(1001CJA102120047)

Test Pattern

ALLEN CAREER INSTITUTE KOTA (RAJASTHAN) path to success

CLASSROOM CONTACT PROGRAMME

JEE(Advanced)
FULL SYLLABUS

SAMPLE PAPER-5

ANSWER KEY PAPER-1

PART-1 : PHYSICS

SECTION-I	Q.	1	2	3	4	5	6	7	8	9	10
	A.	D	A	B	B	A	B	C	B,C	A,B,C,D	A,B,C,D
	Q.	11	12								
	A.	A,B	A,C								
SECTION-II	Q.	1	2	3	4	5	6				
	A.	3.00	0.66 to 0.67	286.66 to 286.67	3.00	0.56 to 0.57	2.18				

PART-2 : CHEMISTRY

SECTION-I	Q.	1	2	3	4	5	6	7	8	9	10
	A.	C	A	C	A	D	A	A,B,C,D	A,B,C	A,B,C,D	C,D
	Q.	11	12								
	A.	A,B,D	A,B,C,D								
SECTION-II	Q.	1	2	3	4	5	6				
	A.	3.00	5.00	1.91	4.00	4.00	–38.85 to –38.95				

PART-3 : MATHEMATICS

SECTION-I	Q.	1	2	3	4	5	6	7	8	9	10
	A.	B	A	A	A	C	A	A,D	B,C	A,B,C	A,D
	Q.	11	12								
	A.	A,C,D	A,B,C,D								
SECTION-II	Q.	1	2	3	4	5	6				
	A.	5.00	1.00	1.87 or 1.88	15.00	0.75	12.56 OR 12.57				

SAMPLE PAPER-5

PAPER-1

PART-1 : PHYSICS — SOLUTION

SECTION-I

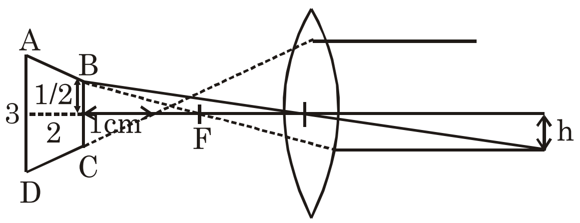
1. Ans. (D)

Sol. The initial velocity of smoke particles die out quickly due to air resistance. The Buoyant force is vertically up. So they move in upward direction.

But train moves forwards, so smoke trail is at an acute angle behind the train.

2. Ans. (A)

Sol.



$$\frac{h}{1} = m_1 = \frac{f}{-x_0}$$

$$\frac{h}{3} = m_2 = \frac{f}{-(x_0+2)}$$

$$\frac{f}{3x_0} = \frac{f}{x_0+2}$$

$\Rightarrow\ x_0 + 2 = 3x_0$

$x_0 = 1$ cm

$x_0 x_I = -f^2$

$1 \times x_I = +f^2 \Rightarrow x_I = f^2$

$$x_{I'} = \frac{f^2}{3}$$

$$x_I - I'_I = \frac{2}{3}f^2 = h = f$$

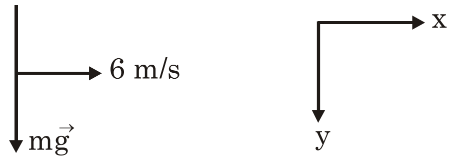
$f = 1.5$ cm

$h = 1.5$ cm

3. Ans. (B)

Sol. $mg = C \times 8$

$-C \times \vec{v}_{m\omega}$



$-C \times \vec{v}_{m\omega} = mg$

$v_{m\omega}$ should be vertically down.

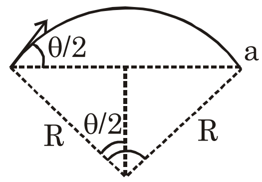
$\Rightarrow \vec{v}_{m\omega} = 8\text{m/s}\ \hat{i}$

$\vec{v}_m - \vec{v}_\omega = 8\hat{i}$

$v_m = 8\hat{i} + 6\hat{j}$

4. Ans. (B)

Sol.



$$\theta = \omega t = \frac{qBt}{m}$$

$$\sin\frac{\theta}{2} = \frac{a}{2R}$$

$$\Rightarrow\ \frac{\theta}{2} = \sin^{-1}\left(\frac{aqB}{2p_0}\right)$$

$$t = \frac{2m}{qB}\sin^{-1}\left(\frac{aqB}{2p_0}\right) = \frac{2m}{qB} \times \frac{\pi}{6}$$

$$= \frac{12}{\pi \times 1/5} \times \frac{\pi}{6} = 20 \text{ sec}$$

5. Ans. (A)

Sol. $P = i^2R = \dfrac{v_0^2}{2(x_0^2 + (R_1 + R_2)^2)} \times R_2$

$= \dfrac{900 \times 8}{2\left(\dfrac{1}{5000 \times 2\pi} \times 10^6\right)^2 + 20^2}$

$= \dfrac{3600}{1000 + 400} = \dfrac{3600}{1400} = 2.57W$

6. Ans. (B)

Sol. $\dfrac{df}{f^2} = \dfrac{du}{u^2} + \dfrac{dv}{v^2}$, du = dv

$\dfrac{1}{v} - \dfrac{1}{u} = \dfrac{1}{f} \Rightarrow \dfrac{1}{u^2} + \dfrac{1}{v^2} - \dfrac{2}{uv} = \dfrac{1}{f^2}$

$\dfrac{df}{f^2} = du\left(\dfrac{1}{f^2} + \dfrac{2}{uv}\right) = du\left(\dfrac{1}{f^2} + \dfrac{2}{u}\left(\dfrac{1}{f} + \dfrac{1}{u}\right)\right)$

$df = du\left(1 + \dfrac{2f}{u} + \dfrac{2f^2}{u^2}\right)$

$\Rightarrow$ df is minimum when

$0 = -\dfrac{2f}{u^2} - \dfrac{4f^2}{u^3}$

u = –2f

(for u = ∞, 2nd derivative is zero so it is not a minimum.)

7. Ans. (C)

Sol. $\dfrac{h}{\mu v_{rel}} = \dfrac{5}{8}\dfrac{h}{m_1v_1} = \dfrac{5}{2}\dfrac{h}{m_2v_2}$

$m_2v_2 = \pm 4m_1v_1$

Taking +ve signs

$\dfrac{m_1m_2}{m_1 + m_2}(v_1 - v_2) = \dfrac{8}{5}m_1v_1$

$m_2v_1 - m_2v_2 = \dfrac{8}{5}m_1v_1 + \dfrac{8}{5}m_2v_1$

$\dfrac{3}{5}m_2v_1 = -\dfrac{7}{5}m_1v_1$

takign $v_{rel} = v_2 - v_1$

$\Rightarrow \dfrac{m_2(v_2 - v_1)}{m_1 + m_2} = \dfrac{8}{5}v_1$

$m_2v_2 - m_2v_1 = \dfrac{8}{5}m_1v_1 + \dfrac{8}{5}m_2v_1$

$\dfrac{3}{5}m_2v_2 = \dfrac{13}{5}m_2v_1$

$v_2 = \dfrac{13}{3}v_1$

$m_2v_2 = 4m_1v_1$

$\dfrac{v_2}{v_1} = \dfrac{4m_1}{m_2} = \dfrac{13}{3}$

$\dfrac{m_1}{m_2} = \dfrac{13}{12}$ not possible as $m_1 < m_2$

Taking –ve sign

$m_2v_2 = -4m_1v_1$

$m_2v_1 - m_2v_2 = +\dfrac{8}{5}m_1v_1 + \dfrac{8}{5}m_2v_1$

$-\dfrac{3}{5}m_2v_1 = \dfrac{3}{5}m_2v_2$

$v_1 = -v_2$

or

$m_2v_2 - m_2v_1 = \dfrac{8}{5}m_1v_1 + \dfrac{8}{5}m_2v_1$

$m_2v_2 - \dfrac{8}{5}m_1v_1 = \dfrac{13}{5}m_2v_1$

$\dfrac{7}{5}m_2v_2 = \dfrac{13}{5}m_2v_1$

$\dfrac{v_2}{v_1} = \dfrac{13}{7}$ × not possible as $\dfrac{m_2}{m_1} = -ve$

8. Ans. (B,C)

Sol. Applying Bernaullis theorem from the frame of car.

$P_{in} + 0 = P_{out} + \dfrac{1}{2}\rho v^2$

$(P_{in} - P_{out}) \times (0.4)^2 = \sigma \times 1.6 \times t$

$\Rightarrow \dfrac{1}{2}\rho v^2 \times 0.16 = 4 \times 10^7 \times 1.6t$

$$\Rightarrow \frac{1}{2}\times\frac{1.25\times 40\times 40\times 0.16}{1.60\times 4\times 10^{7}} = t$$

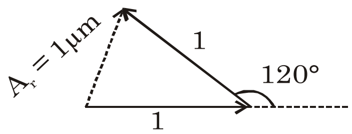
$t = 2.5\ \mu m$

9. Ans. (A,B,C,D)

Sol. $S_1 = 1\mu m \sin\left(\frac{\pi}{4}x - 100\pi t - \frac{\pi}{3}\right)$

$= 1\mu m \sin\left(\frac{\pi}{4}x - 100\pi t + \pi - \frac{\pi}{3}\right)$

$= 1\mu m \sin\left(\frac{\pi}{4}x - 100\pi t + \frac{2\pi}{3}\right)$



10. Ans. (A,B,C,D)

Sol. $Q = nC_v \times 15 = 600$

$\Rightarrow nC_v = 40$

$Q = nC_P \times 10 = 600$

$nC_P = 60$

$n(C_p - C_v) = 20$

$n\times\frac{25}{3} = 20$

$n = 2.4$

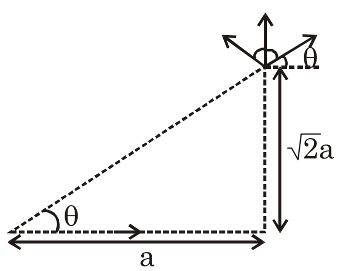
$\frac{C_P}{C_v} = 1.5$

$C_v = 2R = \frac{n_1\times\frac{5}{2}R + n_2\times\frac{3}{2}R}{n_1 + n_2}$

$\Rightarrow n_1 = n_2$

11. Ans. (A,B)

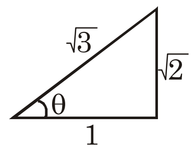
Sol.



$$\frac{2kp\cos^2\theta}{r^3} = \frac{kp\sin^2\theta}{r^3}$$

$\tan\theta = \sqrt{2}$

$$\frac{kp\cos\theta\sin\theta}{\sqrt{3}a^3} = \frac{kq}{2a^2}$$



12. Ans. (A,C)

Sol. $R_1 + R_2 = 4R_0$

$\Rightarrow \frac{R_1}{R_0} + \frac{R_2}{R_0} = 4 \Rightarrow x + y = 4$

$$\Rightarrow \frac{\frac{R_1R_2}{R_1 + R_2}}{R_0} = \frac{40}{60} = \frac{2}{3}$$

$\Rightarrow R_1R_2 = \frac{2}{3}4R_0^2$

$\Rightarrow xy = \frac{2}{3}\times 4 = \frac{8}{3}$

$x + \frac{8}{3x} = 4$

$3x^2 - 12x + 8 = 0$

$$x = \frac{12 \pm \sqrt{144 - 4\times 8\times 3}}{6} = \frac{R_1}{R_0} = \frac{x}{100 - x}$$

$0.84 = \frac{x}{100 - x} \Rightarrow x \approx 46$ cn

SECTION-II

1. Ans. 3.00

Sol. $P_{\alpha\text{-decay}} = 0.25\ \alpha\lambda_\alpha = \frac{\ln 2}{T_\alpha}$

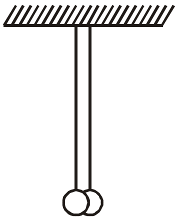
$P_\beta = 0.75\alpha\lambda_\beta = \frac{\ln 2}{T_\beta}$

$\Rightarrow \frac{1}{3} = \frac{T_\beta}{T_\alpha}$

$$\Rightarrow \frac{T_\alpha}{T_\beta} = 3$$

2. Ans. 0.66 to 0.67

Sol.



$$T = 2\pi\sqrt{\frac{\ell}{g}} = \text{Same for both.}$$

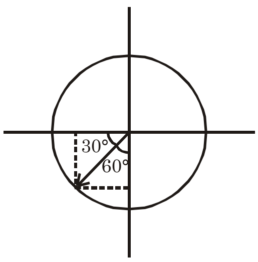
$\Rightarrow$ collision at mean position of the bobs.

$\Rightarrow$ velocities are exchanged

$\Rightarrow$ Amplitude are exchanged

$A_{\text{left bob}} = 6°$

$$T_1 = \frac{T}{4}$$



$$\omega T_2 = \frac{\pi}{6}$$

$$T_2 = \frac{T}{12}$$

$$\Rightarrow t = \frac{T}{4} + \frac{T}{12} = \frac{4T}{12} = \frac{T}{3} = 0.66 \text{ sec}$$

3. Ans. 286.66 to 286.67

Sol. Apply the condition for constructive interference twice:

$$\frac{(4\lambda_1)}{d} = \frac{x}{D} \text{ and } \frac{(6\lambda_2)}{d} = \frac{x}{d},$$

So that $4\lambda_1 = 6\lambda_2$.

If $\lambda_1 = 430$ nm, then $\lambda_2 = 287$ nm.

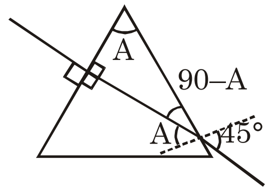
4. Ans. 3.00

Sol. $v_1 = -L_1\frac{dI_1}{dt} - M\frac{dI_2}{dt}$

$= -(0.050)(120) - (0.015)(-200) = -3V$

5. Ans. 0.56 to 0.57

Sol.



nsinA = 1sin45°

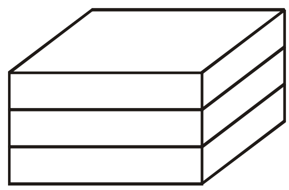
$$n = \frac{C}{v} = \frac{3\times10^8}{2.4\times10^8} = \frac{5}{4}$$

$$\sin A = \frac{2\sqrt{2}}{5} = \frac{2.82}{5}$$

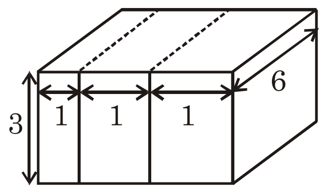
$= 0.56$ to 0.57

6. Ans. 2.18

Sol.



$$C_1 = \frac{\epsilon_0 \times 18}{\frac{1}{1}+\frac{1}{2}+\frac{1}{3}} = \frac{\epsilon_0 \times 18}{\frac{6+3+2}{6}} = \epsilon_0 \times \frac{108}{11}$$



$$C_2 = \frac{1\times\epsilon_0\times1\times6}{3} + \frac{2\times\epsilon_0\times1\times6}{3} + \frac{3\times\epsilon_0\times1\times6}{3}$$

$= \epsilon_0 \times 12$

$$\Rightarrow \frac{C_2 - C_1}{\epsilon_0} = 12\left[1 - \frac{9}{11}\right] = \frac{24}{11} = 2.18$$

PART–2 : CHEMISTRY SOLUTION

SECTION-I

1. **Ans.(C)**
2. **Ans.(A)**

Sol. At isoelectric point Amino acid has least viscosity. Therefore option **(A)** is correct.

3. **Ans.(C)**

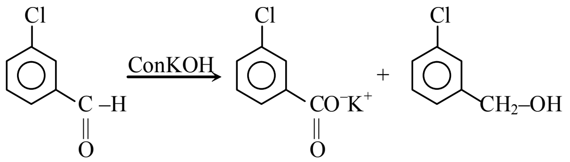
$Na_xA \rightarrow xNa^+ + A^{x-}$

$\Delta T_f = ik_f.m$

$3.255 = (x + 1) \times 1.86 \times 0.35$

$x = 4$

4. **Ans.(A)**
5. **Ans.(D)**

Sol. 

Metachloro benzaldehyde — Potassium meta-chloro benzoate — m-chloro-benzyl alcohol

It is an example of cannizzaro reaction. Therefore option (D) is correct.

6. **Ans.(A)**

$3Cu + 8\,HNO_3 \rightarrow 3Cu(NO_3)_2 + 2NO + 4H_2O$

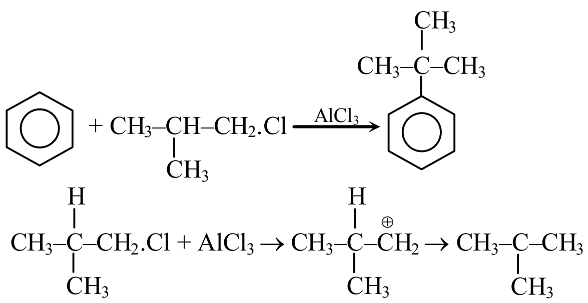
$\frac{95.25}{63.5} = 1.5\text{mol} \qquad \frac{8}{3} \times \frac{3}{2} = 4\text{mol}$

$= M \times V(l)$

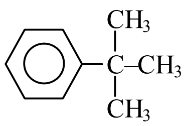
$= 1 \times V(l)$

$\therefore$ V = 4.0 litre.

7. **Ans.(A,B,C,D)**
8. **Ans.(A,B,C)**

Sol. 

→ Carbocation rearrangment

→ 

→ Friedal craft reaction

9. **Ans.(A,B,C,D)**

Initial mass of H_2SO_4 in the solution = 1260 × 0.40 = 504 gm H_2SO_4

Say Wgm H_2SO_4 consumed.

$$\frac{(504 - w)}{\left(1260 - w + \frac{18}{98}w\right)} \times 100 = 28$$

w = 196

$$n_{H_2SO_4} = \frac{196}{98} = 2$$

2 mol H_2SO_4 consumed ≡ 2F charge passed = 2 × 96500 = 1.93 × 10^5 coulomb.

10. **Ans.(C,D)**
11. **Ans.(A,B,D)**
12. **Ans.(A,B,C,D)**

Angular function α cos θ ⇒ orbital p_z

Power of σ is 1 ⇒ l = 1 ⇒ p-orbital

n – l – 1 = 1 ⇒ n = 3

∴ orbital is $3p_z$

Angular node = l = 1

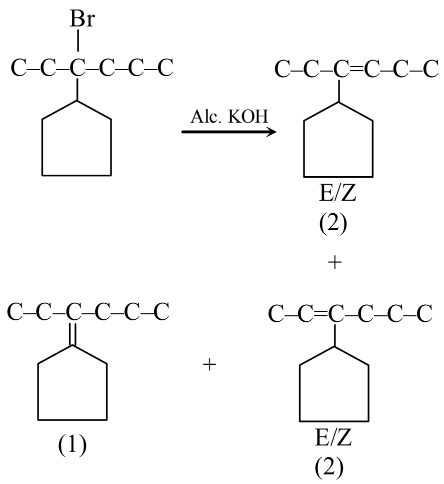
Radial node =n – l – 1 = 1

XY - plane is nodal plane

SECTION-II

1 **Ans.(3.00)**

2. **Ans.(5.00)**

Sol. 

Total five isomer

3. **Ans.(1.91)**

$$\ln 2 = \frac{E_a}{R}\left[\frac{1}{280} - \frac{1}{290}\right]$$

$$\ln \mu = \frac{E_a}{R}\left[\frac{1}{290} - \frac{1}{300}\right]$$

$$\frac{\ln \mu}{\ln 2} = \frac{280 \times 290}{290 \times 300}$$

$\mu = 1.91$

4. Ans.(4.00)

5. Ans.(4.00)

6. Ans.(–38.85 to –38.95)

$\Delta G = \Delta H - \Delta(T.S)$

or $\Delta G = n.C_{p,m}(T_2 - T_1) - (T_2.S_2 - T_1.S_1)$ ......(1)

Now , $\Delta S = S_{H_2(g,600K,2bar)} - S_{H_2(g,300K,1bar)}$

$= n.C_{p,m.} \ln \frac{T_2}{T_1} - nR.\ln \frac{P_2}{P_1}$

or, $S_{H_2(g,600K,2bar)} - 130.0$

$= 1 \times 28.314 \times \ln \frac{600}{300} - 1 \times 8.314 \times \ln \frac{2}{1}$

$\therefore S_{H_2(g,600K,2bar)} = 144.0 J/K - mol$

Eq(1) : $\Delta G = 1 \times 28.314 (600 - 300)$

$- (600 \times 144 - 300 \times 130)$

$= -38905.8$ J/mol = –38.9058 kJ/mol.

PART-3 : MATHEMATICS SOLUTION

SECTION–I

1. Ans. (B)

$a_1\ a_2\ a_3\ ....\ a_i\ a_{i+1}\ a_{i+2}\ .....\ a_n$

$$a_{i+1} = \frac{a_i + a_{i+2}}{2}$$

$a_i + a_{i+2} = 2a_i + 1$

$$S_n = \sum_{i=1}^{n} \frac{a_i a_{i+1} a_{i+2}}{2a_{i+1}} = \sum_{i=1}^{n} \frac{a_i a_{i+2}}{2}$$

$$S_n = \frac{1}{2}\sum_{i=1}^{n}\left[(5+(i-1)3)(5+(i+1)3)\right]$$

$$S_n = \frac{1}{2}\sum_{i=1}^{n}(3i+2)(8+3i)$$

$$S_n = \frac{1}{2}\sum_{i=1}^{n}(9i^2+30i+16)$$

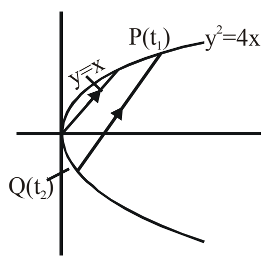
$$S_n = \frac{1}{2}\left[9.\sum_{i=1}^{n} i^2 + 30\sum_{i=1}^{n} i + \sum_{i=1}^{n} 16\right]$$

$$S_n = \frac{1}{2}\left[9.\frac{n(n+1)(2n+1)}{6} + 30\frac{n(n+1)}{2} + 16n\right]$$

$$S_{10} = \frac{1}{2}\left[\frac{9 \times 10 \times 11 \times 21}{6} + \frac{30 \times 10 \times 11}{2} + 16 \times 10\right]$$

$$S_{10} = \frac{1}{2}[3465 + 1650 + 160] = 2637.50$$

2. Ans. (A)



Let $P(t_1^2, 2t_1)$ and $Q(t_2^2, 2t_2)$

slope of chord $PQ = \frac{2}{t_1 + t_2}$

$\frac{2}{t_1 + t_2} = 1 \quad \therefore t_1 + t_2 = 2$ ...(1)

Equation of normal at P

$y = -t_1 x + 2t_1 + t_1^3$

equation of normal at Q

$y = -t_2 x + 2t_2 + t_2^3$

point of intersection of normals at P &Q

$x = t_1^2 + t_2^2 + t_1 t_2 + 2$ ...(3)

$y = -t_1 t_2 (t_1 + t_2)$ ...(4)

using (1) $y = -2t_1 t_2$ ...(2)

$x = (t_1 + t_2)^2 - 2t_1 t_2 + t_1 t_2 + 2$

$x = (2)^2 - t_1 t_2 + 2 = 6 - t_1 t_2 = 6 - \left(-\frac{y}{2}\right)$

$x = 6 + \frac{y}{2}$

$2x = 12 + y$

$2x - y - 12 = 0$

it is satisfied by point (7,2)

3. Ans. (A)

$\lim_{x\to 0^+}\left[\frac{x}{\pi}\right] = \lim_{h\to 0}\left[\frac{h}{\pi}\right] = 0$

$$\lim_{x\to 0^+}\frac{\sin^2 x}{\left[\frac{x}{\pi}\right]+\frac{x^2}{\pi^2}}\times\frac{\sin(\sin x)-\sin x}{ax^5+bx^3+c} = -\frac{\pi^2}{12}$$

$x = 0 + h$

$$\lim_{h\to 0}\frac{\sin^2 h}{\frac{h^2}{\pi^2}}\times\frac{2\cos\left(\frac{\sin h+h}{2}\right)\sin\left(\frac{\sin h-h}{2}\right)}{ah^5+bh^3+c} = -\frac{-\pi^2}{12}$$

$$\lim_{h\to 0}\frac{\pi^2.\sin^2 h}{h^2}\times 2\cos\left(\frac{\sin h+h}{2}\right)$$

$$\frac{\sin\left(\frac{\sin h-h}{2}\right)}{\frac{\sin h-h}{2}}\times\frac{\frac{\sin h-h}{2}}{ah^5+bh^3+c} = -\frac{\pi^2}{12}$$

$$\Rightarrow \lim_{h\to 0}\frac{\sin h-h}{ah^5+bh^3+c} = -\frac{1}{12}$$

$$\lim_{h\to 0}\frac{\sin h-h}{h^3\left(ah^2+b\right)} = -\frac{1}{12}\quad (c = 0)$$

$$-\frac{1}{6(b)} = -\frac{1}{12}\quad \therefore b = 2$$

$a \in R$, $b = 2$, $c = 0$

$b + c = 2 + 0 = 2$

4. Ans. (A)

$(x - 1)(x^2 - 2)(x^3 - 3) \ldots (x^{12} - 12)$

Highest power of x in product

is $1 + 2 + 3 \ldots + 12 = \frac{12\times 13}{2} = 78$

(1) $(x^{70}+\ldots)(x^1 - 1)(x^7 - 7) = 7x^{70}$

(2) $(x^{70}+\ldots)(x^2 - 2)(x^6 - 6) = 12x^{70}$

(3) $(x^{70}+\ldots)(x^3 - 3)(x^5 - 5) = 15x^{70}$

(4) $(x^{70}+\ldots)(x - 1)(x^2 - 2)(x^5 - 5) = -10x^{70}$

(5) $(x^{70}+\ldots)(x - 1)(x^3 - 3)(x^4 - 4) = -12x^{70}$

(6) $(x^{70}+\ldots)(x^8 - 8) = -8x^{70}$

coefficient of $x^{70} = 7 + 12 + 15 - 10 - 12 - 8 = 4$

5. Ans. (C)

Using cross product of three vectors

$\hat{i}\times\left[\left(\vec{a}-\hat{j}\right)\times\hat{i}\right]$

$\Rightarrow \left(\hat{i}.\hat{i}\right)\left(\vec{a}-\hat{j}\right)-\left(\hat{i}.\left(\vec{a}-\hat{j}\right)\right)\hat{i}$

$\Rightarrow \vec{a}-\hat{j}-\left(\vec{a}.\hat{i}\right)\hat{i}$

Let $\vec{a} = a_1\hat{i}+a_2\hat{j}+a_3\hat{k}$

$\Rightarrow \vec{a}-\hat{j}-\left(a_1\hat{i}\right)$ ...(1)

similarly :

$\hat{j}\times\left[\left(\vec{c}-\hat{k}\right)\times\hat{j}\right]$

$\Rightarrow \vec{a}-\hat{k}-a_2\hat{j}$ ...(2)

And : $\hat{k}\times\left[\left(\vec{a}-\hat{i}\right)\times\hat{k}\right]$

$\vec{a}-\hat{i}-a_3\hat{k}$ ...(3)

Addning (1) (2) and (3)

$3\vec{a}-\left(\hat{i}+\hat{j}+\hat{k}\right)-\left(a_1\hat{i}+a_2\hat{j}+a_3\hat{k}\right) = 0$

$3\vec{a}-\left(\hat{i}+\hat{j}+\hat{k}\right)-\vec{a} = 0$

$$\vec{a} = \frac{\hat{i}+\hat{j}+\hat{k}}{2}$$

$$\therefore\ \vec{a}.\vec{b} = \left(\frac{\hat{i}+\hat{j}+\hat{k}}{2}\right).2\left(\hat{i}+\hat{j}+\hat{k}\right) = 3$$

6. Ans. (A)

$2|z^2 + |z|^2| + 3|z^2 - |z|^2| = 6|z|$

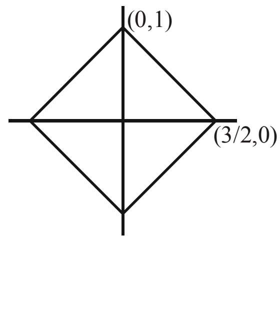
$2|z(z + \bar{z})| + 3|z(z - \bar{z})| = 6|z|$

$2|z|\ |z + \bar{z}| + 3|z|\ |z - \bar{z}| = 6|z|$

$2|z + \bar{z}| + 3|z - \bar{z}| = 6$

$4|x| + 6|y| = 6$

$$\frac{|x|}{\frac{3}{2}}+\frac{|y|}{2} = 1$$



Area $= 4\times\frac{1}{2}\times\frac{3}{2}\times 1$

$\Rightarrow 3$ unit^2

7. Ans. (A,D)

$$\frac{dy}{dx}+\frac{2xy}{1+x^2} = \frac{4x^2}{1+x^2}$$

IF : $e^{\int\frac{2x}{1+x^2}dx} = e^{\ell n\left(1+x^2\right)} = \left(1+x^2\right)$

solution of equation :

$$y\left(1+x^2\right) = \int\frac{4x^2}{\left(1+x^2\right)}\left(1+x^2\right)dx$$

$y\left(1+x^2\right)=\frac{4x^3}{3}+c$

since curve passes through origin

x = 0, y = 0

$0 = 0 + c \therefore c = 0$

$y(1 + x^2) = \frac{4x^3}{3}$

$f(x)=y=\frac{4x^3}{3\left(1+x^2\right)}$

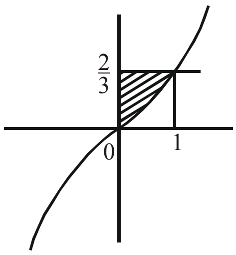
$\frac{dy}{dx}=\frac{\left(1+x^2\right)\left(12x^2\right)-4x^3(2x)}{3\left(1+x^2\right)^2}$

$\frac{dy}{dx}=\frac{12x^2+4x^4}{3\left(1+x^2\right)^2}=\frac{4x^2\left(3+x^2\right)}{3\left(1+x^2\right)^2}$

$\frac{dy}{dx}>0\ \forall x-\{0\}$ and Also $y=\frac{4x^3}{3\left(1+x^2\right)}$

As $x\to\infty, y\to\infty$

$x\to-\infty, y\to-\infty$



$A=\frac{2}{3}-\int_0^1\left(\frac{4x^3}{3\left(1+x^2\right)}dx\right)$

$A=\frac{2}{3}-\frac{2}{3}\int_0^1\frac{2x.x^2}{1+x^2}dx$

$\int_0^1\frac{2x.x^2}{1+x^2}dx$

$1 + x^2 = t$

$2xdx = dt$

$\int_1^2\frac{(t-1)}{t}dt$

$\left[t-\ell nt\right]_1^2$

$(2-\ell n2)-(1-0)\ \ 1-\ell n2$

$A=\frac{2}{3}-\frac{2}{3}\left[1-\ell n2\right]$

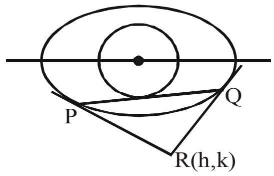
$A=\frac{2}{3}\ell n2$

8. **Ans. (B,C)**

Let the equation of tangent to circle $x^2 + y^2 = 1$ be $x\cos\theta + y\sin\theta = 1$ ...(1)

This tangent intersection ellipse in P and Q.

Let point of intersection of tangent at P and Q be R(h,k)



PQ will be chord of contact w.r.t R(h,k)

$xh + 2yk = 4$ ...(2)

comparing (1) and (2)

$\frac{\cos\theta}{h}=\frac{\sin\theta}{2k}=\frac{1}{4}$

$\cos\theta=\frac{h}{4},\sin\theta=\frac{2k}{4}$

$\therefore\ h^2+4k^2=16$ or $\frac{x^2}{16}+\frac{y^2}{4}=1$

$\therefore\ e=\sqrt{1-\frac{4}{16}}$

$e=\frac{\sqrt{3}}{2}$

9. **Ans. (A,B,C)**

Let $\angle B = \theta$

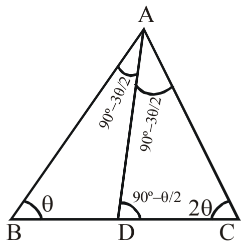
$\therefore \angle C = 2\angle B = 2\theta$

$\angle A + \angle B + \angle C = 180^\circ$

$\angle A + \theta + 2\theta = 180^\circ$

$\angle A = 180^\circ - 3\theta$

$\frac{\angle A}{2}=90^\circ-\frac{3\theta}{2}$



Now in ΔABD $\frac{AD}{\sin\theta}=\frac{BD}{\sin\left(90-\frac{3\theta}{2}\right)}$

$AD=\frac{BD\sin\theta}{\cos\frac{3\theta}{2}}$ ...(1)

In ΔADC $\frac{AD}{\sin2\theta}=\frac{AC}{\sin\left(90-\frac{\theta}{2}\right)}$

since AC = BD $AD=\frac{BD\sin2\theta}{\cos\frac{\theta}{2}}$ ...(2)

From (1) and (2)

$\frac{\sin\theta}{\cos\frac{3\theta}{2}}=\frac{\sin2\theta}{\cos\frac{\theta}{2}}$

$\sin\theta\cos\frac{\theta}{2}=\sin2\theta.\cos\frac{3\theta}{2}$

$\sin\theta\cos\frac{\theta}{2}=2\sin\theta\cos\theta.\cos\frac{3\theta}{2}$

$$\cos\frac{\theta}{2} = \cos\frac{5\theta}{2} + \cos\frac{\theta}{2}$$

$\cos\frac{5\pi}{2} = 0 \qquad \therefore \frac{5\theta}{2} = 90^\circ$

$\theta = 36^\circ$

Hence

$\angle B = 36^\circ, \angle C = 72^\circ$

$\angle A = 180^\circ - 3\theta = 180^\circ - 108$

$= 72$

$\therefore \angle A = \angle C$

Now check the options

10. Ans. (A,D)

E_1 : Person owns merceds independently

$$P(E_1) = \frac{3}{10}; P(\bar{E}_1) = \frac{7}{10}$$

E_2 : Person owns audi independently

$$P(E_2) = \frac{4}{10}; P(\bar{E}_2) = \frac{6}{10}$$

D : Event that person keeps driver

$$P(D) = P(E_1 \cap \bar{E}_2)P\left(\frac{D}{E_1 \cap \bar{E}_2}\right) + P(\bar{E}_1 \cap E_2).P\left(\frac{D}{\bar{E}_1 \cap E_2}\right) + P(E_1 \cap E_2).P\left(\frac{D}{E_1 \cap E_2}\right)$$

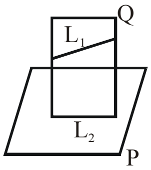
$$P(D) = \frac{3}{10}\times\frac{6}{10}\times\frac{6}{10} + \frac{7}{10}\times\frac{4}{10}\times\frac{7}{10} + \frac{3}{10}\times\frac{4}{10}\times\frac{9}{10}$$

$$P(D) = \frac{412}{1000}$$

$$P\left(\frac{E_2}{D}\right) = \frac{\frac{7\times4\times7}{1000} + \frac{3\times4\times9}{1000}}{\frac{412}{1000}}$$

$$= \frac{304}{412} = \frac{76}{103}$$

11. Ans. (A,C,D)



Let $L_1 : \frac{x}{2} = \frac{y-1}{2} = \frac{z-1}{1}$

and projection of L_1 on plane P is $L_2 : \frac{x}{1} = \frac{y-1}{1} = \frac{z-1}{-1}$

vector normal to plane Q containing line L_1 and L_2 is

$$\vec{n}_1 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 2 & 1 \\ 1 & 1 & -1 \end{vmatrix} = -3\hat{i} + 3\hat{j}$$

Now direction Ratios of normal to plane 'P' is

$$\vec{n}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -3 & 3 & 0 \\ 1 & 1 & -1 \end{vmatrix} = -3\hat{i} - \hat{j} - 6\hat{k}$$

Hence equation of plane P is

$-3(x - 0) -3(y - 1) - 6(z - 1) = 0$

$-3x - 3y + 3 - 6z + 6 = 0$

$3x - 3y - 6z + 9 = 0$

$x + y + 2z - 3 = 0$

Hence option (A)

Distance of plane 'P' from origin

$$\left|\frac{0+0+0-3}{\sqrt{6}}\right| = \frac{3}{16} = \sqrt{\frac{3}{2}}$$

Hence (C)

Distance of plane P from (1,1,–1)

$$\left|\frac{1+1-2-3}{\sqrt{6}}\right| = \frac{3}{\sqrt{6}}$$

12. Ans. (A,B,C,D)

Since Rolle's theorem is applicable in [–3,3] therefore it is continuous and differentiable in [–3,3]

Now At x = 1

using continuity

$a + b = 1 = c$ ...(1)

Also differentiable at x = 1

LHD $\lim_{h\to0} \frac{f(1-h) - f(1)}{-h}$

$$\lim_{h\to0} \frac{a(1-h)^2 + b - 1}{-h}$$

$$\lim_{h\to0} \frac{a + b - 2ha + h^2 - 1}{-h}$$

$$\lim_{h\to0} \frac{1 - 2ah + h^2 - 1}{-h} = 2a$$

Now LHD = RHD

$2a = -1$

$\therefore a = -\frac{1}{2}$

$a + b = 1$

$b = \frac{3}{2}; c = 1$

Hence $a=-\frac{1}{2}, b=\frac{3}{2}; c=1$

All options are correct.

SECTION–II

1. Ans. (5.00)

Sol. $f(x) = (x^2-1)2\ h(x);\ h(x)$

$= a_0 + a_1x + a_2x^2 + a_3x^3$

Now, $f(1) = f(-1) = 0$

$\Rightarrow\ f'(\alpha) = 0\ ,\ \alpha \in (-1, 1)$

[Rolle's Theorem]

Also, $f'(1) = f'(-1) = 0 \Rightarrow f'(x) = 0$ has at least 3 root, $-1, \alpha, 1$ with $-1 < \alpha < 1$

$\Rightarrow\ f''(x) = 0$ will have at least 2 root, say β, γ such that

$-1 < \beta < \alpha < \gamma < 1$ [Rolle's Theorem]

So, $\min(m_{f''}) = 2$

and we find $(m_{f'} + m_{f''}) = 5$ for

$f(x) = (x^2-1)^2\ h(x)$

Thus, Ans. 5

2. Ans. 1.00

$|f(x)| \le |e^x - 1|$

By taking limit $x \to -\infty$ on

Both sides

$\lim_{x\to-\infty}|f(x)| \le |1|$

$\lim_{x\to-\infty}|f(x)| \le 1$

$\Rightarrow$ degree of $f(x)$ cannot be ≥ 1

(Because if degree of $f(x) \ge 1$ then $\lim_{x\to-\infty}|f(x)| = \infty$)

so $f(x)$ must be constant function

Now for $x = 0$

$|f(0)| \le |e^{\circ} - 1|$

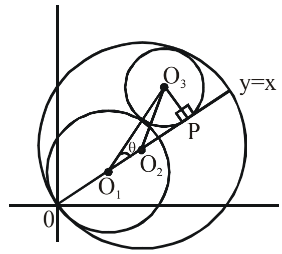
$|f(0)| \le 0$

$\therefore\ f(0) = 0$

Hence $f(x) = 0\ \forall\ x \in R$

$f'(0) = 0$

3. Ans. 1.87 or 1.88



$S_1 = x^2 + y^2 + 3\sqrt{2}x + 3\sqrt{2}y = 0$

$r = 3$

$S_2 : x^2 + y^2 + 5\sqrt{2}x + 5\sqrt{2}y = 0$

$R = 5$

Let O_1, O_2 be centre of circle with radius 3 and 5 respectively.

Let O_3 be the centre of circle with radius x which is tangent to two given circles and common diametre

$O_1O_2 = 5 - 3 = 2;\ O_1O_3 = 3 + x,$

$O_2O_3 = 5 - x$

Apply cosine rule in D O_1, O_2, O_3

$$\cos\theta = \frac{2^2 + (3+x)^2 - (5-x)^2}{2.2.(3+x)} = \frac{4x-3}{3+x}$$

Also in ΔO_1PO_3 $\quad \sin\theta = \frac{x}{3+x}$

$\cos^2\theta + \sin^2\theta = 1$

$$\left(\frac{4x-3}{3+x}\right)^2 + \left(\frac{x}{3+x}\right)^2 = 1$$

$x = \frac{15}{8}$

4. Ans. 15.00

We have five distinct balls and to be distributed in 3 persons. we can distribute by forming groups.

Case I : 1,1,3

$$\frac{5!}{(1!)^2\,2!3!} \times 3! = 60$$

Case II : 1,2,2

$$\frac{5!}{1!(2!)^2\,2!} \times 3! = 90$$

Total ways = 60 + 90 = 150

$\frac{150}{10} = 15.00$

5. Ans. 0.75

$F(x) + F\left(x+\frac{1}{2}\right)=1$

$F\left(x+\frac{1}{2}\right)+F(x+1)=1$

F(x) – F(x + 1) = 0

$\therefore$ F(x) = F(x + 1)

F(x) is periodic with period 1.

$I=\int_0^{1500} F(x)=1500\int_0^1 F(x)$

$\Rightarrow 1500\left[\int_0^{\frac{1}{2}} F(x)+\int_{\frac{1}{2}}^{1} F(x)dx\right]$

$\Rightarrow 1500\left[\int_0^{\frac{1}{2}} F(x)+\int_0^{\frac{1}{2}} F\left(x+\frac{1}{2}\right)\right]$

$\Rightarrow 1500\left[\int_0^{\frac{1}{2}}\left(F(x)+F\left(x+\frac{1}{2}\right)\right)dx\right]$

$1500\left[\int_0^{\frac{1}{2}} dx\right]$

$1500\times\frac{1}{2}=750$

$\therefore\ I=\frac{750}{1000}=0.75$

6. Ans. 12.56 OR 12.57

$\tan^4x + \tan^4y + 2\cot^2x.\cot^2y = 3 + \sin^2(x + y)$

$(\tan^2x - \tan^2y)^2 + 2\tan^2x.\tan^2y + 2\cot^2x\cot^2y = 3 + \sin^2(x + y)$

$(\tan^2x - \tan^2y)^2 + 2(\tan x\tan y - \cot x\cot y)^2 + 4 = 3 + \sin^2(x + y)$

$(\tan^2x - \tan^2y)^2 + 2(\tan x\tan y - \cot x\cot y)^2 + \cos^2(x + y) = 0$

$\tan^2x = \tan^2y$ and $\tan x\tan y = \cot x\cot y$ and $\cos(x + y) = 0$

$\tan x = \pm\tan y$; $\tan x\tan y = \pm 1$, $\cos(x + y) = 0$

Case I : tanx = tany

then $\tan^2x = 1 \Rightarrow \tan x = \pm 1$

$\tan x = 1 \quad x=\frac{\pi}{4},\frac{5\pi}{4}$

$\tan x = -1 \quad x=\frac{3\pi}{4},\frac{7\pi}{4}$

sum of possible values of

$\Sigma x = 4\pi = 4 \times 3.14$

$\Sigma x = 12.56$