ALLEN® CAREER INSTITUTE KOTA (RAJASTHAN) path to success

CLASSROOM CONTACT PROGRAMME

JEE(Advanced)
FULL SYLLABUS

SAMPLE PAPER-4

ANSWER KEY PAPER-2

PART-1 : PHYSICS

SECTION-I	Q.	1	2	3	4	5	6	7	8	9	10
	A.	B,C,D	A,B,D	A,B,C,D	A,B,D	B,C	A,B,C	A,B,C	A,B,C,D	B	D
	Q.	11	12								
	A.	C	D								
SECTION-II	Q.	1	2	3	4	5	6				
	A.	1.56 to 1.58	6.00	1.72 to 1.74	2.21 to 2.23	24.50 to 24.51	1.80				

PART-2 : CHEMISTRY

SECTION-I	Q.	1	2	3	4	5	6	7	8	9	10
	A.	B,D	A,C,D	A,B,C	A,D	A,C,D	B,C,D	B,C	A,B,C	B	D
	Q.	11	12								
	A.	B	C								
SECTION-II	Q.	1	2	3	4	5	6				
	A.	6.00	5.00 to 5.10	86.00	0.84 or 0.85	2.00	0.50				

PART-3 : MATHEMATICS

SECTION-I	Q.	1	2	3	4	5	6	7	8	9	10
	A.	A,B,D	B,C,D	A,B,D	B,C	B,C	B,D	A,C	A,B,C	C	D
	Q.	11	12								
	A.	D	B								
SECTION-II	Q.	1	2	3	4	5	6				
	A.	13.00	6.00	2.00	1.00	0.00	1.42 or 1.43				

SAMPLE PAPER-4

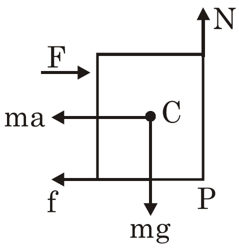
PAPER-2

PART-1 : PHYSICS SOLUTION

SECTION-I

1. Ans. (B,C,D)

Sol.



For no relative motion $a = \left(\frac{F}{2m+m}\right)$

For topping about P w.r.t. 2m

$$F \times 2b \geq ma \cdot \frac{3b}{2} + mg\frac{b}{2}$$

$$2Fb - \frac{3}{2}mb \cdot \frac{F}{3m} \geq mg\frac{b}{2}$$

$$\frac{3F}{2} \geq \frac{mg}{2}$$

$$\boxed{F \geq \frac{mg}{3}}$$

$\therefore f = 2m \times a$

$= 2m \times g/9$

$$= \frac{2mg}{9}$$

2. Ans. (A,B,D)

Sol. $\theta = 60°$

$$\therefore h = \frac{2T\cos\theta}{r\rho g} = \frac{T}{r\rho g}$$

$$W_{ST} = (2\pi r T\cos\theta)h$$

$$= \frac{4\pi T^2\cos^2\theta}{\rho g} = \frac{\pi T^2}{\rho g}$$

$$\Delta U = m g y_{com}$$

$$= (\pi r^2 h\rho g)(h/2)$$

$$= \frac{\pi r^2 \rho g h^2}{2}$$

$$= \frac{2\pi T^2\cos^2\theta}{\rho g} = \frac{\pi T^2}{2\rho g}$$

From first law of thermodynamics, heat dissipated

$Q = W - \Delta U$

$$= \frac{\pi T^2}{\rho g} - \frac{\pi T^2}{2\rho g}$$

$$= \left(\frac{\pi T^2}{\rho g}\right)$$

3. Ans. (A,B,C,D)

Sol. From conservation of energy

$$mg\sin\theta \cdot x = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 + \frac{1}{2}kx^2$$

$$mg\sin\theta \cdot x = \frac{3}{4}mv^2 + \frac{1}{2}kx^2$$

On differentiating

$$mg\sin\theta \cdot v = \frac{3}{4}m \cdot 2v\frac{dv}{dt} + \frac{1}{2}k \cdot 2x\frac{dx}{dt}$$

$$mg\sin\theta = \frac{3}{2}m\left(\frac{dv}{dt}\right) + kx$$

At mean position $\frac{dv}{dt} = 0$

$$mg\sin\theta = kA$$

$A = \left(\frac{mg\sin\theta}{K}\right)$

Acceleration $= \frac{2}{3}g\sin\theta - \left(\frac{2k}{3m}\right)x$

$\therefore \omega = \sqrt{\frac{2k}{3m}}$

$T = 2\pi\sqrt{\frac{3m}{2k}}$

Energy of oscillation

$= \frac{1}{2}kA^2 = \frac{1}{2}k\left(\frac{mg\sin\theta}{k}\right)^2$

$= \frac{1}{2}\frac{m^2g^2\sin^2\theta}{k}$

At extreme position

$2kA - f - mg\sin\theta = ma_{max}$

$2mg\sin\theta - f - mg\sin\theta = m\cdot\omega^2 A$

$f = mg\sin\theta - m\cdot\frac{2k}{3m}\frac{mg\sin\theta}{k}$

$f = \frac{mg\sin\theta}{3}$

4. Ans. (A,B,D)

Sol. From KCL at O

$2 + i = 1 + 3$

$i = 2A$

$V_G - V_H = 2\times4 - 3 + 1\times2 - 2\times2$

$= 8 - 3 + 2 - 4$

$= 3V$

Power consumed by cell B = 4 × 5 = 20 watt

Terminal voltage = 3 – 2 × 1 = 1 volt

5. Ans. (B,C)

Sol. Zero error = – (10 – 7) × 0.1 = – 0.3 mm

Diameter = 7.7 + 8 × 0.01 – (– 0.03)

= 7.81 cm

6. Ans. (A,B,C)

Sol. $U = 2x^3 - 9x^2 + 12x$

$F = -\frac{dU}{dx} = \left(-6x^2 + 18x - 12\right)$

For equilibrium F = 0 x = 1 & 2

at x = 0 F = –12 N

$\frac{d^2U}{dx^2} = 12x - 18$

x = 1 $\frac{d^2U}{dx^2} = -6$

maxima

x = 2 $\frac{d^2U}{dx^2} = +6$

minima

$U_{max} = 2(1)^3 - 9\times1^2 + 12\times1 = 5J$

$U_{min} = 2\cdot(2)^3 - 9\times(2)^2 + 12\times2 = 4J$

For oscillatory motion 4J < M.E. < 5J

7. Ans. (A, B, C)

Sol. $eV = \frac{hc}{\lambda} - \phi$ ...(1)

$e(V+1) = = \frac{2hc}{\lambda} - \phi$ ....(2)

$\frac{eV}{2} = \frac{hC}{\lambda+100} - \phi$ ...(3)

$\Rightarrow \frac{hc}{\lambda} = 1eV \Rightarrow \lambda = 1240nm$

$\frac{1}{2} - \frac{\phi}{2} = \frac{eV}{2} = \frac{1240}{1340} - \phi$

$\frac{\phi}{2} = \frac{62}{67} - \frac{1}{2} \Rightarrow \phi = 0.85eV$

8. Ans. (A,B,C,D)

Sol. $\phi = \int_{b-x}^{x}\frac{\mu_0 Iadr}{2\pi r} = \frac{\mu_0 Ia}{2\pi}\ln\left(\frac{x}{b-x}\right)$

$x \to b$ $\phi \to \infty$

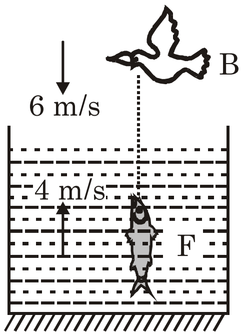
$x = \frac{b}{2}$ $\phi = 0$

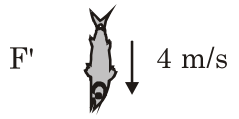
$E_{in} = -\frac{d\phi}{dt} = -\frac{d}{dt}\left\{\frac{\mu_0 a}{2\pi}\ln(1/3)\cdot 2t\right\}$

$= \left(\frac{\mu_0 a\ln 3}{\pi}\right)$

9. Ans. (B)

Sol.





$$V_{FB} = V_F \times \frac{\mu_B}{\mu_F} + V_B$$

$$= 4 \times \frac{1}{4/3} + 6$$

$= 9$ m/s

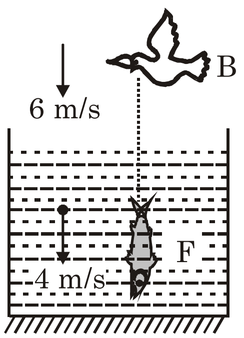
$$V_{F'B} = \frac{4}{(4/3)} - 6 = -3\text{ m/s}$$

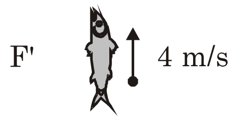
$$V_{BF} = 6 \times \frac{4}{3} + 4 = 12\text{ m/s}$$

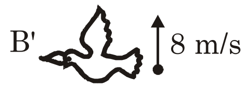
$$V_{B'F} = (8-4) = 4\text{ m/s}$$

10. Ans. (D)

Sol.







$$V_{FB} = \frac{4}{4/3} - 6 = -3\text{ m/s}$$

$$V_{F'B} = \frac{4}{4/3} + 6 = 9\text{ m/s}$$

$$V_{BF} = 6 \times 4/3 - 4 = 4\text{ m/s}$$

$$V_{B'F} = (8+4) = 12\text{ m/s}$$

11. Ans. (C)

Sol. $W_{gas} = nR\Delta T$ (isobaric)

$$= 2 \times \frac{25}{3} \times 60$$

$= 1000$ J

$$\Delta U = \frac{f}{2} nR\Delta T$$

$$= \frac{3}{2} \times 2 \times \frac{25}{3} \times 60$$

$= 1500$ J

$Q = W + \Delta U = 2500J$

$$\Delta U_g = mgh = 1 \times 10 \times \left(\frac{2 \times \frac{25}{3} \times 60}{2 \times 10^5 \times 10^{-4}} \right)$$

$= 500$ J

12. Ans. (D)

Sol. Piston is fixed (isochoric)

$W_{gas} = 0$

$$\Delta U_g = \frac{f}{2} nR\Delta T = \frac{3}{2} \times 2 \times \frac{25}{3} \times 60$$

$= 1500$ J

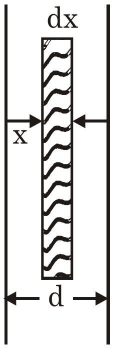
$Q = W + \Delta U = 1500J$

$Wg = mgh = 0$

SECTION-II

1. Ans. 1.56 to 1.58

Sol.



$$dc = \frac{A \in_0 k}{dx}$$

$$= \frac{A \in_0 \left(1 + \sin\frac{\pi x}{d}\right)}{dx}$$

All strips are connected in series hence

$$\frac{1}{Ceq} = \int \frac{1}{dc} = \int_0^d \frac{dx}{A \in_0 \left(1 + \sin\frac{\pi x}{d}\right)}$$

$$= \frac{2d}{A \in_0 \pi}$$

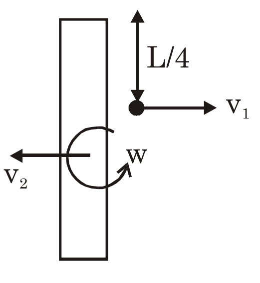
$$Ceq = \frac{A \in_0 \pi}{2d}$$

Hence $\lambda = \frac{\pi}{2} = \frac{3.14}{2} = 1.57$

$\boxed{\lambda = 1.57}$

2. Ans. 6.00

Sol.



$$v_1 = \left(u - v_2 - \frac{L\omega}{4}\right)$$

From conservation of linear momentum

$$m\left(u - v_2 - \frac{L}{4}\omega\right) = m \cdot v_2 \quad ..... \text{(i)}$$

From conservation of angular momentum

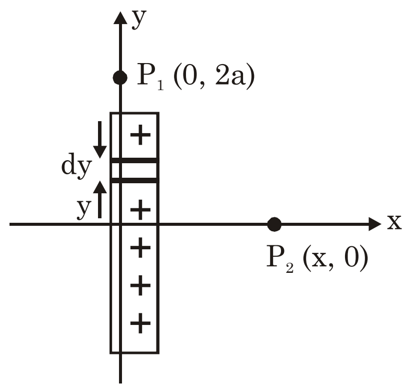
$$m\left(u - v_2 - \frac{\ell}{4}\omega\right) \cdot \ell/4 = \frac{m\ell^2}{12} \cdot \omega \quad ..... \text{(ii)}$$

on solving $\omega = \frac{12u}{11\ell}$

$$= \frac{12 \times 11}{11 \times 2} = 6 \text{ rad/sec}$$

3. Ans. 1.72 to 1.74

Sol.



$$v_1 = k\int_{-a}^{a} \frac{\lambda dy}{(2a - y)} = k\lambda \ln 3$$

$$v_2 = k\int_{-a}^{+a} \frac{\lambda dy}{\sqrt{y^2 + x^2}}$$

$$= 2k\lambda \ln\left(\frac{a + \sqrt{a^2 + x^2}}{x}\right)$$

$$\because v_1 = v_2$$

$$k\lambda \ln 3 = 2k\lambda \ln\left(\frac{a + \sqrt{a^2 + x^2}}{x}\right)$$

$$3 = \left(\frac{a + \sqrt{a^2 + x^2}}{x}\right)^2$$

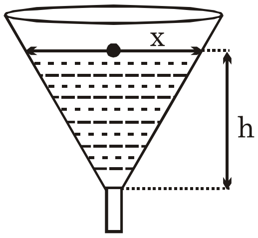
$$\sqrt{3}x = a + \sqrt{a^2 + x^2}$$

$$\frac{x}{a} = \sqrt{3} = 1.732$$

$\boxed{1.73}$

4. Ans. 2.21 to 2.23

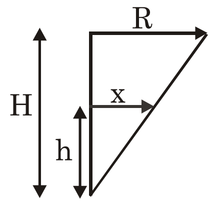
Sol.



Speed of efflux $v = \sqrt{2gh}$

From equation of continuity

$$\pi r^2 \cdot \sqrt{2gh} = \pi x^2 \left(-\frac{dh}{dt}\right)$$



$$\because \frac{R}{H} = \frac{x}{h}$$

$$x = \frac{Rh}{H}$$

$$r^2\sqrt{2gh} = \frac{R^2h^2}{H^2}\left(-\frac{dh}{dt}\right)$$

$$\int_0^T dt = -\frac{R^2}{r^2H^2\sqrt{2g}}\int_H^0 h^{3/2}dh$$

$$T = \frac{2}{5}\frac{R^2}{r^2}\sqrt{\frac{H}{2g}}$$

$$T = \frac{2}{5} \times \frac{4}{10^{-4}} \times \sqrt{\frac{5}{2 \times 10}}$$

$$= \frac{4}{5} \times 10^4$$

$= 2.22$ hr.

5. Ans. 24.50 to 24.51

Sol. Initial activity $A_0 = \lambda N_0$

$$N = N_0 e^{-\lambda t}$$

$$\therefore A = \lambda N = \lambda N_0 e^{-\lambda t}$$

Fractional decrease in activity = 4% per hour

$$\frac{\lambda N_0 - \lambda N_0 e^{-\lambda t}}{\lambda N_0} = \frac{4}{100} = \frac{1}{25}$$

$$1 - e^{-\lambda t} = \frac{1}{25}$$

$$e^{-\lambda t} = \frac{24}{25}$$

$$\lambda t = \ln\left(\frac{25}{24}\right)$$

$$\lambda = \frac{1}{t}\ln\left(\frac{25}{24}\right)$$

$$\tau = \frac{1}{\lambda} = \frac{t}{\ln(25/24)}$$

$$= \frac{1}{0.0408}$$

$= 24.5$ hr

$\boxed{\tau = 24.5 hr}$

6. Ans. 1.80

Sol. Velocity of wave $v = \sqrt{\frac{T}{\mu}}$

$$\frac{dx}{dt} = \sqrt{\frac{Kt}{\mu}}$$

$$dx = \sqrt{\frac{K}{\mu}} \cdot t^{1/2}dt$$

On integration

$$L = \sqrt{\frac{K}{\mu}} \cdot t^{3/2} \cdot \frac{2}{3}$$

$$2 = \sqrt{\frac{K \times 10^{-2}}{2 \times 10^{-3}}} \times (L)^{3/2} \times 2/3$$

$$9 = \frac{K}{2} \times 10$$

$\boxed{K = 1.80}$

PART–2 : CHEMISTRY SOLUTION

SECTION-I

1. Ans. (B,D)

Sol.: d – 2 orbitals – $4e^-$

f – 2 orbitals – $4e^-$

$\therefore$ total 8 e^-

For Ma_2b_2cd, total 6 G.I. and

4 are optically active. Hence total S.I. = 8

2. Ans. (A,C,D)

O_2 is paramagnetic having 2 unpaired electrons, O_3 is diamagnetic, B_2 is paramagnetic having 2 unpaired electrons, C_2 is diamagnetic.

up to N_2 sp intermixing takes place.

3. Ans. (A, B,C)

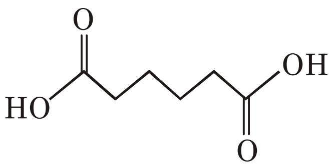
Sol. For 1 mole Vander Waal's gas

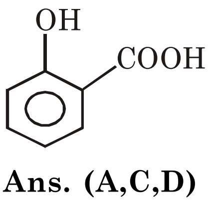
$$P = \frac{RT}{V-b} - \frac{a}{V^2}$$

If $P_{ext} = P$, means process is reversible. For Vanderwaal gas, expression is correct for all reversible process.

4. Ans. (A,D)

Sol.:





5. Ans. (A,C,D)

Sol.: For the given hydrogenic wave function n = 3 and $\ell = 1$

$\therefore$ Number of Angular node = $\ell = 1$ and number of radial node = $n - \ell - 1 = 1$

ψ = 0 in xy-plane.

6. Ans. (B,C,D)

7. Ans. (B,C)

8. Ans. (A,B,C)

Sol.: (A) $Z = \frac{PM}{dRT} = \frac{10 \times 32}{20 \times \frac{1}{12} \times 300} = \frac{16}{25}$

$\therefore$ O_2 shows negative deviation

(B) $Z = \frac{6}{11.2} \Rightarrow Z < 1$

$\therefore$ N_2 shows negative deviation.

(C) A shows negative deviation at T = T_C and P < P_C.

(D) P = low, T = T_B

$\therefore$ Z = 1 or PV = nRT

$\because$ T > T_C, so gas cannot be liquefied at any pressure at given temperature.

9. Ans. (B)

Sol.: Alkene will be obtained if Cu/300°C is the reagent used.

10. Ans. (D)

Sol.: MnO_2 oxidises only allylic or benzylic –OH 1° on oxidises to Aldehyde.

11. Ans. (B)

Sol.: A–RT, B–QT, C–PT, D–S

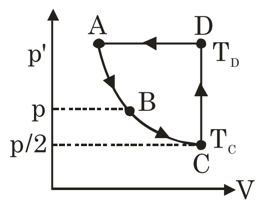
12. Ans. (C)

Sol.: A–RT, B–QT, C–PT, D–S

SECTION-II

1. Ans. (6.00)

Sol.: (A, B, C, D, E, G are correct)



$$\Delta S_{CD} = 4\ln 16 = nC_v \ln\frac{T_0}{T_C}$$

$$\frac{T_D}{T_C} = 16 = \frac{p'}{p/2}$$

p' = 8p

$$\frac{T_B}{T_A}=\left(\frac{P_A}{P_B}\right)^{\frac{1-\gamma}{\gamma}}$$

$$\frac{T_B}{T_A}=(8)^{\frac{-1}{3}}=\frac{1}{2}$$

$\therefore$ $T_B = 150$

$T_D = 150 \times 16 = 2400$

2. Ans. (5.00 to 5.10)

Sol.: Radius of ball = $\frac{0.1}{2}=0.05$

wt of jwellery $=\frac{4}{3}\pi r^3\times 10000\times 10.5=54.95g$

Electricity in faradey = $\frac{54.95\times 96500\times 100}{96.5\times 108}$

$= 50879.63$

$= 5.09 \times 10^4$ coloumb

3. Ans. (86.00)

Sol.: A. $CH_3{-}CH_2{-}CH_2Cl$ A* $-CH_3{-}CH(Cl){-}CH_3$

B. $CH_3{-}CH_2$ Mg Br 133 g

C. $CH_3{-}CH_2{-}CH_2{-}CH_2{-}CH_3 \xrightarrow[hv]{Cl_2}$ C–C–C–C–C 4 monochlorinated product

D. $CH_3{-}C{\equiv}CH$ 40g

E. $CH_3{-}CH_3$

4. Ans. (0.84 or 0.85)

Sol.: $\frac{0.693}{8}=\frac{2.303}{4}\log\frac{N_0}{N}$

$\therefore \frac{N}{N_0}=\frac{1}{\sqrt{2}}$

$\frac{60\sqrt{2}}{100}\times 1$

$= 0.8485$ mg

5. Ans. (2.00)

Sol.: 5, 9 are true statements.

6. Ans. (0.50)

Sol.:

$\Rightarrow \underset{(X)}{FeS(s)} \xrightarrow{dil.HCl} \underset{(Y)}{H_2S\uparrow} \xrightarrow[NaOH]{Na_2[Fe(CN)_5(NO)]} \underset{\text{Purple solution}}{Na_4[Fe(CN_5)(NO)]}$

(a) $2FeCl_3 + 2H_2S\uparrow \xrightarrow[Neutral]{Acidic} 2Fe^{2+} + S\downarrow + 4H^+ + 6Cl^-$

(b) $CuSO_4 + H_2S\uparrow \xrightarrow[Neutral]{Acidic} \underset{Black}{CuS\downarrow} + 2H^+ + SO_4^{2-}$

(c) $BaCl_2 + H_2S\uparrow \xrightarrow[Neutral]{Acidic} \underset{\text{No observable change}}{Ba^{2+} + 2Cl^- + 2H^+ + S^{2-}}$

(d) $SO_2\uparrow + 2H_2S\uparrow \rightarrow 2S\downarrow + 2H_2O$

(e) $Cr_2O_7^{2-} + 8H^+ + 3H_2S\uparrow \xrightarrow[Neutral]{Acidic} 2Cr^{3+} + S\downarrow + 7H_2O$

(f) $2CH_3COONa + H_2S\uparrow \xrightarrow[Neutral]{Acidic} \underset{\text{No observable change}}{2CH_3COOH + 2Na^+ + S^{2-}}$

(g) $Hg^{2+} + H_2S \xrightarrow[Neutral]{Acidic} HgS\downarrow + 2H^+$

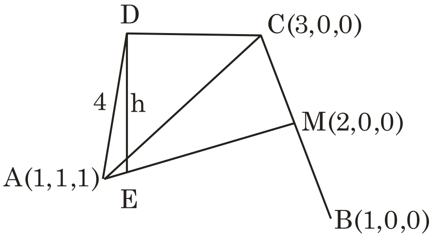
$\therefore$ P = 3; Q = 2; R = 2

PART-3 : MATHEMATICS SOLUTION

SECTION-I

1. Ans. (A,B,D)

Sol. $v = \frac{2\sqrt{2}}{3}$



$\Rightarrow \frac{1}{3}\cdot\frac{1}{2}\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 1 & 1 \\ 2 & -1 & -1 \end{vmatrix} h = \frac{2\sqrt{2}}{3}$

$\Rightarrow h = 2$

Also ΔABC is right angled.

Now let E divides AM in the ratio $\lambda : 1$.

$E = \left(\frac{2\lambda+1}{\lambda+1}, \frac{1}{\lambda+1}, \frac{1}{\lambda+1}\right)$ and $AE^2 + DE^2 = AD^2$

$\Rightarrow \left(\frac{2\lambda+1}{\lambda+1} - 1\right)^2 + \left(1 - \frac{1}{\lambda+1}\right)^2 + \left(1 - \frac{1}{\lambda+1}\right)^2 + 4 = 16$

$\Rightarrow$ Two values of λ

$\frac{3\lambda^2}{(\lambda+1)^2} = 12$ $\quad\quad \therefore E = (3, -1, -1)$

$\lambda^2 = 4(\lambda+1)^2$

$EB = \sqrt{4+1+1} = \sqrt{6}$

$\lambda = \pm 2(\lambda+1)$, $\lambda = -2$, $\lambda = \frac{-2}{3}$

$\therefore E = (3, -1, -1)$ $\quad\quad \overline{BC} = 2\hat{i}$

$\overline{BE} = 2\hat{i} - \hat{j} - \hat{k}$

$ar(\Delta EBC) = \frac{1}{2}\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 0 & 0 \\ 2 & -1 & -1 \end{vmatrix} = \frac{1}{\sqrt{2}}$

2. Ans. (B,C,D)

Sol. $f(x) = \tan^{-1}\left(\frac{|x|}{\sqrt{1-x^2}}\right) + \cos^{-1}\left(\frac{1}{\sqrt{1+x^2}}\right)$

Put $|x| = \sin\theta$ $\quad\quad$ Put $|x| = \tan\theta$

$\theta \in \left(0, \frac{\pi}{2}\right)$ $\quad\quad$ $\theta \in \left(0, \frac{\pi}{2}\right)$

$\therefore f(x) = \sin^{-1}|x| + \tan^{-1}|x|$

Since $\sin^{-1}|x| + \tan^{-1}|x|$ are both increasing function in [0, 1].

$\therefore f(x) \in \left[0, \frac{3\pi}{4}\right)$

Integer's in range = {0, 1, 2}

f(x) = 2 has two solution.

$f(\sin\theta) > f(\cos\theta)$

$\therefore \sin\theta > \cos\theta$

$\theta \in \left(\frac{\pi}{4}, \frac{\pi}{2}\right)$

3. Ans. (A,B,D)

Sol. $x^2 - y^2 - 2x + 4y - 7 = 0$

$x^2 - 2x + 1 - (y^2 - 4y + 4) = 4$

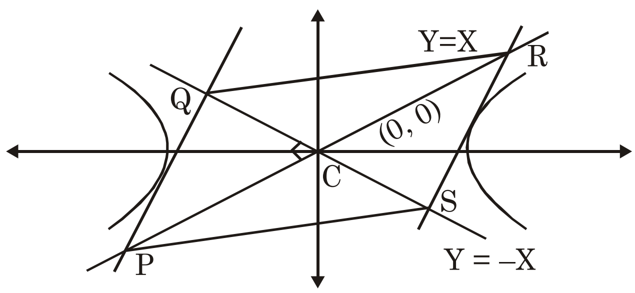
$(x-1)^2 - (y-2)^2 = 4$

Perpendicular tangents from the centre are Y = X and Y= –X which are its asymptotes.

$\therefore Ar(\Delta PCQ) = ab = 4$

$\therefore (\square PQRS) = 16$

Equation of tangent to the hyperbola with slope = 2.



$Y = 2X \pm \sqrt{4\cdot 4 - 4}$

$Y = 2X \pm 2\sqrt{3}$

$$\left.\begin{array}{l} PQ: Y = 2X + 2\sqrt{3} \\ QS: Y = -X \end{array}\right\} \Rightarrow Q \equiv \left(\frac{-2}{\sqrt{3}}, \frac{2}{\sqrt{3}}\right)$$

$$PR: Y = X \Rightarrow P \equiv \left(-2\sqrt{3}, -2\sqrt{3}\right)$$

$$PQ = \sqrt{24 + \frac{8}{3}}$$

$$= \sqrt{\frac{80}{3}}$$

$$= 4\sqrt{\frac{5}{3}}$$

$$Ar(\Delta PCQ) = \frac{1}{2} \times \sqrt{24} \cdot \sqrt{\frac{8}{3}} = 4$$

$\therefore$ $Ar(\Delta PQR) = 8$ sq.units

$Ar(\square PQRS) = 16$ sq.units

4. Ans. (B,C)

Sol. x, 12 y, in H.P.

So, $12 = \frac{2xy}{x+y}$

$\Rightarrow$ 12 (x + y) = 2xy ...(1)

x, 12, z, y are in increasing A.P.

So, 24 = x + z (option B) ...(2)

2z = 12 + y ...(3)

12 + z = x + y ...(4)

From (2), (3) and (4) put values in (1)

12(12 + z) –2(24 – z) (2z – 12)

We get, z = 15

Put z = 15 in equation (2) and (3), we get

x = 9, y = 18

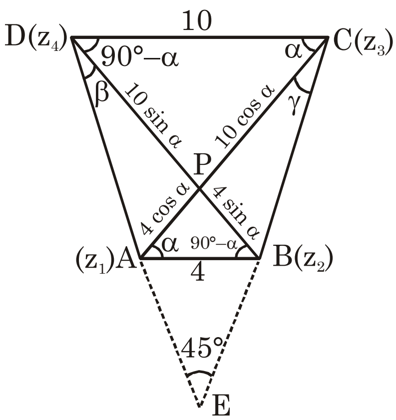
So, maximum value of

$$\sqrt{(x-3)\sin\alpha - (y-10)\cos\beta + 2} = \sqrt{6\sin\alpha - 8\cos\beta + 2}$$

Maximum when $\sin\alpha = 1$ and $\cos\beta = -1$ is 4.

5. Ans. (B,C)

Sol.



We have $\angle C + \angle D + \angle E = 180°$

$\therefore$ $\beta + \gamma = 45°$

From the figure we can say that

$$\tan\beta = \frac{2}{5}\cot\alpha, \; \tan\gamma = \frac{2}{5}\tan\alpha$$

Since $\beta + \gamma = 45°$

$\therefore$ $(1+\tan\beta)(1+\tan\gamma) = 2$

$$\left(1 + \frac{2}{5}\cot\alpha\right)\left(1 + \frac{2}{5}\tan\alpha\right) = 2$$

$$\therefore \sin\alpha\cos\alpha = \frac{10}{21}$$

$$\therefore \sin 2\alpha = \frac{20}{21}$$

Area of trapezium is the sum of area of 4 triangle.

$$= \frac{1}{2}(16\sin\alpha\cos\alpha + 40\sin\alpha\cos\alpha + 100\sin\alpha\cos\alpha + 40\sin\alpha\cos\alpha)$$

$$= \frac{196}{2}\sin\alpha\cos\alpha = \frac{196}{2} \times \frac{10}{21} = \frac{140}{3}$$

$$|CP - DP| = |10(\cos\alpha - \sin\alpha)| = \frac{10}{\sqrt{21}}$$

$$\text{In } (\Delta PCB) = \frac{1}{2} 40\sin\alpha\cos\alpha = \frac{200}{21}$$

6. Ans. (B,D)

Sol. $BI = r\,\text{cosec}\frac{B}{2} = 4R\sin\frac{A}{2}\sin\frac{C}{2}$

$$BI_1 = r_1\sec\frac{B}{2} = 4R\sin\frac{A}{2}\cos\frac{C}{2}$$

$$\Rightarrow II_1 = \sqrt{(BI)^2 + (BI')^2} = 4R\sin\frac{A}{2}$$

$$\Rightarrow \sum II_1 = 4R\sum\sin\frac{A}{2} \Rightarrow R = \frac{15}{8}$$

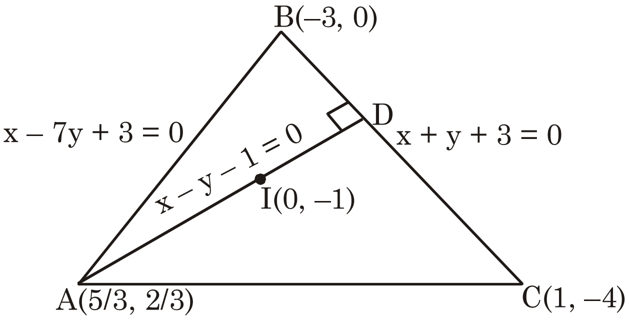
We know $\sin\frac{A}{2} + \sin\frac{B}{2} + \sin\frac{C}{2} - 1 = 4$

$$\sin\left(\frac{\pi - A}{4}\right)\cdot\sin\left(\frac{\pi - B}{4}\right)\cdot\sin\left(\frac{\pi - C}{4}\right)$$

$$\therefore \sin\left(\frac{\pi - A}{4}\right)\cdot\sin\left(\frac{\pi - B}{4}\right)\cdot\sin\left(\frac{\pi - C}{4}\right) = \frac{1}{20}$$

7. **Ans. (A,C)**

Sol. Solve AB and BC to gel B(–3, 0)
Solve AD and AB to get A(5/3, 2/3)
Take the image of B in $x - y - 1 = 0$



(which is altitude as well as angle bisector through A) to get C(1, –4), now get AC

8. **Ans. (A,B,C)**

Sol. $(\bar{x}\times\bar{y})\times\bar{b}=\bar{a}\times\bar{b}$

$\Rightarrow\ (\bar{x}\cdot\bar{b})\cdot\bar{y}-(\bar{y}\cdot\bar{b})\bar{x}=\bar{a}\times\bar{b}$

$\Rightarrow\ \gamma\cdot\bar{y}-(\bar{y}\cdot\bar{b})\cdot\bar{x}=\bar{a}\times\bar{b} \qquad (\because\ \bar{x}\cdot\bar{b}=\gamma)$

$\bar{y}\cdot\bar{b}=\bar{y}\cdot(\bar{y}\times\bar{z})=0$

$\Rightarrow\ \bar{y}=\dfrac{\bar{a}\times\bar{b}}{\gamma}$

$\Rightarrow\ (\bar{x}\times\bar{y})\times\bar{y}=\bar{a}\times\bar{y}$

$\Rightarrow\ (\bar{x}\cdot\bar{y})\cdot\bar{y}-(\bar{y}\cdot\bar{y})\bar{x}=\bar{a}\times\bar{y}$

$\Rightarrow\ \bar{y}-|\bar{y}|^2\bar{x}=\bar{a}\times\bar{y} \qquad (\because\ \bar{x}\cdot\bar{y}=1)$

$\Rightarrow\ \bar{x}=\dfrac{1}{y^2}[\bar{y}-\bar{a}\times\bar{y}]$

$(\bar{y}\times\bar{z})\times\bar{y}=\bar{b}\times\bar{y}$

$\Rightarrow\ y^2\bar{z}-(\bar{z}\cdot\bar{y})\cdot\bar{y}=\bar{b}\times\bar{y} \quad\Rightarrow\ y^2\bar{z}-\bar{y}=\bar{b}\times\bar{y}$

$\Rightarrow\ \bar{z}=\dfrac{1}{y^2}[\bar{y}+\bar{b}\times\bar{y}]$

9. **Ans. (C)**

10. **Ans. (D)**

I. Number of functions, that satisfy condition $i + f(i) < 10$; where i = 1, 3, 5, 7 is

Sol. $f(1) < 9$ & $f(3) < 7$ & $f(5) < 5$ & $f(7) < 3$
$= 4 \times 3 \times 2 \times 1 = 24$

II. Number of functions, that satisfy $f(i) \neq 1 + i$, where i = 1, 3, 5, 7 is

Sol. $f(1) \neq 2$; $f(3) \neq 4$; $f(5) \neq 6$; $f(7) \neq 8$,
$3 \times 3 \times 3 \times 3 = 81$

III. Number of one-one and onto function, that satisfy condition $f(i) \neq 1 + i$ where i = 1, 3, 5, 7 is

Sol. $f(1)\neq 2;\ f(3)\neq 4;\ f(5)\neq 6,\ f(7)\neq 8$
Number of ways = $D_4 = 9$

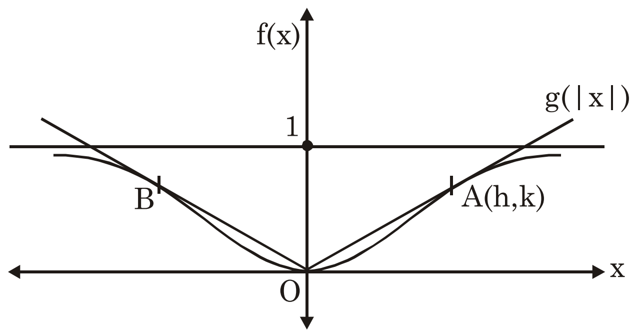
IV. If number of many one function from $A\rightarrow B$ is k, k is divisible by

Sol. No. of many are function
$= 4^4 - 24 = 256 - 24 = 232$

11. **Ans. (D)**

12. **Ans. (B)**

Sol.



$f(x)=\dfrac{x^2}{1+x^2},\ g(x)=px$

$f(x)=1-\dfrac{1}{1+x^2} \quad\Rightarrow\ f'(x)=\dfrac{2x}{(1+x^2)^2}$

Slope of the tangent at A from the origin

$\dfrac{2h}{(1+h^2)^2}=\dfrac{k}{h}$

$\Rightarrow\ \dfrac{2h}{(1+h^2)^2}=\dfrac{h^2}{(1+h^2)h}$

$\Rightarrow\ 1+h^2=2 \quad\Rightarrow\ h=\pm 1$

$\therefore\ A\equiv\left(1,\dfrac{1}{2}\right)$

SECTION-II

1. **Ans. (13.00)**

Sol. $\displaystyle\sum_{k=0}^{7}\left(\frac{{}^7C_k}{\lfloor 14}\lfloor k\,\lfloor 14-k\sum_{r=K}^{14}\frac{\lfloor r}{\lfloor k\,\lfloor r-k}\,\frac{\lfloor 14}{\lfloor r\,\lfloor 14-r}\right)$

$\displaystyle=\sum_{k=0}^{7}\left({}^7C_k\sum_{r=k}^{14}{}^{14-K}C_{r-k}\right)=\sum_{k=0}^{7}{}^7C_k\,2^{14-k}$

$\displaystyle=2^{14}\sum_{k=0}^{7}{}^7C_k\left(\frac{1}{2}\right)^k=6^7$

2. Ans. (6.00)

Sol. $p(x) = x^2(x^4 - x^3 - x^2 - 1) + (x^4 - x^3 - x^2 - 1) + x^2 - x + 1$

$\Rightarrow \quad P(\alpha) + P(\beta) + P(\gamma) + P(\delta)$

$= (\alpha^2 + \beta^2 + \gamma^2 + \delta^2) - (\alpha + \beta + \gamma + \delta) + 4 = 6$

3. Ans. (2.00)

Sol.

$$9x^4 - 4\sqrt{3}x + 3 = (3x^2)^2 - 2 \times 3x^2 + 1 + 6x^2 - 4\sqrt{3}x + 2$$

$$= (3x^2 - 1)^2 + 2(3x^2 - 2\sqrt{3}x + 1)$$

$$= (\sqrt{3}x - 1)^2\left[(\sqrt{3}x + 1)^2 + 2\right]$$

$$\frac{1}{6}\int \frac{(4 + 6x^2)dx}{\left((\sqrt{3}x + 1)^2 + 2\right)(\sqrt{3}x - 1)^2} = \frac{1}{6}\int \frac{(\sqrt{3}x + 1)^2 + 2 + (\sqrt{3}x - 1)^2}{\left[(\sqrt{3}x + 1)^2 + 2\right]\left[\sqrt{3}x - 1\right]^2}dx$$

$$= \frac{1}{6}\int \frac{dx}{(\sqrt{3}x + 1)^2 + 2} + \frac{1}{6}\int \frac{dx}{(\sqrt{3}x - 1)^2}$$

$$= \frac{1}{6} \cdot \frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{\sqrt{3}x + 1}{\sqrt{2}}\right) \cdot \frac{1}{\sqrt{3}} + \frac{1}{6}\left(\frac{-1}{\sqrt{3}x - 1}\right)\left(\frac{+1}{\sqrt{3}}\right) + C$$

$$= \frac{1}{6\sqrt{6}}\tan^{-1}\left(\frac{\sqrt{3}x + 1}{\sqrt{2}}\right) - \frac{1}{6\sqrt{3}} \cdot \frac{1}{(\sqrt{3}x - 1)} + C$$

$f(x) = \sqrt{3}x - 1$

4. Ans. (1.00)

Sol. $f(x).f(yf(x)) = f(x + y)$

$\Rightarrow \quad f'(x) \cdot f(yf(x)) + f(x) \cdot f'(yf(x)) \cdot yf'(x) = f'(x + y)$

Putting x = 0 and then replacing y by x

$\Rightarrow \quad -f(x) - xf'(x) = f'(x)$

$(\because\ f(0) = 1,\ f'(0) = -1)$

$$\Rightarrow \quad \frac{f'(x)}{f(x)} = -\frac{1}{1 + x}$$

Integrating $\ln|f(x)| = -\ln|1 + x| + \ln|c|$

$$\Rightarrow \quad f(x) = \frac{c}{1 + x} \quad \Rightarrow f(x) = \frac{1}{1 + x} \quad (\because\ f(0) = 1)$$

$$\therefore \quad \text{Required area } = \int_0^\infty \frac{1}{(1 + x)^2}dx = 1$$

5. Ans. (0.00)

Sol. $$\lim_{x \to \infty} \frac{\left(xe^{-x} + x^2e^{-2x} + x^3e^{-3x} + ...\infty\right)}{(\ln(1 + x))^{-1}}$$

$$= \lim_{x \to \infty}\left(\frac{xe^{-x}}{1 - xe^{-x}}\right)\ln(1 + x)$$

$$\lim_{x \to \infty} \frac{x \cdot \ln(1 + x)}{e^x - x}\left(\frac{\infty}{\infty}\right)$$

Using L'Hospital rule

$$\lim_{x \to \infty} \frac{x \cdot \frac{1}{x + 1} + \ln(1 + x)}{e^x - 1}$$

$$= \lim_{x \to \infty} \frac{\frac{1}{(x + 1)^2} + \frac{1}{(x + 1)}}{e^x} = 0$$

6. Ans. (1.42 or 1.43)

Sol. $f(3) = 4 \Rightarrow g(4) = 3$

For inverse functions,

$f(g(x)) = x$

Differentiating both sides w.r.t. we get,

$$f'(g(x)) \cdot g'(x) = 1 \ \Rightarrow \ g'(x) = \frac{1}{f'(g(x))}$$

Putting x =4

$$g'(4) = \frac{1}{f'(g(4))} \quad \Rightarrow g'(4) = \frac{1}{f'(3)} = \left(\frac{5}{2}\right)^{1/3}$$

Differentiating again.

$$f'(g(x)) \cdot g''(x) + f''(g(x)) \cdot g'(x) \cdot g'(x) = 0$$

Putting again x = 4 we finally get,

$$g''(4) = \left(\frac{5}{2}\right)^{\frac{2}{3}+\frac{1}{3}} \cdot \left(\frac{4}{7}\right) = \frac{10}{7}$$