

SAMPLE PAPER-4**ANSWER KEY****PAPER-1****PART-1 : PHYSICS**

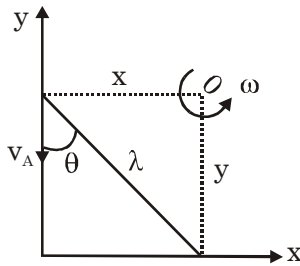
	Q.	1	2	3	4	5	6	7	8	9	10
SECTION-I	A.	A	D	C	A	B,C,D	A,D	B,D	C,D	A,D	A,B,C,D
	Q.	11	12								
	A.	B,C	A,B,C,D								
SECTION-II	Q.	1	2	3	4	5	6				
	A.	0.30	24.00	5.89	4.00	41.50	14.58				

PART-2 : CHEMISTRY

	Q.	1	2	3	4	5	6	7	8	9	10
SECTION-I	A.	B	D	D	D	B,D	A,B,D	A,B,D	A,C,D	B	A,B
	Q.	11	12								
	A.	B,C,D	A,B,C,D								
SECTION-II	Q.	1	2	3	4	5	6				
	A.	8.00	2.00	2.25	5.00	2.00	1.25				

PART-3 : MATHEMATICS

	Q.	1	2	3	4	5	6	7	8	9	10
SECTION-I	A.	B	A	D	A	A,C	A,D	A,B,D	A,D	B,C	B,C,D
	Q.	11	12								
	A.	A,C	A,C,D								
SECTION-II	Q.	1	2	3	4	5	6				
	A.	7.00	0.31 to 0.32	2.00 OR 4.30	0.75	9.00	9.00				

SAMPLE PAPER-4
PAPER-1
PART-1 : PHYSICS
SOLUTION
SECTION-I
1. Ans. (A)
Sol. At any instant,


$$\omega = \frac{v_A}{x}$$

$$= \frac{v_A}{l \sin \theta}$$

2. Ans. (D)
Sol. $30 = mC\Delta T + W'$ (i)

 and $90 = (4m)C\Delta T + W'$ (ii)

After solving above equations, we get

$$W' = 10 \text{ W}$$

3. Ans. (C)
Sol. The flux of this charge will pass through such four identical cubes, and so

$$\phi_{\text{total}} = \frac{q}{\epsilon_0}$$

$$\therefore \phi_{\text{each}} = \frac{q}{4 \epsilon_0}$$

4. Ans. (A)
Sol. $I_1 = I_0$

$$I_2 = 4 I_0$$

$$I_{\text{max}} = (\sqrt{I_1} + \sqrt{I_2})^2 = (\sqrt{I_0} + \sqrt{4I_0})^2 = 9I_0$$

$$I_{\text{min}} = (\sqrt{I_0} - \sqrt{4I_0})^2 = I_0$$

5. Ans. (B,C,D)

Sol.
$$\frac{n_2}{v_1} = \frac{n_1}{-u} + \frac{(n_2 - n_1)}{R}$$
 (1)

$$\frac{n_3}{v} = \frac{n_2}{v_1} + \frac{(n_3 - n_2)}{-R}$$
 (2)

Eqn. (1) + Eqn. (2)

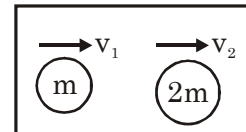
$$\frac{n_3}{v} = -\frac{n_1}{u} + \frac{1}{R} [2n_2 - (n_1 + n_3)]$$

$$u = \infty, v = f$$

$$\frac{n_3}{f} = \frac{1}{R} [2n_2 - (n_1 + n_3)]$$

$$f = +ve \quad n_2 > \left(\frac{n_1 + n_3}{2} \right)$$

$$f = -ve \quad n_2 < \left(\frac{n_1 + n_3}{2} \right)$$

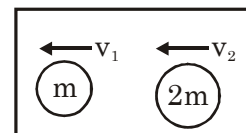
6. Ans. (A,D)
Sol. For 1st collision between 2 balls.


after collision

$$mv_1 + 2mv_2 = mu$$

$$\frac{v_2 - v_1}{0 - u} = -\frac{1}{2} \Rightarrow v_2 - v_1 = \frac{u}{2}$$

After collision of ball B with wall direction of velocity is interchanged only finally after all collision.



$$mv_1' + 2mv_2' = 2m \times \frac{u}{2}$$

$$\Rightarrow v_1' + 2v_2' = u$$
 (iii)

Also
$$\frac{v_1' - v_2'}{0 - \frac{u}{2}} = -\frac{1}{2}$$

$$\Rightarrow v_1' - v_2' = \frac{u}{2}$$
 (iv)

From (iii) and (iv),

$$v'_2 = \frac{u}{4}, v'_1 = \frac{u}{2}$$

i.e., $v_A = \frac{u}{2}, v_B = \frac{u}{4}$

7. **Ans. (B,D)**

Sol. The effective emf the two cells in parallel is

$$\epsilon = \frac{\frac{1}{1} - \frac{2}{2}}{\frac{1}{1} + \frac{2}{2}} = 0$$

So null point will be at zero distance from A. When jockey is touched to B, the current flows through 2 V cell towards B.

8. **Ans. (C,D)**

Sol. $f' = \frac{v - v_0}{v - v_s} f \Rightarrow f' < f$

$$\lambda' = \lambda - v_s T \Rightarrow \lambda' < \lambda$$

$$v_{rel} = v - v_0 \Rightarrow v_{rel} < v$$

9. **Ans. (A,D)**

Sol. Since sun rays fall on the black body, it will absorb more radiation and since, its temperature is constant it will emit more radiation. The temperature will remain same only when energy emitted is equal to energy absorbed.

10. **Ans. (A,B,C,D)**

Sol. On x-axis two conductors produce equal and opposite fields, so net field becomes zero.

11. **Ans. (B,C)**

Sol. In ground state $n = 1$ and for first excited state $n = 2$

$$KE = \frac{1}{4\pi\epsilon_0} \frac{e^2}{2r} (z=1) = \frac{14.4 \times 10^{-10}}{2r} \text{ eV}$$

$$(\because \tau = 0.53n^2 A^\circ (z=1))$$

$$(KE)_1 = \frac{14.4 \times 10^{-10}}{2 \times 0.53 \times 10^{-10} \times 4} \text{ eV} = 13.58 \text{ eV}$$

and

$$(KE)_1 = \frac{14.4 \times 10^{-10}}{2 \times 0.53 \times 10^{-10} \times 4} \text{ eV} = 3.39 \text{ eV}$$

\therefore KE decreases by = 10.2 eV

\therefore PE increases by

= Excitation energy + Loss in kinetic energy

$$= 10.2 + 10.2 = 20.4 \text{ eV}$$

Now Angular momentum;

$$L = mvr = \frac{nh}{2\pi}$$

$$\Rightarrow L_1 - L_1 = \frac{h}{2\pi} = \frac{6.6 \times 10^{-34}}{6.28}$$

$$= 1.05 \times 10^{-34} \text{ J-sec}$$

12. **Ans. (A,B,C,D)**

Sol. (B) when circuit under resonance

(C) there will always be a phase difference between current and voltage across AC

SECTION-II

1. **Ans. 0.30**

Sol. The least count of given Vernier Calipers is

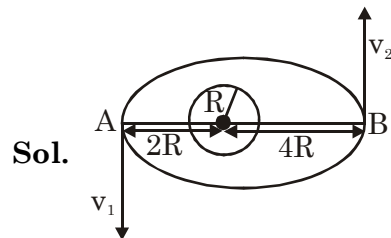
$$LC = MSD - VSD = 1 - (9/10) = 0.1 \text{ mm}$$

The main scale reading is $MSR_0 = 0 \text{ mm}$ and the Vernier scale reading is $VSR_0 = 3$.

Thus,

$$\text{Zero Error} = MSR_0 + VSR_0 \times LC = 0 + 3 \times 0.1 = 0.3 \text{ mm}$$

2. **Ans. 24.00**



Sol.

Applying conservation of angular momentum

$$mv_1 (2R) = mv_2 (4R)$$

$$v_1 = 2v_2 \quad \dots (i)$$

From conservation of energy

$$\frac{1}{2}mv_1^2 - \frac{GMm}{2R} = \frac{1}{2}mv_2^2 - \frac{GMm}{4R} \quad \dots (ii)$$

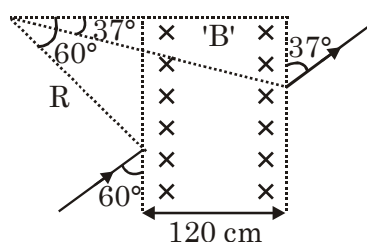
Solving Eqs. (i) and (ii), we get

$$v_2 = \sqrt{\frac{GM}{6R}}$$

3. **Ans. 5.89**

Sol. $T_\alpha = \frac{(Q - \Delta E) Y}{Y + 4} = \frac{(8 - 2) 220}{224} = \boxed{5.89 \text{ MeV}}$

4. Ans. 4.00



Sol.:

$$d = R \cos 37^\circ - R \cos 60^\circ$$

$$R = 4\text{m}$$

5. Ans. 41.50

Sol. According to law of equipartition of energy, energy is equally distributed among its degree of freedom. Let translational and rotational degree of freedom be f_1 and f_2 .

$$\therefore \frac{K_T}{K_R} = \frac{3}{2} \text{ and } K_T + K_R = U$$

Hence the ratio of translational to rotational degrees of freedom is 3 : 2. Since translational degrees of freedom is 3, the rotational degrees of freedom must be 2.

$$\therefore \text{Internal energy (U)} = 1 \times (f_1 + f_2) \times \frac{1}{2} RT$$

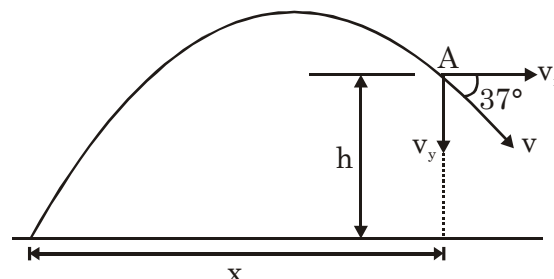
$$U = \frac{1 \times 5 \times 8.3 \times 200}{2} = \boxed{U = 4150\text{J}}$$

6. Ans. 14.58

$$\text{Sol. } u_x = 20 \cos 53^\circ = 12 \text{ m/s}$$

$$u_y = 20 \sin 53^\circ = 16 \text{ m/s}$$

When the ball enters the pipe its velocity vector makes an angle of 37° with the horizontal.



$$\tan 37^\circ = \frac{v_y}{v_x}$$

$$\frac{3}{4} = \frac{v_y}{12}$$

$$v_y = 9 \text{ m/s}$$

$$v_y^2 = u_y^2 - 2gh$$

$$\therefore 9^2 = 16^2 - 2 \times 10 \times h$$

$$\Rightarrow h = 8.75\text{m}$$

$$\text{Now } \frac{h}{L} = \sin 37^\circ \Rightarrow L = \frac{8.75 \times 5}{3} = 14.58\text{m}$$

PART-2 : CHEMISTRY

SOLUTION

SECTION-I

1. Ans. (B)

$$\text{Sol. } E_{\text{back}} = \frac{0.059}{2} \log \frac{0.12}{0.08} = 5.19 \text{ mV} = 5.2 \text{ mV}$$

2. Ans. (D)

Sol.: Information based

3. Ans. (D)

Any orbital can accommodate a maximum of 2 electrons.

Successive ionisation gets difficult.

Number of unpaired electrons in Co^{2+} cation = 3.

Number of unpaired electrons in Co^{3+} cation = 4.

4. Ans. (D)

Sol.: Covalent character depends on polarizing part of cation.

reducing power $\text{F}^- < \text{Cl}^- < \text{Br}^- < \text{I}^-$

Boiling point increases with increment in Vander Wall force.

5. Ans. (B,D)

6. Ans. (A,B,D)

Sol.: A – Structure of lactose

B – Instead of β 1, 4 Glycosidic, it should be α 1, 4 Glycosidic.

D – Thymine is pyrimidine not purine.

(\therefore No imidazole ring)

7. Ans. (A,B,D)

Sol.: $k_1 = Ae^{\frac{-E_1}{RT_a}}$ $k_2 = Ae^{\frac{-E_2}{RT_a}}$

$k'_1 = Ae^{\frac{-E_1}{RT_b}}$ $k'_2 = Ae^{\frac{-E_2}{RT_b}}$

$$\frac{k_1}{k'_1} = \frac{Ae^{\frac{-E_1}{RT_a}}}{Ae^{\frac{-E_1}{RT_b}}}$$

$$\frac{k_1}{k'_1} = e^{\frac{-E_1}{R} \left(\frac{1}{T_a} - \frac{1}{T_b} \right)}$$

$$\Rightarrow \frac{k'_1}{k_1} = e^{\frac{E_1}{R} \left(\frac{1}{T_a} - \frac{1}{T_b} \right)}$$

Similarly $\frac{k'_2}{k_2} = e^{\frac{E_2}{R} \left(\frac{1}{T_a} - \frac{1}{T_b} \right)}$

$E_1 < E_2$

$$\Rightarrow e^{\frac{E_1}{R} \left(\frac{1}{T_a} - \frac{1}{T_b} \right)} < e^{\frac{E_2}{R} \left(\frac{1}{T_a} - \frac{1}{T_b} \right)}$$

$$\Rightarrow \frac{k'_1}{k_1} < \frac{k'_2}{k_2}$$

8. Ans. (A,C,D)

9. Ans. (B)

Sol.: XeF_4 is square planer and donates F^- to PF_5 to give PF_6^- .

10. Ans. (A,B)

11. Ans. (B,C,D)

Sol.: Electrical conductance reciprocally depends on covalent character.

Ionic character increases with increment in size of cation.

F-F has least bond energy due to repulsion between lone pairs of F atoms.

As the molar mass of isotope increases, the bond energy increases.

Reducing character of hydride increases with increment in ionic character.

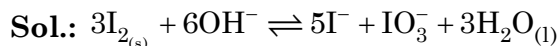
12. Ans. (A,B,C,D)

Sol.: When the one of Bromine containing species is too low the intermediate of A and B revert to the initial species and the allytic substitution completes successfully and in NBS their is ions of Bromine.

D - Although bridgehead FR substitution is possible. This reaction demonstrates that the free radical need not be planar, and the yield of the product is 87%.

SECTION-II

1. Ans. (8.00)



$$\Delta G^\circ = 5(-50) + (-123.5) + 3(-233) - 0 - 6(-150)$$

$$\Delta G^\circ = -172.5 \text{ kJ / mol}$$

$$= \frac{-25}{3} \times 300 \times 2.303 \times 10^{-3} \log k$$

$$\log k = 30$$

$$10^{30} = \frac{10^{-5} \times 10^{-1}}{[OH^-]^6}$$

$$OH^- = 10^{-6}$$

$$p^{OH} = 6, \quad p^H = 8$$

2. Ans. (2.00)

Sol.: 1, 3, are correct.

3. Ans. (2.25)

Sol.: $x = 3, y = 6, z = 4$

4. Ans. (5.00)

Sol.: a, b, c, d & e

5. Ans. (2.00)

Sol.: 2 ppm

$$M_1 V_1 = M_2 V_2$$

$$50 \times M_1 = \frac{1}{50} \times 0.1$$

$$M_1(HCO_3^-) = \frac{1}{25 \times 10^3}$$

$$\text{m moles of } HCO_3^- = \frac{1}{25 \times 10^3} \times 50 = \frac{1}{500}$$

$$\text{m moles of } Ca^{2+} = \frac{1}{1000}$$

$$\text{m moles of } CaCO_3 = \frac{1}{1000}$$

$$\text{weight of } CaCO_3 = \frac{1}{1000} \times 10^{-3} \times 100 = 10^{-4}$$

$$\text{hardness} = [\text{weight of } CaCO_3 / \text{weight of water} \times 10^6]$$

$$= \frac{10^{-4}}{50} \times 10^6 = 2 \text{ ppm}$$

6. Ans. (1.25)

Sol.: $x = 6, y = 2$

$$\frac{x^2 + y^2}{x^2 - y^2} = \frac{36 + 4}{36 - 4} = \frac{40}{32} = \frac{10}{8} = 1.25$$

SECTION-I

1. **Ans. (B)**

Sol. The total number of coins in the box is $N + 7$. The total value of the selected coins will be greater than or equal to one rupee and fifty paise if the selected coins are

- 1 fifty paise coin, 4 twenty-five paise coins
- 2 fifty paise coins, 3 twenty-five paise coins
- 2 fifty paise coins, 2 twenty-five coins and 1 coin out of the N five paise and ten paise coins.

If E = total value of the 5 elected coins is greater than or equal to one rupee and fifty paise, then

$$n(E) = {}^2C_1 \cdot {}^5C_4 \cdot {}^NC_0 + {}^2C_2 \cdot {}^5C_3 \cdot {}^NC_0 + {}^2C_2 \cdot {}^5C_2 \cdot {}^NC_1 \\ = 10(N + 2)$$

Hence, required probability

$$= 1 - P(E) = 1 - \frac{n(E)}{n(S)} = 1 - \frac{10(N + 2)}{N + {}^7C_5}$$

2. **Ans. (A)**

Sol. $8x^2 - 10x + 3 = 0$

$$8x^2 - 6x - 4x + 3 = 0$$

$$(2x - 1)(4x - 3) = 0$$

$$x = \frac{1}{2}, \frac{3}{4}$$

$$\alpha = \frac{1}{2}, \beta = \frac{\sqrt{3}}{2}$$

$$\left(\frac{1}{2} + \frac{\sqrt{3}}{2}i\right)^{100} \quad \text{and} \quad \left(\frac{1}{2} - \frac{\sqrt{3}}{2}i\right)^{100}$$

$$(-w^2)^{100} \quad \text{and} \quad (-w)^{100}$$

$$w^{200} \quad \text{and} \quad w^{100}$$

$$w^2 \quad \text{and} \quad w$$

$$x^2 + x + 1 = 0$$

3. **Ans. (D)**

Sol. $\Delta = 0$

$$\Delta = -\left[(a_0^3 + a_1^3 + a_2^3) - 3a_0a_1a_2\right] = 0$$

$$\text{given } a_0 + a_1 + a_2 \neq 0$$

$$\text{then } a_0 = a_1 = a_2$$

$$\text{Putting } x = 0$$

$$a_0 = 1$$

$$\text{Coefficient of } x = 4a$$

$$\text{Coefficient of } x^2 = 4b + 6a^2$$

$$1 = 4a = 6a^2 + 4b$$

$$\Rightarrow a = \frac{1}{4}; b = \frac{5}{32} \quad 5 \times \left(\frac{1/4}{5/32}\right) = 8$$

4. **Ans. (A)**

Sol. $\therefore f(1 + x) = f(1 - x)$

$$f'(1 + x) = -f'(1 - x)$$

$$|f'(1 + x)| = |f'(1 - x)|$$

$$g'(x) = (1 + x)|f'(x + 1)| + (1 - x)|f'(1 - x)|$$

$$= |f'(1 + x)|(1 + x + 1 - x) > 0 \quad \forall x \in \mathbb{R}$$

5. **Ans. (A,C)**

$$\text{Sol. } I = \int_{-\infty}^a \frac{(\sin^{-1} e^x + \cos^{-1} e^x) \left(\frac{e^x}{e^{2x} + 1}\right) dx}{(\tan^{-1} e^a + \tan^{-1} e^x)}$$

$$= \frac{\pi}{2} \int_{-\infty}^a \frac{1}{(\tan^{-1} e^a + \tan^{-1} e^x)} \left(\frac{e^x}{e^{2x} + 1}\right) dx$$

$$\tan^{-1}(e^x) = t$$

$$e^x = \tan(t)$$

$$e^x dx = \sec^2 t dt$$

$$= \frac{\pi}{2} \int_0^{\tan^{-1} e^a} \frac{dt}{(t + \tan^{-1} e^a)}$$

$$= \frac{\pi}{2} \left[\ln |t + \tan^{-1} e^a| \right]_0^{\tan^{-1} e^a}$$

$$= \frac{\pi}{2} \ln 2$$

6. **Ans. (A,D)**

$$\text{Sol. R.H.D.} = f'(2^+) = \lim_{h \rightarrow 0^+} \frac{\int_0^{2+h} (5 + |1 - t|) dt - 11}{h}$$

$$= \lim_{h \rightarrow 0^+} \frac{\int_0^1 (6 - t) dt + \int_1^{2+h} (4 + t) dt - 11}{h}$$

$$= \lim_{h \rightarrow 0^+} \frac{\left[\frac{(6-t)^2}{2} \right]_1^0 + \left[\frac{(t+4)^2}{2} \right]_1^{2+h} - 11}{h}$$

$$= \lim_{h \rightarrow 0^+} \frac{h(12+h)}{2h} = 6$$

L.H.D. = 5 \Rightarrow Continuous but non differentiable at $x = 2$.

7. Ans. (A,B,D)

Sol. Dividing throughout by $\sin^2 y \cos^2 x$, we get

$$3 \tan x \sec^2 x \, dx - 7 \sec^2 x \, dx - 7 \cot y \operatorname{cosec}^2 y \, dy + 5 \operatorname{cosec}^2 y \, dy + 4(\cot y \sec^2 x \, dx - \tan x \operatorname{cosec}^2 y \, dy) = 0$$

Integrating

$$3 \frac{\tan^2 x}{2} - 7 \tan x + 7 \frac{\cot^2 y}{2} - 5 \cot y + 4(\cot y \tan x) = c$$

So $a = 7, b = 5, d = 4$

8. Ans. (A,D)

Sol. $|a| = |b| = 1$

$a \cdot b = 0$; (as $a \perp b$)

$$c = \alpha a + \beta b + \gamma(a \times b) \quad \dots(i)$$

Taking dot product by a ,

$$a \cdot c = \alpha |a|^2 + \beta(a \cdot b) + \gamma[a \cdot (a \times b)]$$

$$\Rightarrow |a| \cdot |c| \cos \theta = \alpha \cdot 1 + 0 + 0$$

$$\Rightarrow 1 \cdot |c| \cdot \cos \theta = \alpha$$

Taking dot product of (i) by b

$$b \cdot c = b \cdot a + \beta |b|^2 + \gamma[b \cdot (a \times b)]$$

$$\Rightarrow |b| |c| \cos \theta = 0 + \beta \cdot 1 + 0$$

$$\therefore \beta = 1 \cdot 1 \cdot \cos \theta = \cos \theta$$

$$|c|^2 = 1 \Rightarrow \alpha^2 + \beta^2 + \gamma^2 = 1$$

$$\Rightarrow \cos^2 \theta + \cos^2 \theta + \gamma^2 = 1$$

$$\therefore \gamma^2 = 1 - 2 \cos^2 \theta = -\cos 2\theta$$

Hence, $\alpha = \beta = \cos \theta, \gamma^2 = -\cos 2\theta$

9. Ans. (B,C)

Sol. Equation of chord of contact from $(-2, 0)$ is $3x + 4y - 34 = 0$

Solving this with circle we get $x = 6, -\frac{2}{5}$

10. Ans. (B,C,D)

Sol. Let $(x_1, y_1) = (at^2, 2at)$ tangent at this points $ty = x + at^2$

Any point on this tangent is $\left(h, \frac{h+at^2}{t} \right)$

Chord of contact of this point w.r.t. the circle $x^2 + y^2 = a^2$ is

$$hx + \left(\frac{h+at^2}{t} \right) y = a^2 \Rightarrow (at y - a^2) + h \left(x + \frac{y}{t} \right) = 0$$

Which is a family of st. lines passing through point of intersection of

$$ty - a = 0 \text{ and } x + \frac{y}{t} = 0$$

So the fixed point is $\left(-\frac{a}{t^2}, \frac{a}{t} \right)$

$$\therefore x_2 = -\frac{a}{t^2}, y_2 = \frac{a}{t}$$

$$\therefore x_1 x_2 = -a^2, y_1 y_2 = 2a^2$$

11. Ans. (A,C)

Sol. $PA = x, PB = y, PC = z$

Applying cosine laws :

$$\text{cosine} \Rightarrow \begin{cases} x^2 = z^2 + b^2 - 2bz \cos \alpha \\ y^2 = x^2 + c^2 - 2cx \cos \alpha \\ z^2 = y^2 + a^2 - 2ay \cos \alpha \end{cases}$$

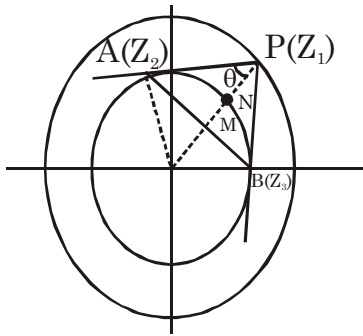
$$\text{Adding } 2(cx + ay + bz) \cos \alpha = a^2 + b^2 + c^2$$

$$\text{Also for } \Delta ABC \Delta = \frac{1}{2}(cx + ay + bz) \sin \alpha$$

$$\tan \alpha = \frac{4\Delta}{a^2 + b^2 + c^2} = \frac{168}{295} \Rightarrow m + n = 463$$

12. Ans. (A,C,D)

Sol.



OA = OB = 1; OP = 2

$$\sin \angle OPA = \frac{1}{2}, \quad \angle OPA = \frac{\pi}{6}, \quad \angle APB = \frac{\pi}{3}$$

OM ⊥ AB so ΔAPB is equilateral.

So (D) is true.

OM = 1/2, MN = 1/2, PN = 1 so

centroid on |z| = 1(A)

$$\left| \frac{z_1 + z_2 + z_3}{3} \right| = 1$$

$$|z_1 + z_2 + z_3|^2 = 9$$

$$(z_1 + z_2 + z_3)(\bar{z}_1 + \bar{z}_2 + \bar{z}_3) = 9$$

$$z_1 \bar{z}_1 = 4 \quad z_2 \bar{z}_2 = 1 \quad z_3 \bar{z}_3 = 1$$

$$\text{or } \left(\frac{4}{z_1} + \frac{1}{z_2} + \frac{1}{z_3} \right) \left(\frac{4}{z_1} + \frac{1}{z_2} + \frac{1}{z_3} \right) = 9 \quad \dots(C)$$

$$\angle AOB = \pi - \frac{\pi}{3} = \frac{2\pi}{3} \quad \dots(B)$$

SECTION-II

1. Ans. 7.00

Sol. There are Maths-I, Maths-II and 6 other books.

If Maths-II is selected, Maths-I also selected. So number of ways = 6C_1

If Maths-II not selected then number of ways = 7C_3

$$\lambda = {}^6C_1 + {}^7C_3 = 6 + 35 = 41$$

Perfect squares less than 41 are 0, 1, 4, 9, 16, 25, 36.

2. Ans. 0.31 or 0.32

Sol. Let E_1 = Event that A wrote a plus sign.

E_2 = Event that A wrote a minus sign.

E = Event that the reference observes a plus sign.

$$\text{Given } P(E_1) = \frac{1}{3} \Rightarrow P(E_2) = \frac{2}{3}$$

$P(E/E_1)$ = Probability that none of B, C, D change sign + Probability that exactly two of B, C, D change sign.

$$= \frac{1}{27} + 3 \left(\frac{1}{3} \times \frac{2}{3} \times \frac{2}{3} \right) = \frac{13}{27}$$

$P(E/E_2)$ = Probability that all of B, C, D change the sign + Probability that exactly one of them change the sign.

$$= \frac{8}{27} + 3 \times \left(\frac{2}{3} \times \frac{1}{3} \times \frac{1}{3} \right) = \frac{14}{27}$$

$$\therefore P(E_1 / E) = \frac{13}{41} \quad \text{Using Baye's theorem.}$$

3. Ans. 2.00 OR 4.30

Sol. $|A| = |Q|^{(n-1)^2} = |Q|^4 = 10^4$

$$P^{-1}BP = A \Rightarrow |P^{-1}BP| = |A|$$

$$\Rightarrow |B| = |A|$$

$$\text{Hence } |A| + |B| = 2 \times 10^4$$

$$\therefore \frac{b}{a} = \frac{4}{2} = 2.00$$

4. Ans. 0.75

Sol. $f'(x) = 0 \Rightarrow x = \pm \sqrt{\frac{a}{3b}}$

$$f\left(-\sqrt{\frac{a}{3b}}\right) = \frac{-2a}{3} \sqrt{\frac{a}{3b}}$$

$$f\left(\sqrt{\frac{a}{3b}}\right) = \frac{2a}{3} \sqrt{\frac{a}{3b}}$$

$$f(-1) = b - a$$

$$f(1) = a - b$$

Given that $\left| \frac{2a}{3} \sqrt{\frac{a}{3b}} \right| = \left| -\frac{2a}{3} \sqrt{\frac{a}{3b}} \right|$

$$= |b - a| = |a - b| = 1$$

$$\Rightarrow \frac{4a^3}{27b} = 1 \Rightarrow b = \frac{4a^3}{27}$$

$$\Rightarrow a - b = 1 \Rightarrow a - \frac{4a^3}{27} = 1$$

$$\Rightarrow 4a^3 - 27a + 27 = 0$$

$$a = -3, \frac{3}{2}$$

$$\Rightarrow a = -3 - 1 : b = \frac{3}{2} - 1 = \frac{1}{2}$$

$$= -4$$

Also, $b - a = 1 \Rightarrow \frac{4a^3}{27} - a$

$$\Rightarrow 4a^3 - 27a - 27 = 0$$

$$\Rightarrow (a - 3)(2a + 3)^2 = 0$$

$$\Rightarrow a = 3 \Rightarrow b = 4$$

Rejecting -ve values,

therefore $a = 3, b = 4$

5. Ans. 9.00

Sol. $h(x) = \frac{1}{(x-\omega)(x-\omega^2)} = \frac{1}{(\omega-\omega^2)} \left(\frac{1}{x-\omega} - \frac{1}{x-\omega^2} \right)$

$$h^{(36)}(x) = \frac{1}{(\omega-\omega^2)} \left[\frac{(-1)^{36} \times 36!}{(x-\omega)^{37}} - \frac{(-1)^{36} (36!)}{(x-\omega^2)^{37}} \right]$$

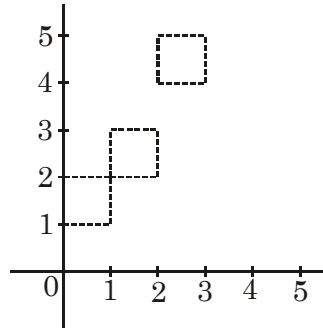
$$= \frac{1}{\omega-\omega^2} (36!) \left(\frac{-1}{\omega} + \frac{1}{\omega^2} \right) = 36!$$

$$36 = 2^2 \cdot 3^2$$

so, number of divisors = 9.

6. Ans. 9.00

Sol.



If $x \in [0, 1) \Rightarrow y \in [1, 2)$

If $x \in [1, 2) \Rightarrow y \in [2, 3)$

If $x \in [2, 3) \Rightarrow y \in [4, 5)$

If $x \in [3, 4) \Rightarrow y \in [8, 9)$

$$\begin{matrix} \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \end{matrix}$$

If $x \in [8, 9) \Rightarrow y \in [256, 257)$

Hence,

total area = $1 + 1 + 1 \dots = 9$