ALLEN CAREER INSTITUTE KOTA (RAJASTHAN)

CLASSROOM CONTACT PROGRAMME

JEE(Advanced)
FULL SYLLABUS

SAMPLE PAPER-3

ANSWER KEY PAPER-2

PART-1 : PHYSICS

SECTION-I	Q.	1	2	3	4	5	6
	A.	A,B,D	A,C,D	B,C	A,D	A,C	A,D
SECTION-II	Q.	1	2	3	4	5	6
	A.	15.00	8.00	0.00	7.66 to 7.67	35.00	8.00
SECTION-III	Q.	1	2	3	4	5	6
	A.	5	2	8	4	8	5

PART-2 : CHEMISTRY

SECTION-I	Q.	1	2	3	4	5	6
	A.	A,C,D	A,B,C	A,B,D	A,B,C,D	A,B,C,D	B,C
SECTION-II	Q.	1	2	3	4	5	6
	A.	5.00	0.00	0.11	0.02	2.00	6.00
SECTION-III	Q.	1	2	3	4	5	6
	A.	5	2	3	9	8	7

PART-3 : MATHEMATICS

SECTION-I	Q.	1	2	3	4	5	6
	A.	B,D	A,C	A,B,C	A,B,D	C,D	B,C,D
SECTION-II	Q.	1	2	3	4	5	6
	A.	0.45	45.95 TO 45.96	31.25	1.25	1.00	1.75
SECTION-III	Q.	1	2	3	4	5	6
	A.	0	1	0	3	7	2

SAMPLE PAPER-3

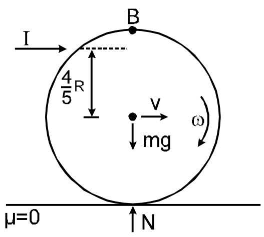
PAPER-2

PART-1 : PHYSICS SOLUTION

SECTION-I

1. Ans. (A,B,D)

Sol. Using linear impulse momentum equation



$I = mv \Rightarrow v = \dfrac{I}{M}$

Using angular impulse momentum equation wrt centre

$I.\dfrac{4}{5}R = \dfrac{2}{5}MR^2\omega$

$\omega = \dfrac{2I}{MR}$ ....(2)

Point B has to traverse angle p to reach ground.

As w is const., time required $t = \dfrac{\theta}{\omega} = \dfrac{\pi}{2I}$

$MR = \dfrac{MR\pi}{2I}$

2. Ans. (A,C,D)

Sol. At max. amp. v = 0 ; $E = Ax^2 \Rightarrow x = \sqrt{\dfrac{E}{A}}$

For max. vel. x = 0 ; $E = Bv^2 \Rightarrow v = \sqrt{\dfrac{E}{B}}$

$U = Ax^2$

$F = -\dfrac{\partial E}{\partial x} = -2Ax \Rightarrow F \text{ (at } x = 0) = 0$

$\therefore$ x = 0 is eq. position.

$F = -2Ax \Rightarrow a = \dfrac{-2Ax}{m}$

$\dfrac{1}{2}mv^2 = Bv^2 \Rightarrow m = 2B$

$\therefore a = -\dfrac{A}{B}x$

3. Ans. (B,C)

Sol. U = Fr

[Using U = Potential energy and v = velocity, to avoid confusion between their symbols]

$\Rightarrow$ Force $= \dfrac{-dU}{dr} = -F$

$\Rightarrow$ Magnitude of force = Constant = F

$\Rightarrow F = \dfrac{mv^2}{R}$ ......(1)

$\Rightarrow mvR = \dfrac{nh}{2\pi}$ ......(2)

$\Rightarrow F = \dfrac{m}{R} \times \dfrac{n^2h^2}{4\pi^2} \times \dfrac{1}{m^2R^2}$

$\Rightarrow R = \left(\dfrac{n^2h^2}{4\pi^2 mF}\right)^{1/3}$ ......(3)

$\Rightarrow v = \dfrac{nh}{2\pi mR}$

$\Rightarrow v = \dfrac{nh}{2\pi m}\left(\dfrac{4\pi^2 mF}{n^2h^2}\right)^{1/3}$

$\Rightarrow \boxed{v = \dfrac{n^{1/3}h^{1/3}F^{1/3}}{2^{1/3}\pi^{1/3}m^{2/3}}}$ ......(4)

(B) is correct

$\Rightarrow E = \dfrac{1}{2}mv^2 + U$

$= \dfrac{1}{2}mv^2 + FR$

$$\Rightarrow E = \frac{1}{2}m\left(\frac{n^{2/3}h^{2/3}F^{2/3}}{2^{2/3}\pi^{2/3}m^{4/3}}\right) + F\times\left(\frac{n^2h^2}{4\pi^2 mF}\right)^{1/3}$$

$$\Rightarrow E = \left(\frac{n^2h^2F^2}{4\pi^2 m}\right)^{1/3}\left[\frac{1}{2}+1\right]$$

$$= \frac{3}{2}\left(\frac{n^2h^2F^2}{4\pi^2 m}\right)^{1/3}$$

Answer is (B,C)

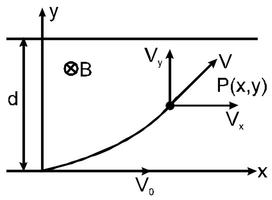
4. Ans. (A,D)

Sol. Let at time t particle be at point P (x, y) and its velocity be

$\vec{V} = \left(V_x\hat{i} + V_y\hat{j}\right)$.

$\left|\vec{V}\right| = \left|\vec{V}_0\right| \Rightarrow V_0^2 = V_x^2 + V_y^2$.

(work done by magnetic field is always zero so change in magnitude of velocity is zero) Then, magnetic force on the particle at point P is



$$\vec{F} = q\left(V_x\hat{i} + V_y\hat{j}\right)\times B_0\left(1+\frac{y}{d}\right)\left(-\hat{k}\right)$$

$$\Rightarrow -qB_0\left[1+\frac{y}{d}\right]dy = mdv_x$$

Now when the particle will be coming out of the at that point y = d. Let the velocity in x-direction be V_x then integrating we get,

$$\int_{v_0}^{v_x} dv_x = -\frac{qB_0}{m}\int_0^d\left[1+\frac{y}{d}\right]dy$$

$$= -\frac{qB_0}{m}\left[d+\frac{d^2}{2d}\right] = -\frac{3qB_0 d}{2m}$$

so $V_x = V_0 - \frac{3qB_0 d}{2m}$

Now

$$\Rightarrow V_y = \sqrt{V_0^2 - V_x^2}$$

$$\Rightarrow V_y = \sqrt{V_0^2 - \left(V_0 - \frac{3qB_0}{2m}d\right)^2}$$

5. Ans. (A,C)

Sol. Charge on capacitor before insertion of dielectric slab = 100 mC

Charge on capacitor after insertion of dielectric slab = 300 mC

Increase in charge on the capacitor

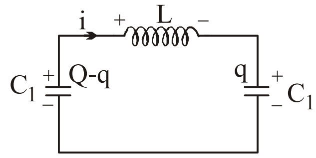
= 300 – 100 = 200 mC

Heat produced = 0

Energy supplied by the cell = increase in stored potential energy + work done on the person who filling the dielectric slab.

6. Ans. (A,D)

Sol. at any time 't'



using KVL :

$$\frac{Q-q}{C_1} - L\frac{di}{dt} - \frac{q}{C_1} = 0$$

$$\text{or } \frac{Q-q}{C_1} - L\left(\frac{d^2q}{dt^2}\right) - \frac{q}{C_1} = 0$$

$$\text{or } L\frac{d^2q}{dt^2} = \frac{Q}{C_1} - \frac{2q}{C_1}$$

$$\text{or } \frac{d^2q}{dt^2} = -\left(\frac{2}{LC_1}\right)q + \frac{Q}{LC_1}$$

$$\Rightarrow q = \frac{Q}{2}\sin(wt+\theta) + \frac{Q}{2}\quad\left\{\omega = \sqrt{\frac{2}{LC_1}}\right\}$$

$$Q_1 = Q - q = \frac{Q}{2}(1+\cos\omega t)$$

$$= \frac{CV_0}{2}(1+\cos\omega t)$$

$$Q_2 = q = \frac{Q}{2}(1-\cos\omega t)$$

$$= \frac{CV_0}{2}(1-\cos\omega t)$$

SECTION-II

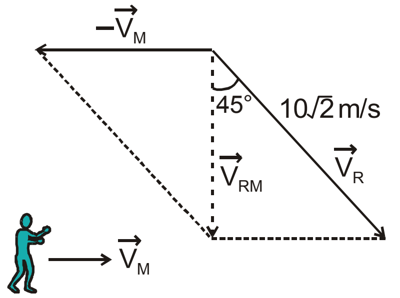
1. Ans. 15.00

Sol. In the first case :

From the figure it is clear that

$\vec{V}_{RM}$ is 10 m/s downwards and

$\vec{V}_M$ is 10 m/s towards right.



In the second case :

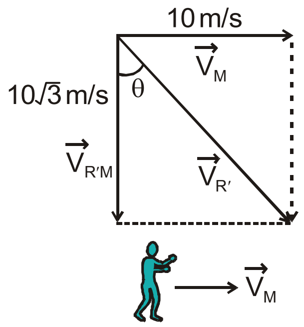
Velocity of rain as observed by man becomes $\sqrt{3}$ times in magnitude.

$\therefore$ New velocity of rain

$\vec{V}_{R'} = \vec{V}_{R'M} + \vec{V}_M$

$\therefore$ The angle rain makes with vertical is

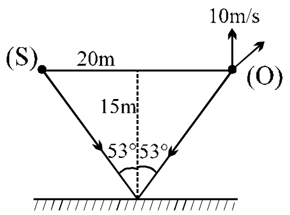
$\tan\theta = \frac{10}{10\sqrt{3}}$ or $\theta = 30°$



$\therefore$ Change in angle of rain = 45 – 30 = 15°.

2. Ans. 8.00

Sol.



$f_{direct} = 440$ Hz

$$f_{indirect} = 440\left[\frac{330 - 10\cos 53°}{330}\right]$$

$$= 440\,[1] - \frac{40}{3}\left(\frac{3}{5}\right)$$

$= 440 - 8 \Rightarrow 432$ Hz

$\Delta f = 440 - 432 = 8$ Hz

3. Ans. 0.00

Sol. Let x is the vertical distance covered by the ring. Then $x = L \tan 37° = 0.7 \times \frac{3}{4}$

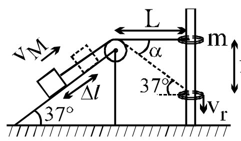
$\Delta l = L \sec 37° - L = L(\sec 37° - 1)$

$\Rightarrow \frac{L}{4} = D\,l$

Δl = distance moved by block M

Now, from constraint relation

$v_M = v_r \sin 37° = \frac{3}{5} v_r$ ...(1)



v_r = velocity of ring, v_M = velocity of the block at this instant

From mechanical energy conservation

$\Delta PE + \Delta KE = 0$

$$-mgx + Mg\,\Delta l \sin 37° + \frac{1}{2} m v_r^2 + \frac{1}{2} M v_M^2 = 0 \quad ...(2)$$

on solving eq. (1) and (2), we get

$v_r = 0$ m/s

Hence instantaneous power will also be zero.

4. Ans. 7.66 to 7.67

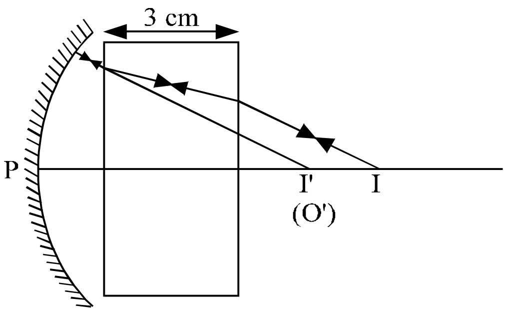
Sol. The rays starting from the object *O* would appear to come from the point *O'*, due to refraction through the glass plate.

This displacement *OO'* is given by $OO' = t(1 - 1/\mu)$ where *t* is the thickness of the plane and μ its refractive index.

$$\therefore OO' = 3\left(1 - \frac{1}{1.5}\right) = 1 \text{ cm}$$

$\therefore PO' = 21 - 1 = 20$ cm

Now if the object were at *O'* (apparent position) its image formed by the mirror would have been at a distance *v* from the mirror (at *I'*) so that



$v = \dfrac{uf}{u-f}$, u = 20 cm,

$f = \dfrac{R}{2} = \dfrac{10}{2} = 5$ cm

$v = \dfrac{20\times5}{20-5}$

$v = \dfrac{100}{15} = \dfrac{20}{3}$ cm

However, the reflected rays have to pass through the glass slab and would converge at *I*.

But $II' = 3(1-\dfrac{1}{1.5}) = 1$ cm

$\therefore$ The final position of the image will be at a distance $PI = \dfrac{20}{3}+1 =$ 7.67 cm from the mirror.

5. Ans. 35.00

Sol. Total resistance = $\int_0^L \lambda x\,dx = \dfrac{\lambda}{2}L^2$

For balance condition

$$\frac{R_1}{R_2} = \frac{(\lambda/2)l_1^2}{(\lambda/2)(L^2-l_1^2)}$$

Similarly, $\dfrac{R_2}{R_1} = \dfrac{l_2^2}{L^2-l_2^2}$

$$\therefore \frac{R_1}{R_2} = \frac{l_1^2}{L^2-l_1^2} = \frac{l_1^2}{L^2-l_1^2} = n$$

Solving, $l_1 = L\sqrt{\dfrac{n}{n+1}}$ and $l_2 = L\sqrt{\dfrac{1}{n+1}}$

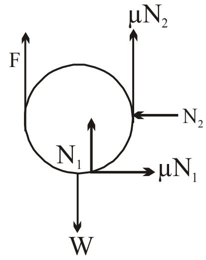
$$\therefore \Delta = l_1 - l_2 = \frac{L}{\sqrt{n+1}}(\sqrt{n}-1)$$

Putting L = 100 cm and n = 3

$$D = \frac{100}{2}(\sqrt{3}-1) = 35 \text{ cm}$$

6. Ans. 8.00

Sol.



$\Sigma F_y = 0 \Rightarrow F + \mu N_2 + N_1 = W$

$\Sigma F_x = 0 \Rightarrow F = \mu N_1$

$\Sigma t = 0 \Rightarrow Fr = \mu N_1 r + \mu N_2 r$

Solving above equations with $\mu = 0.5$

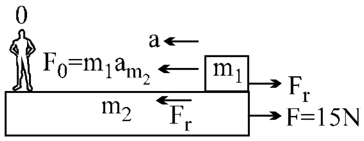
$\Rightarrow F = \dfrac{3W}{8} = 0.375W$

after rounding-off upto 2 decimal places, it will be 0.38W.

SECTION-III

1. Ans. 5

Sol.



Fr = mmg = 0.1 × 1 × 10 = 1N

$$a_{m_2} = \frac{F-Fr}{m_2} = \frac{15-1}{10} = 1.4 \text{ m/sec}^2$$

$$a_{m_1,m_2} = a = \frac{m_1 a_{m_2} - Fr}{m_1} = \frac{1.4-1}{1}$$

$= 0.4$ m/sec^2

Time taken by m_1 to move 1 m

$$t = \sqrt{\frac{2\times1}{0.4}} = \sqrt{5}$$

2. Ans. 2

Sol. Using the energies for the photons for two given transitions,

$$E_{2n} - E_n = 40.8 \Rightarrow 13.6z^2\left[\frac{1}{n^2} - \frac{1}{4n^2}\right] = 40.8$$

and

$$E_{2n} - E_1 = 204 \Rightarrow 13.6z^2\left[\frac{1}{1^2} - \frac{1}{4n^2}\right] = 204$$

3. Ans. 8

Sol. $\dfrac{PT^2}{V} = k \Rightarrow \dfrac{nRT}{V}.\dfrac{T^2}{V} = k \Rightarrow T^3 = \dfrac{kV^2}{nR}$

Now, Differentiating,

$$3T^2\frac{dT}{dV}=\frac{2kV}{nR}\Rightarrow\frac{dV}{dT}=\frac{3nRT^2}{2kV}=\frac{3}{2}\frac{nR}{P}$$

Molar Heat capacity,

$$C=C_V+\frac{P}{n}\frac{dV}{dT}=\frac{5R}{2}+\frac{P}{n}.\frac{3}{2}\frac{nR}{P}=\frac{8R}{2}$$

4. Ans. 4

Sol. For constant velocity,

$a = 0$

$F_0 = F_m$

$$= i\ell B=\left(\frac{\varepsilon}{R}\right)\ell B=\left(\frac{B\ell v_0}{R}\right)\ell B$$

$v_0=\frac{F_0R}{B^2\ell^2}$ velocity at point 'P'

Now, retardation $a=\frac{F_m}{m}=\frac{i\ell B}{m}$

$$a=\frac{B^2\ell^2}{mR}v$$

$$\Rightarrow -v\frac{dv}{ds}=\frac{B^2\ell^2}{mR}v$$

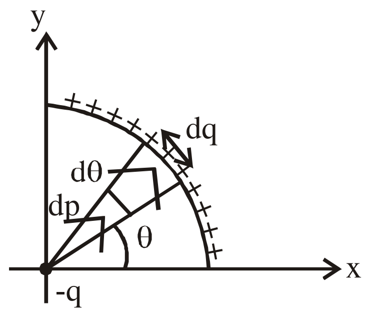
$$\text{or } -\int_{v_0}^{0}dv=\frac{B^2\ell^2}{mR}\int_0^s ds$$

$$\text{or } v_0=\frac{B^2\ell^2}{mR}s$$

$$\text{or } s=\frac{mRv_0}{B^2\ell^2}=\frac{F_0mR^2}{B^4\ell^4}=320\text{ m}$$

5. Ans. 8

Sol. Dipole moment $\frac{2\sqrt{2}qR}{\pi}$



$$\left|\vec{dp}\right|=Rdq=R\times\frac{q}{\frac{\pi}{2}R}(Rd\theta)=\frac{2q}{\pi}Rd\theta$$

$$\vec{p}_x=\int_0^{\pi/2}dp\cos\theta=\frac{2q}{\pi}R\int_0^{\pi/2}\cos\theta d\theta=\frac{2qR}{\pi}\hat{i}$$

$$\vec{p}_y=\int_0^{\pi/2}dp\sin\theta=\frac{2q}{\pi}R\int_0^{\pi/2}\sin\theta d\theta=\frac{2qR}{\pi}\hat{j}$$

$$|p|=\sqrt{p_x^2+p_y^2}=\sqrt{2}\times\frac{2qR}{\pi}=\frac{2\sqrt{2}\times1\times\sqrt{8}\pi}{\pi}=8\text{cm}$$

6. Ans. 5

Sol. By Energy Conservation

$$mg\frac{R}{\sqrt{2}}=\frac{1}{2}\frac{m(\sqrt{2}R)^2\omega^2}{3}$$

$$\Rightarrow w^2=\frac{3g}{\sqrt{2}R}$$

Now, $2N\cos 45° - mg = m\times\frac{3g}{\sqrt{2}R}\times\frac{R}{\sqrt{2}}$

$$\Rightarrow N=\frac{5mg}{2\sqrt{2}}=50$$

PART–2 : CHEMISTRY SOLUTION

SECTION-I

1. Ans. (A,C,D)

2. Ans.(A,B,C)

3. Ans.(A,B,D)

4. Ans. (A,B,C,D)

(i) $H_2A+NaOH\longrightarrow NaHA+H_2O$

0.1M,1mole   0.1M,1mole

–   –   0.05M,200ml

$$PH=\frac{PKa_1+PKa_2}{2}=\frac{3+6}{2}=4.5$$

(ii) $H_2A+2NaOH\longrightarrow Na_2A+H_2O$

0.1 M, 100 ml 0.1 M, 200ml $M=\frac{10}{300}=\frac{1}{30}$

$$\underset{C(1-\alpha)}{A^{-2}}+H_2O\rightleftharpoons\underset{C\alpha}{HA^-}+\underset{C\alpha}{OH^-}$$

$$K_h=\frac{[HA^-][OH^-][H^+]}{[A^{-2}][H^+]}=\frac{K_W}{Ka_2}=\frac{10^{-14}}{10^{-6}}=10^{-8}$$

$$K_h = \frac{C\alpha^2}{(1-\alpha)} = 10^{-8}$$

$$\Rightarrow \frac{10}{300}\alpha^2 = 10^{-8}$$

$$\alpha^2 = 30 \times 10^{-8}$$

$$\alpha^2 = \sqrt{30} \times 10^{-4}$$

$$\left[OH^-\right] = \alpha = \frac{1}{30}.\sqrt{30} \times 10^{-4} = \frac{1}{\sqrt{30}} \times 10^{-4}$$

$$\left[OH^+\right] = \frac{10^{-14}}{10^{-4} \times \frac{1}{\sqrt{30}}} = \sqrt{30} \times 10^{-10}$$

$$PH = 10 - \frac{1}{2}\log 30 = 10.738 = 9.26$$

(iii)

H_2A	+	$2NaOH$	$\longrightarrow$	Na_2A	+	$2H_2O$
0.1×100		0.1×400				
10		40				
0		20		20		

$Na_2A + NaOH$

$$\left(OH^-\right) = \frac{1}{25}$$

$$POH = -\log\frac{1}{25} = \log 25$$

POH = 1.3979

PH = 12.602

(iv) Buffer solution

5. **Ans.(A,B,C,D)**

6. **Ans.(B,C)**

SECTION-II

1. **Ans. (5.00)**

2. **Ans. (0.00)**

3. **Ans.(0.11)**

No. of eq. of oxalic acid = No. of eq. of NaOH

$$\text{or } \frac{5.00 \times 0.10}{1000} \times 2 = \frac{9.0 \times M}{1000} \times 1$$

Molarity of NaOH solution = $\frac{1}{9} = 0.11M$

4. **Ans. (0.02)**

$$Z = 1 + \left(b - \frac{a}{RT}\right)\frac{1}{Vm}$$

$$PVm = ZRT \qquad \Rightarrow \frac{1}{Vm} = \frac{P}{ZRT}$$

$$Z = 1 + \frac{P}{ZRT}\left(b - \frac{a}{RT}\right) \quad \left(T_B = \frac{a}{Rb}\right)$$

$$Z = 1 + \frac{P}{ZRT}\left(b - \frac{bT_B}{T}\right)$$

$$\frac{Z(Z-1)RT}{P} = b\left(1 - \frac{T_B}{T}\right) = b\left(\frac{T - T_B}{T}\right)$$

$$b = \left(\frac{T}{T - T_B}\right).\frac{(Z-1)ZRT}{P}$$

$$b = \left(\frac{300}{300-200}\right).\frac{0.0003 \times 1.0003}{1bar} \times \frac{0.08 litre\, bar x^{-1}\, mole^{-1}}{x\, 300}$$

$$b = 3 \times 3 \times 10^{-4} \times 1.003 \times 8 \times 10^{-2} \times 300$$

b = 0.02160 litre/mole

$b \approx 0.02$ litre / mole

5. **Ans.(2.00)**

6. **Ans. (6.00)**

SECTION-III

1. **Ans. (5)**

$$\underset{\frac{25}{3}mmoles}{5KI} + KIO_3 + \underset{10mmoles}{6HCl} \longrightarrow \underset{5mmoles}{3I_2} + 6KCl + 3H_2O$$

$$\underset{5 millimoles}{I_2} + \underset{10 millimoles}{2Na_2S_2O_3} \longrightarrow 2NaI + Na_2S_4O_6$$

$x = \frac{25}{3}$ and y = 10

3X – 2Y = 5

2. **Ans.(2)**

3. **Ans.(3)**

4. **Ans.(9)**

5. **Ans.(8)**

6. **Ans.(7)**

PART-3 : MATHEMATICS SOLUTION

SECTION-I

1. Ans. (B,D)

Sol. There will be three points in a plane at a time and a plane parallel to the plane containing three points and equidistant from point and plane. No. of such plane '4'. Now consider set of skew lines passing through two points at a time. Three sets of two skew lines can be obtained. Now consider plane bisecting line of shortest distance between them, such three planes are possible. Total planes '7'.

2. Ans. (A,C)

Sol. Divide throughout by x^3y, we get

$$2\frac{dx}{x}+2\frac{dy}{y}-2\frac{y^3}{x^3}dx+3\frac{y^2}{x^2}dy=0$$

$$\Rightarrow 2\frac{dx}{x}+2\frac{dy}{y}+\frac{3xy^2dy-2y^3dx}{x^3}=0$$

$$\Rightarrow 2\frac{dx}{x}+2\frac{dy}{y}+\frac{3x^2y^2dy-2xy^3dx}{x^4}=0$$, on integrating we get

$$\Rightarrow 2(\ln x+\ln y)+\frac{y^3}{x^2}=C$$ Now put x = 1, y = 1

$$\Rightarrow 2\ell n(1)+1=C\Rightarrow C=1\Rightarrow 2\ell n(xy)+\frac{y^3}{x^2}=1,\ \begin{matrix}m=3\\n=2\end{matrix}$$

3. Ans. (A,B,C)

Sol. We have $\int_0^x f(t)dt+x\int_0^x f(t)dt-\int_0^x t\cdot f(t)dt=\left(-1+e^{-x}\right)$

Differentiate both sides with respect to x, we get

$$f(x)+\int_0^x f(t)dt=-e^{-x} \qquad ...(i)$$

Put $x=0,$ wet get $f(0)=-1$

Again differentiating both sides of equation (i) with respect to 'x', we get

$e^x\left(f(x)+f'(x)\right)=1$

Now integrating both sides with respect to x, we get

$e^xf(x)=x+C, f(0)=-1$

$-1=0+C\Rightarrow C=-1$

$f(x)=(x-1)e^{-x},$ Now we can check options.

4. Ans. (A,B,D)

Sol. $\phi(x)=f(x)+Ag(x)$

clearly $\phi(x)$ is continuous and differentiable in $[x_1,x_2]$ and (x_1,x_2) respectively.

If $f(x_1)+Ag(x_1)=f(x_2)+Ag(x_2)$

$$\Rightarrow\frac{f(x_1)-f(x_2)}{g(x_2)-g(x_1)}=A$$

$$\Rightarrow A=\frac{4-6}{2-1}=\frac{-2}{1}=-2$$

We can apply Rolle's theorem on $\phi(x)=f(x)-2g(x)$

which states that $\phi'(c)=0$ for some $c\in(x_1,x_2)$

$f'(c)=2g'(c)\Rightarrow\lambda=2$

(A) Clearly prime number has two divisors

(B) No. of ways of putting 5 identical balls in three identical boxes (3,1,1) & (2,2,1) two ways.

(C) $e^{\sin x}-x=0\Rightarrow\sin x=\ln x$

only '1' solution so this option is incorrect.

(D) No. of into functions are two.

5. Ans. (C,D)

Sol. $f(0)=\lim_{h\to0^+}f(0+h)=\lim_{h\to0^-}f(0-h)$

$a=\ell n(1)+b(0)\Rightarrow\boxed{a=0}$

$$f'(x)=\begin{cases}(\cos(\sin x))\cos x\ ;\ x\ge0\\ -\dfrac{\sin x}{\cos x}+b\ ;\ x<0\end{cases}$$

LHD = (cos(0)) cos0 = 1

RHD = a + b = b

$\Rightarrow$ b = 1

6. Ans. (B,C,D)

Sol. T_r (r^{th} term of the series) = $(-1)^{r-1}\left(1+\frac{1}{2}+\frac{1}{3}+....+\frac{1}{r}\right)\cdot{}^nC_r$

Now $(-1)^{r-1}\left(1+x+x^2+....+x^{r-1}\right)\cdot{}^nC_r=\left(\frac{1-x^r}{1-x}\right)(-1)^{r-1}\cdot{}^nC_r$

Now $\frac{1}{(1-x)}\sum_{r=1}^{n}\left[(-1)^{r-1}\cdot{}^nC_r-(-1)^{r-1}\cdot x^r\cdot{}^nC_r\right]$

$$=\frac{1}{(1-x)}\left[(-1)\sum_{r=1}^{n}(-1)^r\cdot{}^nC_r+\sum_{r=1}^{n}(-1)^r\cdot x^r\cdot{}^nC_r\right]$$

$$=\frac{1}{(1-x)}\left[(-1)\left\{(1-1)^n-1\right\}+\left\{(1-x)^n-1\right\}\right]$$

$$=\frac{(1-x)^n}{(1-x)}=(1-x)^{n-1}$$

$$\Rightarrow\sum_{r=1}^{n}(-1)^{r-1}\left(1+x+x^2+....+x^{r-1}\right)\cdot{}^nC_r=(1-x)^{n-1}$$

Integrating above equation, we get

$$\int_0^1\left[\sum_{r=1}^{n}(-1)^{r-1}\left(1+x+x^2+....+x^{r-1}\right)\cdot{}^nC_r\right]dx=\int_0^1(1-x)^{n-1}dx$$

$$\Rightarrow\sum_{r=1}^{n}(-1)^{r-1}\cdot{}^nC_r\int_0^1\left(1+x+x^2+....+x^{r-1}\right)dx=\left[-\frac{(1-x)^n}{n}\right]_0^1$$

$$\Rightarrow \sum_{r=1}^{n}(-1)^{r-1}\cdot {}^{n}C_r\left[x+\frac{x^2}{2}+\frac{x^3}{3}+.....+\frac{x^2}{r}\right]_0^1=\frac{1}{n}$$

$$\Rightarrow \sum_{r=1}^{n}(-1)^{r-1}\cdot {}^{n}C_r\left[1+\frac{1}{2}+\frac{1}{3}+....+\frac{1}{r}\right]=\frac{1}{n}$$

SECTION-II

1. Ans. 0.45

Sol. Applying $R_1 \to R_1 - R_2\sec x$ and $R_2 \to R_2 - R_3\cos^2 x$

$$f(x)=\begin{vmatrix} 0 & 0 & \sec^2 x+\dfrac{\cos x}{\sin^2 x}-\dfrac{1}{\cos x\sin^2 x} \\ 0 & \cos^2-\cos^4 x & \operatorname{cosec}^2 x-\cos^4 x \\ 1 & \cos^2 x & \cos^2 x \end{vmatrix}$$

Expanding along C_1

$$f(x)=-\left(\cos^2 x-\cos^4 x\right)\left(\sec^2 x+\frac{\cos x}{\sin^2 x}-\frac{1}{\cos x\cdot\sin^2 x}\right)$$

$$=-\cos^2 x\cdot\sin^2 x\left(\frac{1}{\cos^2 x}+\frac{\cos x}{\sin^2 x}-\frac{1}{\cos x\sin^2 x}\right)$$

$$=-\cos^2 x\cdot\sin^2 x\left(\frac{\sin^2 x+\cos^3 x-\cos x}{\cos^2 x\sin^2 x}\right)$$

$$=-\left(\sin^2 x+\cos x\left(\cos^2 x-1\right)\right)$$

$$=-\left(\sin^2 x-\cos x\cdot\sin^2 x\right)$$

$$=\cos x\cdot\sin^2 x-\sin^2 x$$

$$=\int_0^{\frac{\pi}{2}}\left(\cos x\cdot\sin^2 x-\sin^2 x\right)dx$$

$$=\int_0^{\frac{\pi}{2}}\cos x\cdot\sin^2 x dx-\int_0^{\frac{\pi}{2}}\sin^2 x\cdot dx$$

$\sin x = t$

$\cos x dx = dt$

$$\int_0^1 t^2\cdot dt-\frac{1}{2}\cdot\frac{\pi}{2}$$

$$=\frac{1}{3}-\frac{\pi}{4}$$

$$=\frac{4-3\pi}{12}=-0.45$$

$$|-0.45|=0.45$$

2. Ans. 45.95 TO 45.96

Sol. $x^4-4x^3+6x^2-4x+1=2021$

$(x-1)^4=2021$

$(x-1)^2 = \pm\sqrt{2021}$

$(x-1)^2 = -\sqrt{2021}$ because only non-real roots required

$x^2+1-2x+\sqrt{2021}=0$

$x^2-2x+1+\sqrt{2021}=0$, P=product of roots $=1+\sqrt{2021}$ = 45.95

3. Ans. 31.25

Sol. $\frac{1}{1+a}+\frac{1}{1+b}+\frac{1}{1+c}\le 1 \Rightarrow \frac{1}{1+b}+\frac{1}{1+c}\le 1-\frac{1}{1+a}=\frac{a}{1+a}$

$\frac{a}{1+a}\ge\frac{1}{1+b}+\frac{1}{1+c}$

Similarly $\frac{b}{1+b}\ge\frac{1}{1+c}+\frac{1}{1+a}$ and $\frac{c}{1+c}\ge\frac{1}{1+a}+\frac{1}{1+b}$

Apply $A\cdot M \ge G.M.$ for $\frac{1}{1+b}+\frac{1}{1+c}$

$\therefore \frac{1}{1+b}+\frac{1}{1+c}\ge\frac{2}{\sqrt{1+b}\sqrt{1+c}} \Rightarrow \frac{a}{1+a}\ge\frac{2}{\sqrt{(1+b)(1+c)}}$

similarly $\frac{b}{1+b}\ge\frac{2}{\sqrt{(1+c)(1+a)}}$ and $\frac{c}{1+c}\ge\frac{2}{\sqrt{(1+a)(1+b)}}$

Multiply the results

$$\left(\frac{a}{1+a}\right)\left(\frac{b}{1+b}\right)\left(\frac{c}{1+c}\right)\ge\left(\frac{2}{\sqrt{(1+b)(1+c)}}\right)\left(\frac{2}{\sqrt{1+c}\sqrt{1+a}}\right)\left(\frac{2}{\sqrt{1+a}\sqrt{1+b}}\right)$$

$\Rightarrow abc\ge 8$

Now expand $(1+a^2)(1+b^2)(1+c^2)$

$=1+(a^2+b^2+c^2)+(a^2b^2+b^2c^2+c^2a^2)+a^2b^2c^2$

$\ge 1+3(a^2b^2c^2)^{\frac{1}{3}}+3(a^4b^4c^4)^{\frac{1}{3}}+a^2b^2c^2$

$\ge 1+3(2^2)+3(2^4)+(8)^2$

$\ge 1+12+48+64$

$\ge 125=\lambda$ so $\frac{\lambda}{4}=31.25$

4. Ans. 1.25

Sol. $z_1^3-3z_1z_2^2=2$ **....(1)**

$3z_1^2z_2-z_2^3=11$ **....(2)**

Multiply equation (2) by i and add in equation (1)

$z_1^3-3z_1z_2^2+3iz_2z_1^2-iz_2^3=2+11i$

$z_1^3+i^2 3z_1z_2^2+3iz_2z_1^2+(iz_2)^3=2+11i$

$(z_1+iz_2)^3=2+11i$

similarly $(z_1-iz_2)^3=2-11i$

Multiply the two $((z_1+iz_2)(z_1-iz_2))^3=4+121$

$\left(z_1^2+z_2^2\right)^3=125$

$z_1^2+z_2^2=5$

$\left|z_1^2+z_2^2\right|=5$

$\frac{\left|z_1^2+z_2^2\right|}{4}=\frac{5}{4}=1.25$

5. Ans. 1.00

Sol. A tangent to $\frac{x^2}{16}-\frac{y^2}{9}=1$ is

$$y=m_1x\pm\sqrt{16m_1^2-9} \qquad ....(i)$$

The other hyperbola is $\frac{x^2}{-9}-\frac{y^2}{-16}=1$

then tangent to it is

$$y=m_2x\pm\sqrt{-9m_2^2-(-16)} \qquad ....(ii)$$

(i) & (ii) are same.

$\Rightarrow\quad m_1=m_2 \quad \& \quad 16m_1^2-9=-9m_2^2+16$

$\Rightarrow\quad 25m_1^2=25 \quad\Rightarrow\quad m_1^2=1 \quad\Rightarrow\quad m_1=\pm1$

$\therefore\quad m=1$

6. Ans. 1.75

Sol. Let a circle be $x^2+y^2+2gx+2fy+c=0$, which intersects the ellipse at $(a\cos\phi, b\sin\phi)$, then

$$a^2\cos^2\phi+b^2\sin^2\phi+2ga\cos\phi+2fb\sin\phi+c=0$$

$$a^2\left(\frac{1-\tan^2\frac{\phi}{2}}{1+\tan^2\frac{\phi}{2}}\right)^2+b^2\left(\frac{2\tan\frac{\phi}{2}}{1+\tan^2\frac{\phi}{2}}\right)^2+2ga\left(\frac{1-\tan^2\frac{\phi}{0}}{1+\tan^2\frac{\phi}{2}}\right)+2f\,b\left(\frac{2\tan\frac{\phi}{2}}{1+\tan^2\frac{\phi}{2}}\right)+c=0$$

$$\Rightarrow\ a^2\left(1-2\tan^2\frac{\phi}{2}+\tan^4\frac{\phi}{2}\right)+4b^2\tan^2\frac{\phi}{2}+2ga\left(1-\tan^4\frac{\phi}{2}\right)+2fb\left(2\tan\frac{\phi}{2}+2\tan^3\frac{\phi}{2}\right)$$

$$+c\left(1+2\tan^2\frac{\phi}{2}+\tan^4\frac{\phi}{2}\right)=0$$

$$\Rightarrow\ \left(a^2-2ga+c\right)\tan^4\frac{\phi}{2}+4f\,b\tan^3\frac{\phi}{2}+\left(4b^2-2a^2+2c\right)\tan^2\frac{\phi}{2}\ +4f\,b\tan\frac{\phi}{2}+\left(a^2+2ga+c\right)=0 \quad ...(1)$$

clearly $\phi_1,\phi_2,\phi_3,\phi_4$ are the four values of ϕ obtained from (1)

So, $\sum\tan\frac{\phi_1}{2}=-\frac{4fb}{a^2-2ga+c},\sum\tan\frac{\phi_1}{2}\tan\frac{\phi_2}{2}=\frac{4b^2-2a^2+2c}{a^2-2ga+c}$

$\sum\tan\frac{\phi_1}{2}\cdot\tan\frac{\phi_2}{2}\cdot\tan\frac{\phi_3}{2}=-\frac{4fb}{a^2-2ga+c}$

and $\tan\frac{\phi_1}{2}\cdot\tan\frac{\phi_2}{2}\cdot\tan\frac{\phi_3}{2}\cdot\tan\frac{\phi_4}{2}=\frac{a^2+2ga+c}{a^2-2ga+c}$

$\tan\left(\frac{\phi_1}{2}+\frac{\phi_2}{2}+\frac{\phi_3}{2}+\frac{\phi_4}{2}\right)=\frac{s_1-s_3}{1-s_2+s_4}=0 \qquad$ since $s_1=s_3$

$\frac{\phi_1}{2}+\frac{\phi_2}{2}+\frac{\phi_3}{2}+\frac{\phi_4}{2}=n\pi \qquad \therefore\ \phi_1+\phi_2+\phi_3+\phi_4=2n\pi$

$\cos(\phi_1+\phi_2+\phi_3+\phi_4)=1$

SECTION-III

1. Ans. 0

Sol. Equation of normal at $P(3\cos\theta, 2\sin\theta)$ is

$3x\sec\theta-2y\,\text{cosec}\,\theta=5$

Now it is tangent to circle so

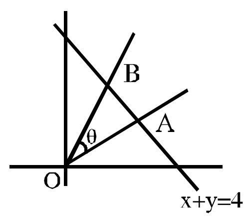
$$\frac{5}{\sqrt{9\sec^2\theta+4\,\text{cosec}^2\theta}}=\sqrt{3}$$

But $\{9\sec^2\theta+4\,\text{cosec}^2\theta\}=(3+2)^2=25$

$\therefore$ No such θ exist.

2. Ans. 1

Sol. Let θ be the angle between OA and OB, then



$$\cos\theta=\frac{OA^2+OB^2-AB^2}{2\cdot OA\cdot OB}$$

Since OA, AB and OB are in A.P.

2AB = OA + OB

$\Rightarrow\ 3AB^2-2OA\cdot OB=OA^2+OB^2-AB^2$

From (i) and (ii) $\cos\theta=\frac{3AB^2}{2OA\cdot OB}-1$

Using $A\cdot M\ge G\cdot M$; $\frac{AB^2}{OA\cdot OB}\ge 1$

$\therefore\ \cos\theta\ge\frac{1}{2}\Rightarrow\theta\le 60^\circ$

$\tan\theta=\left|\frac{m-n}{1+mn}\right|\le\sqrt{3}\Rightarrow|m-n|\le\sqrt{3}|1+mn|, K=\sqrt{3}, [K]=1$

3. Ans. 0

Sol. $\sin x\cdot\sin\left(\frac{1}{x}\right)=1$

$\sin\left(\frac{1}{x}\right)=\text{cosec}\,x$

clearly $\sin\left(\frac{1}{x}\right)=\text{cosec}\,x=1$ or $\sin\left(\frac{1}{x}\right)=\text{cosec}\,x=-1$

which is not possible for same value of x.

4. Ans. 3

Sol. $x\in$ **prime and** $x<10$

$\Rightarrow\ x=2,3,5,7$

Total no. of order pair n(A) = 4 × 10 = 40

and $x^2-3y^2=1$

$\Rightarrow\ x^2=3y^2+1$

For above condition to be satisfied only two such pairs (2,1) and (7,4) are possible

$\Rightarrow P(A)=\frac{2}{40}=\frac{1}{20}=P$ So 60 P = 3

5. Ans. 7

$AB^2=BA \Rightarrow B^2=A^{-1}BA$ squaring both the sides

$B^4=\left(A^{-1}BA\right)\left(A^{-1}BA\right)=A^{-1}B^2A=A^{-1}\left(A^{-1}BA\right)A$

$B^4=\left(A^{-1}\right)^2BA^2$

Again squarring $B^8=\left(\left(A^{-1}\right)^2BA^2\right)\left(\left(A^{-1}\right)^2BA^2\right)$

$\Rightarrow B^8=\left(A^{-1}\right)^2B^2A^2=\left(A^{-1}\right)^2\left(A^{-1}BA\right)A^2=\left(A^{-1}\right)^3BA^3$

Similarly $B^{64}=\left(A^{-1}\right)^6BA^6 \Rightarrow B^{63}=I$

so $m=63=k, \frac{k}{9}=7$

6. Ans. 2

Let $\frac{I}{8}=\int_0^{\pi/2}\frac{x^2(\cos x-\sin x)dx}{(\cos x+\sin x)^3}$

$\Rightarrow \frac{I}{8}=\int_0^{\pi/2}\frac{x^2.\cos 2xdx}{(1+\sin 2x)^2}$

Let 2x = t

$\Rightarrow I=\int_0^{\pi}\frac{t^2\cos tdt}{(1+\sin t)^2}$

By parts

$=\left(\frac{-t^2}{1+\sin t}\right)_0^{\pi}+2\int_0^{\pi}\frac{t}{1+\sin t}$

$I=-\pi^2+2I_1$

where $I_1=\int_0^{\pi}\frac{t}{1+\sin t}$ ....(i)

From king property

$I_1=\int_0^{\pi}\frac{\pi-t}{1+\sin t}$ ....(ii)

(i) + (ii)

$\Rightarrow 2I_1=\pi\int_0^{\pi}\frac{dt}{1+\sin t}$

$\Rightarrow 2I_1=\pi\int_0^{\pi}\left(\sec^2 t-\sec t-\tan t\right)dt$

$\Rightarrow \frac{2I_1}{\pi}=\left(\tan t-\sec t\right)_0^{\pi} \Rightarrow \frac{2I_1}{\pi}=1+1$

$\Rightarrow I_1=\pi \Rightarrow I=\pi(2-\pi)$