ALLEN CAREER INSTITUTE KOTA (RAJASTHAN)

CLASSROOM CONTACT PROGRAMME

JEE(Advanced)
FULL SYLLABUS

SAMPLE PAPER-3

ANSWER KEY — PAPER-1

PART-1 : PHYSICS

SECTION-I	Q.	1	2	3	4	5	6	7	8	9	10
	A.	D	C	B	A	D	A	A,C	A,B,C or B,C	B,C	A,C
	Q.	11	12								
	A.	A,C,D	B,C,D								
SECTION-II	Q.	1	2	3	4	5	6				
	A.	4.80	1.25	0.02	8.47 to 8.49	24.00	0.50				

PART-2 : CHEMISTRY

SECTION-I	Q.	1	2	3	4	5	6	7	8	9	10
	A.	A	D	D	C	D	D	A,C	B,D	A,C,D	A,C,D
	Q.	11	12								
	A.	A,B,C	A,C								
SECTION-II	Q.	1	2	3	4	5	6				
	A.	126.45	5.90 to 5.92	17.40	234.25	10.00	9.00				

PART-3 : MATHEMATICS

SECTION-I	Q.	1	2	3	4	5	6	7	8	9	10
	A.	C	D	D	B	C	C	A,D	A,B,C,D	B,C,D	A,B
	Q.	11	12								
	A.	B,C,D	A,B								
SECTION-II	Q.	1	2	3	4	5	6				
	A.	2.09	2.00	34.00	50.00 or 60.50	22.00	108.00				

SAMPLE PAPER-3

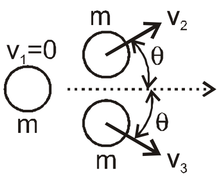
PAPER-1

PART-1 : PHYSICS SOLUTION

SECTION-I

1. Ans. (D)

Sol.



After collision by momentum conservation

Along y-axis

$0 = 0 + mv_2 \sin\theta - mv_3 \sin\theta$

$\Rightarrow v_2 = v_3$

Along x-axis

$mv = 0 + mv_2 \cos\theta + mv_3 \cos\theta$

$mv = 2\,m\,v_2 \cos\theta$

$$v_2 = \frac{v}{2}\frac{1}{\cos\theta}$$

so $v_2 = v_3 > \frac{v}{2}$ $\because \cos\theta < 1$

2 Ans. (C)

Sol. Initially the potential at centre of sphere is

$$V_C = \frac{1}{4\pi\epsilon_0}\frac{Q}{x} + \frac{1}{4\pi\epsilon_0}\frac{2Q}{x} = \frac{1}{4\pi\epsilon_0}\frac{3Q}{x}$$

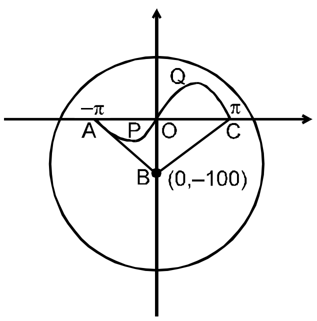
After the sphere is grounded, potential at centre becomes zero. Let the net charge on sphere finally be q.

$$\therefore \frac{1}{4\pi\epsilon_0}\frac{q}{r} + \frac{1}{4\pi\epsilon_0}\frac{3Q}{x} = 0 \text{ or } q = -\frac{3Q}{x}r$$

$\therefore$ The charge flowing out of sphere is $\frac{3Qr}{x}$.

3. Ans. (B)

Sol.



Connect centre B with the two ends points A & C

of the curves, by conducting rods.

$\because$ Electric lines of force will be perpendicular to AB & CB.

$\therefore$ E.M.F. developed in the loop, BAPOQCB

will be = E.M.F. developed in the curve APOQC.

Now Flux in loop ABCQOPA = B

[Area of Δ ABC – Area of loop APOA + Area of loop OCOA] = B [Area of Δ ABC]

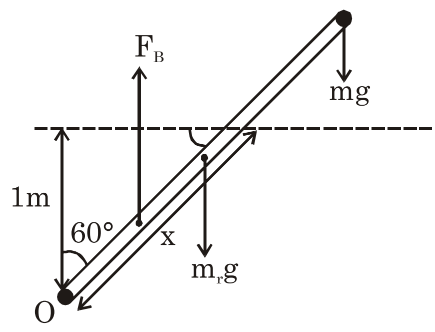
$$= \frac{1}{2} \times (100) \times (2\pi)$$

$= 100\,\pi = 314\ m^2$ $\therefore \phi = 314\ B$

$$\therefore \varepsilon = 314\frac{dB}{dt} = 314\ V.$$

4. Ans. (A)

Sol.



$\tau_{net_0} = 0$

$$m_r g \times \frac{\ell}{2}\sin 60 + \frac{600}{1000} \times g \times \ell \sin 60 - F_B \times \frac{x}{2} = 0$$

$F_B = \rho_L \times (A \times x) \times g$

$m_r = \rho_0^x (A \times \ell)$

By putting the values we get $A = 90\ cm^2$

5. Ans. (D)

Sol. For no ray to emerge out of side PR

$A > 2C \Rightarrow \sin\frac{A}{2} > \sin C \Rightarrow \sin\frac{A}{2} > \frac{\sqrt{3}}{2}$

or $A > 120°$

6. Ans. (A)

Sol. At equilibrium $\frac{P_{radiated}}{A_{sphere}} = \frac{P_{received}}{A_{disc}}$

$$\frac{\sigma A_{sun} T_{sun}^4}{A_{sphere}} = \sigma T_{disc}^4$$

$$T_{disc} = T_{sun} \times \left(\frac{r_{sun}}{r_{sun} + d}\right)^{1/2}$$

$$T_{disc} = 6200 \times \left(\frac{6.8\times10^8}{1.5\times10^{11} + 6.8\times10^8}\right)^{1/2}$$

$T_{disc} = 420.5$ K

7. Ans. (A,C)

Sol. K_α, K_β depends upon energy difference between specific orbit and not on applied potential difference.

8. Ans. (A,B,C OR B,C)

Sol. Here $10 - T_2 = 10a$ ... (i)

$T_2 - T_1 - 0.3\times 2g = 3a$ ... (ii)

$T_1 - 0.3\times 2g = 2a$

Summing up $10g - 0.3\times 4\times g = 15a$

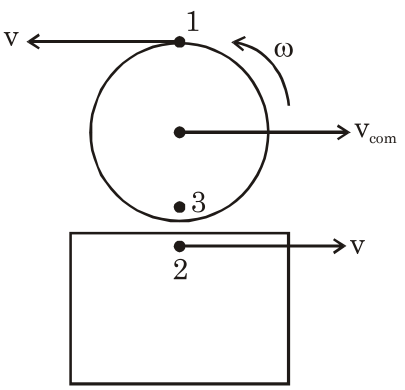
i.e. $a = 5.86$ ms^{-2}

$T_2 = 10\times 9.8 - 10\times 5.86\text{ms}^{-2} = 41.4$ N

$T_1 = 2\times 5.86 + 0.6\times 9.8 = 17.7$ N

9. Ans. (B,C)

Sol.



$v_1 = v_{com} - R_\omega = -v$

$v_2 = v_3 = v_{com} + R_\omega = +v$

$v_{com} = 0$, $\omega = v/R$

10. Ans. (A, C)

Sol. Distance of mean position from water level

= immersed length

= maximum amplitude for equilibrium

$\rho \times 60 \times a \times g = 3\rho Lag$

maximum amplitude = L = immersed length = 20 cm

$$T = 2\pi\sqrt{\frac{m}{3\rho ag}}$$

11. Ans. (A,C,D)

Sol. $\frac{\lambda}{4} = 0.1 \Rightarrow \lambda = 0.4$ m

from graph $\Rightarrow$ T = 0.2 sec. and amplitude of standing wave is 2A = 4 cm.

Equation of the standing wave

$$y(x, t) = -2A\cos\left(\frac{2\pi}{0.4}x\right).\sin\left(\frac{2\pi}{0.2}t\right) \text{ cm}$$

$y\,(x = 0.05,\ \ t = 0.05) = -2\sqrt{2}$ cm

$y(x = 0.04, t = 0.025) = -2\sqrt{2}\ \cos 36°$

speed $= \frac{\lambda}{T} = 2$ m/sec.

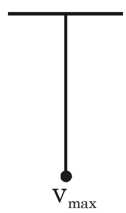
$$V_y = \frac{dy}{dt} = -2A\times\frac{2\pi}{0.2}\cos\left(\frac{2\pi x}{0.4}\right).\cos\left(\frac{2\pi t}{0.2}\right)$$

$V_y = (x = \frac{1}{15}$ m, t = 0.1) = 20 π cm/sec

12. Ans. (B,C,D)

Sol.

$$a_\tau = \frac{F_\tau}{m} = g\,,\ a_c = \frac{v^2}{R} = 0$$



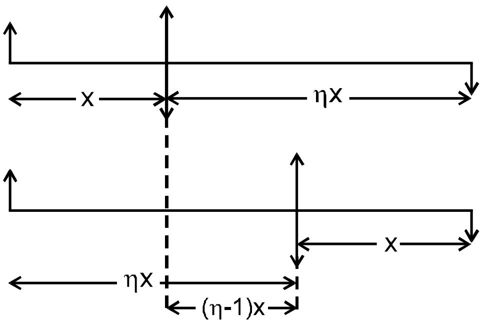
$a_{C_{max}} = \frac{v_{max}^2}{R}$ at lowest point

$$a_t = \frac{F_\tau}{m} = 0$$

SECTION-II

1. Ans. 4.80

Sol.



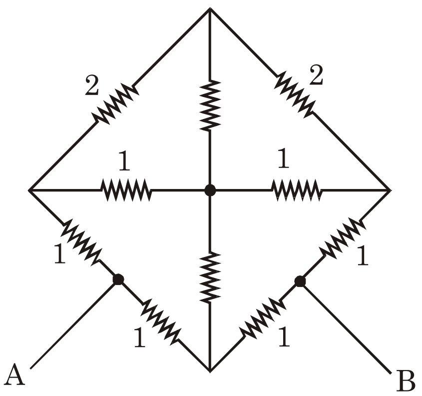
using lens formula :

$$\frac{1}{\eta x}+\frac{1}{x}=\frac{1}{f} \Rightarrow f=\frac{\eta x}{\eta+1}$$

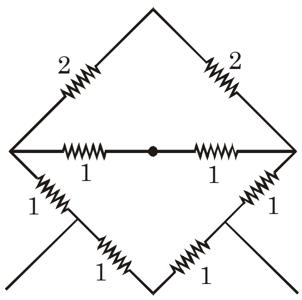
so the given ratio is $\frac{(\eta^2-1)}{\eta}$

2. Ans. 1.25

Sol.



Since it is symmetric about AB



$R_{AB} = 1.25$

3. Ans. 0.02

Sol. Energy stored in capacitor when it is charged upto 2V = $\frac{1}{2}$ 10 2^2 = 20μJ = u_1 (suppose)

Energy stored in capacitor when it is charged upto 4V = $\frac{1}{2}$ 10 4^2 = 80μJ = u_2 (suppose)

Increase in charge = 40 – 20 = 20μC

Energy drawn from cell = 20 x 4 = 80μJ = u (suppose)

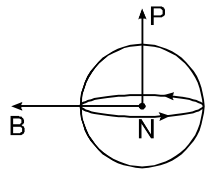
Heat produced = $u_1 + u - u_2$

= 20 + 80 – 80

= 20 μJ

4. Ans. 8.47 to 8.49

Sol.



Torque on the (coil + sphere) due to flow of charge through coil is

= $\left|\vec{p}\times\vec{B}\right|$ (where $\vec{p}$ is the dipole moment of the coil and $\vec{B}$ is the geomagnetic field)

$= i\,N\,\pi\,r^2\;B = I\,\frac{d\omega}{dt}$

$\therefore\; d\omega = \frac{N\,\pi\,r^2\;B}{I}\,i\,dt$

or $\omega = \frac{N\,\pi\,r^2\;B}{\frac{2}{3}m\,r^2}\int_0^{\Delta t} i\,dt = \frac{3\,N\,\pi\,B\,Q}{2}$ **Ans.**

Ans: $\omega = \frac{3}{2}\frac{BN\pi Q}{M}$ **= 2.7 π × 10^{-2} rad/s.**

5. Ans. 24.00

Sol. Shift of fringe pattern $= (\mu - 1)\,\frac{tD}{d}$

$\therefore\; \frac{30\,D\,(4800\times10^{-10})}{d} = (0.6)\,t\,\frac{D}{d}$

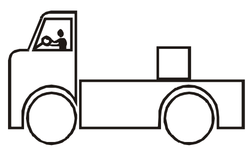
$30 \times 4800 \times 10^{-10} = 0.6$

$t = \frac{30\times4800\times10^{-10}}{0.6}$

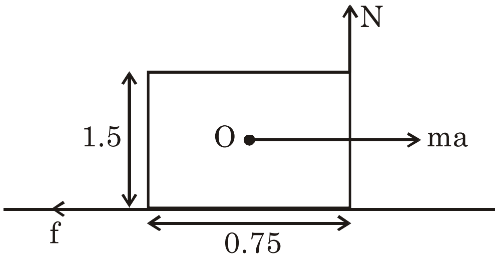
$= \frac{1.44\times10^{-5}}{0.6} = 24\times10^{-6}$

6. Ans. 0.50

Sol.



w.r.t truck F.B.D.



$\tau_0 = 0$

$$f \times \frac{1.5}{2} = N \times \frac{0.75}{2}$$

f = ma, N = mg

$$a = 10 \times \frac{0.75}{1.5} = 5 \text{ m/s}^2$$

$f = \mu N$

$$\mu_{min} = \frac{ma}{mg} = \frac{5}{10} = 0.5$$

PART–2 : CHEMISTRY SOLUTION

SECTION-I

1. Ans.(A)

Sol. BE of reactant $= 6 \times E_{C-H} + 2E_{C-C} + E_{C\equiv C}$

$= 6 \times E_{C-H} + 2 \times 348 + 835$

$= 6 \times E_{C-H} + 1531$

BE of Product = $6E_{C-H} + 3E_{C-C} + E_{C=C}$

$= 6E_{C-H} + 3 \times 348 + 610$

$= 6E_{C-H} + 1654$

$\Delta H = 1531 - 1654 = -123 \text{ kJ}$

2. Ans.(D)

Sol. (i) In a period inert gas has highest 1st ionization energy

(ii) EGE of alkali has negative value

(iii) Metal oxide in higher oxidation state are generally acidic in nature

(iv) In sp^3d^2 hybridization $d_{x^2-y^2}$ and d_{z^2} orbitals are involved

3. Ans.(D)

4. Ans.(C)

Sol. $\frac{-dp}{dt} = KP_{NO}^{a}\ P_{H_2}^{b}$

For 1st experiment, hydrogen pressure is constant $\frac{-dp}{dt} = K^1\ P_{NO}^{a}$

$2.25 \text{ torr/sec} = k^1\alpha(450 \text{ torr})^a$ _____(I)

$2.25 \text{ torr/sec} = k^1\alpha(450 \text{ torr})^a$ _____(II)

On dividing $\frac{2.25}{0.25} = \left(\frac{450}{150}\right)^a = 3^a$

$9 = 3^a \Rightarrow a = 2$

For IInd experiment, NO pressure is constant

$1.90 = k^{ii}(291)^b$ ________(III)

$0.951 = k^{ii}(145)^b$ ________(IV)

On Diving $\frac{1.90}{0.95} = \left(\frac{291}{145}\right)^b$

$2 = 2^b \Rightarrow b = 1$

$$\boxed{\frac{-dp}{dt} = kP_{NO}^{2}\ P_{H_2}}$$

5. Ans.(D)

Sol. Dipole momnet $\mu = e \times d$ and for polyatomic molecules, vector addition is applied.

6. Ans.(D)

Sol. Reaction proceed via SN_1 and E_1 mechanism as carbocation will be the intermediate and so all the products will be feasible.

---

7. Ans.(A, C)

Sol. In lyophilic colloidals dispersion phase shows high affinity with dispersion medium and molecules of DP are large enough to be close to lower limit of colloidal range thus they pass into colloidal state.

8. Ans. (B,D)

9. Ans. (A,C,D)

10. Ans. (A,C,D)

11. Ans. (A,B,C)

12. Ans. (A,C)

Sol. (i) Reaction used to protect certain carbonyl groups

(ii) Reaction is not favourable with monohydric alcohol because ENTROPY decreases.

(iii) In acidic medium acetal is not stable.

SECTION-II

1. Ans.(126.45)

2. Ans.5.90 to 5.92

Sol. In $[Mn(SCN)_6]^{4-}$ Mn^{+2} has d^5 configuration

So that magnetic moment $= \sqrt{5 \times 7}$

$= \sqrt{35}$

$= 5.916$

3. Ans.(17.4)

$$Ph-CH_2-C\equiv C-CH_2-CHO \xrightarrow[\text{(ii)}PBr_3]{\text{(i)}NaNH_2}$$

$$\underset{(P)}{Ph-CH_2-C\equiv C-CH_2-CH_2Br} \xrightarrow{\text{(i)}Mg/ether,\ \text{(ii)}CO_2,\ \text{(iii)}H_3O^+}$$

$$\underset{(Q)}{Ph-CH_2-C\equiv C-CH_2-CH_2-COOH} \xrightarrow{SoCl_2}$$

$$\underset{(R)}{Ph-CH_2-C\equiv C-CH_2-CH_2-COCl} \xrightarrow[Pd/BaSO_4/quinoline]{H_2(g)}$$

$$\underset{(S)}{Ph-CH_2-CH=CH-CH_2-CH_2-CHO}$$

M wt. = 72 + 5 + 14 + 26 + 28 + 29 = 174

M/10 = 17.4

4. Ans.(234.25)

$$A_{2(g)}B_{2(g)} \rightleftharpoons 2AB_{(g)}$$

$$\Delta S^o_{reaction} = [2\times\Delta_f S^o_{AB}] - [\Delta_f S^o_{A_{2(g)}} + \Delta_f S^o_{B_{2(g)}}]$$

$$\Delta S^o = 25\ JK^{-1}\ mole^{-1}\ s$$

$$\Delta H^o_{reaction} = [2\Delta_f H^o_{AB}] - [\Delta_f H^o_{A_2} + \Delta_f H^o_{B_2}]$$

$$\Delta H^o = 270.5\ kJ\ mol^{-1}$$

$$Q_{eq} = \frac{P^2_{AB(g)}}{P_{A_2}\times P_{B_2}}$$

$$Q_{eq} = \frac{(10^{-4})^2}{10^{-2}\times 10^{-1}} = 10^{-5}$$

$$\Delta G = \Delta G^o + RT\ In\,Q$$

$$\Delta G = \Delta H^o - T\Delta S^o + RT\ In\,Q$$

$$\Delta G = 270.5\ kJ - 7.5\ kJ + 5.75\times(-5)$$

$$\Delta G = 234.25\ kJ/mole$$

5. Ans.(10.00)

Sol. $3Ca + N_2 \rightarrow Ca_3N_2$

$$Ca_3N_2 + 6H_2O \rightarrow 3Ca_3(OH)_2 + \underset{(A)}{2NH_3(g)}$$

$$Ca(OH)_2 + CO_2 \rightarrow \underset{(B)}{CaCO_3(s)} + H_2O$$

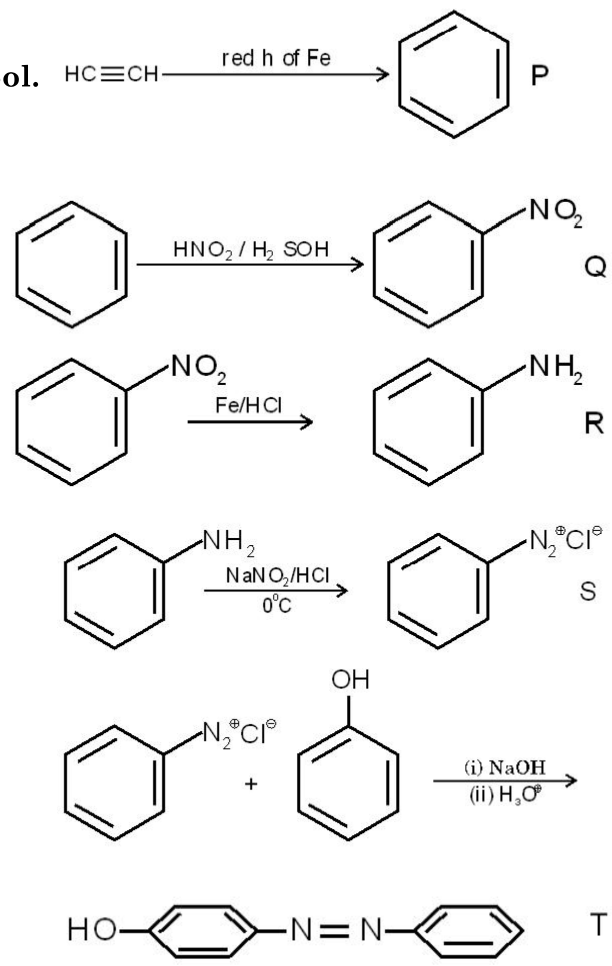
$4gm\ Ca \equiv 0.1 mole\ Ca$

$\equiv 0.1 mole\ CaCO_3$

$\equiv 10\,gm\,CaCO_3$

6. Ans.(9.00)

Sol.



(Dark red)

DU = 9

PART-3 : MATHEMATICS SOLUTION

SECTION-I

1. Ans. (C)

Sol. 2PA = 3PB

$\Rightarrow \quad 2\sqrt{\left(x-9\sqrt{2}\right)^2+\left(y-9\sqrt{2}\right)^2} = 3\sqrt{\left(x-4\sqrt{2}\right)^2+\left(y+4\sqrt{2}\right)^2}$

$\Rightarrow \quad 4\left[x^2+y^2-18\sqrt{2}x-18\sqrt{2}y+324\right] = 9\left[x^2+y^2-8\sqrt{2}x-8\sqrt{2}y+64\right]$

$\Rightarrow \quad 5\left(x^2+y^2\right)=720$

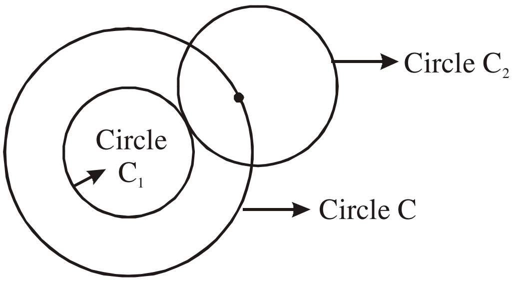
$\Rightarrow \quad x^2+y^2=144$

Let this circle be C_1, then circle C_2 of radius 3 touches C_1 externally.

$\therefore$ locus of centre of circle C_2 will also be a circle 'C' concentric with circle C_1 and of radius = radius of circle C_1 + 3 = 12 + 3

$\therefore$ locus of centre of circle C_2

i.e. equation of circle 'C' is $x^2 + y^2 = 15^2$



2. Ans. (D)

Sol. 2021 = 43 × 47 = abcd

$\therefore$ No.of ways of distributing an object (43 or 47) to 4 distinct places {a,b,c and d} is 4

$\therefore$ No. of positive integral solutions of abcd = 43 × 47 is 4 × 4 = 16

For number of integral solutions

Among a,b,c,d either none or any 2 or all 4 can be negative

$\therefore$ The number of integral solutions of abcd = 2021 = 43 × 47 is

$\left({}^4C_0+{}^4C_2+{}^4C_4\right)\times 16 = 8\times 16 = 128$

3. Ans. (D)

Sol. $x^2 + 20x - 2020 = 0$ has two roots a,b $\in$ R

$x^2 - 20x + 2020 = 0$ has two roots c,d $\in$ complex

$ac\,(a-c) + ad\,(a-d) + bc\,(b-c) + bd\,(b-d)$

$= a^2c - ac^2 + a^2d - ad^2 + b^2c - bc^2 + b^2d - bd^2$

$= a^2\,(c+d) + b^2\,(c+d) - c^2\,(a+b) - d^2\,(a+b)$

$= (c+d)\,(a^2+b^2) - (a+b)\,(c^2+d^2)$

$= (c+d)\,((a+b)^2 - 2ab) - (a+b)\,((c+d)^2 - 2cd)$

$= 20\,[(20)^2 + 4040] + 20\,[(20)^2 - 4040]$

$= 20\,[(20)^2 + 4040 + (20)^2 - 4040]$

$= 20 \times 800 = 16000$

4. Ans. (B)

Sol. $\dfrac{x^2-x+2}{(x+1)(x-1)^3}=\dfrac{(t+1)^2-(t+1)+2}{(t+1+1)t^3}$ $\{\text{Substituting } x-1=t\}$

$$\begin{array}{r|l} & 1+\frac{t^2}{2} \\ 2+t & 2+t+t^2 \\ & 2+t \\ \hline & \text{x} \quad \text{x} \quad t^2 \\ & t^2+\frac{t^3}{2} \\ \hline & \text{x}-\frac{t^3}{2} \end{array}$$

$=\dfrac{2+t+t^2}{(2+t)t^3}$

$=\dfrac{(2+t)\left(1+\frac{t^2}{2}\right)-\frac{t^3}{2}}{(2+t)t^3}=\dfrac{1}{t^3}+\dfrac{1}{2t}-\dfrac{1}{2(2+t)}=\dfrac{1}{(x-1)^3}+\dfrac{1}{2(x-1)}-\dfrac{1}{2(x+1)}$

$\therefore \quad \displaystyle\int \frac{x^2-x+2}{(x+1)(x-1)^3}dx=\int \frac{-1}{2(x+1)}+\frac{1}{2(x-1)}+\frac{1}{(x-1)^3}dx$

$=-\dfrac{1}{2}\ln|x+1|+\dfrac{1}{2}\ln|x-1|+\left(-\dfrac{1}{2}\right)\dfrac{1}{(x-1)^2}+E$

$\therefore \quad A=-\dfrac{1}{2}, B=\dfrac{1}{2}, C=0 \text{ and } D=-\dfrac{1}{2}$

$\therefore \quad A + D = -1 = C - 2B$

5. Ans. (C)

Sol. $\displaystyle\lim_{x\to 0}\left(\frac{f(1+x)}{f(1)}\right)^{\frac{1}{x}}=e^{\lim_{x\to 0}\left\{\frac{f(1+x)}{f(1)}-1\right\}\cdot\frac{1}{x}}=e^{\lim_{x\to 0}\frac{f(1+x)-f(1)}{x}\cdot\frac{1}{f(1)}}$

Applying L Hospital's rule

$=e^{\lim_{x\to 0}\frac{f'(1)}{f(1)}}=e^{\frac{2\sqrt{2}}{\sqrt{2}}}=e^2$

6. Ans. (C)

Sol. $\dfrac{2\sin\theta}{\cos 3\theta}=\dfrac{2\sin\theta\cos\theta}{\cos\theta\cos 3\theta}=\dfrac{\sin(3\theta-\theta)}{\cos\theta\cos 3\theta}=\tan 3\theta-\tan\theta$

similarly

$\dfrac{2\sin 3\theta}{\cos 9\theta}=\tan 9\theta-\tan 3\theta$

$\dfrac{2\sin 9\theta}{\cos 27\theta}=\tan 27\theta-\tan 9\theta$

$\therefore \quad f(\theta)=(\tan 3\theta-\tan\theta)+(\tan 9\theta-\tan 3\theta)+(\tan 27\theta-\tan 9\theta)+\tan\theta$

$\Rightarrow \quad f(\theta)=\tan 27\theta$

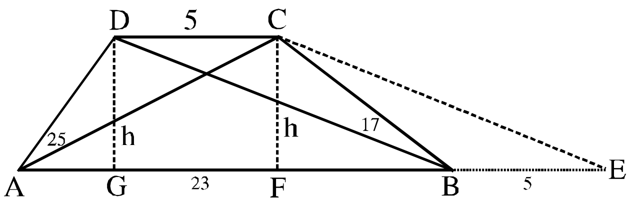
$\Rightarrow \quad \theta \in \left[0, \frac{\pi}{108}\right]$

$\Rightarrow \quad 27\theta \in \left[0, \frac{\pi}{4}\right]$

$\therefore \quad f(\theta) = \tan 27\theta \in [0,1]$

7. Ans. (A,D)

Sol.



In ΔACE

$a = AE = 23 + 5 = 28$

$b = AC = 25$

$c = CE = 17$

$\therefore \; s = \frac{1}{2}(a+b+c) = \frac{1}{2}(28+25+17) = 35$

$\Delta = \frac{1}{2} AE \cdot h = \sqrt{35(35-28)(35-25)(35-17)}$

$\Rightarrow \frac{1}{2} \cdot 28.h = \sqrt{5 \cdot 7 \cdot 7 \cdot 5 \cdot 2 \cdot 2 \cdot 3 \cdot 3}$

$\Rightarrow 14h = 5 \cdot 7 \cdot 2 \cdot 3 \qquad \therefore \; h = 15$

$\therefore$ In ΔCEF: $EF = \sqrt{17^2 - 15^2} = 8$

$\therefore \; BF = 8 - 5 = 3$

$\therefore \; |BC| = \sqrt{h^2 + 3^2}$

$= \sqrt{225 + 9} = \sqrt{234}$

$= 3\sqrt{26}$

From ΔACF

$AF = \sqrt{25^2 - h^2}$

$= \sqrt{25^2 - 15^2} = 20$

$\therefore \; AG = AF - GF = 20 - 5 = 15$

$\therefore \; AD = \sqrt{h^2 + AG^2} = \sqrt{15^2 + 15^2}$

$= 15\sqrt{2}$

8. Ans. (A,B,C,D)

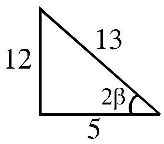
Sol. Let $\alpha = \frac{1}{2}\cos^{-1}\frac{3}{5}$ and $\beta = \frac{1}{2}\tan^{-1}\frac{12}{5}$

then $\cos 2\alpha = \frac{3}{5}$ & $\tan 2\beta = \frac{12}{5}$

$\left\{0 < \alpha, \beta < \frac{\pi}{2}\right\}$

$\therefore \tan\alpha = \sqrt{\frac{1-\cos 2\alpha}{1+\cos 2\alpha}} = \sqrt{\frac{1-\frac{3}{5}}{1+\frac{3}{5}}} = \sqrt{\frac{1}{4}} = \frac{1}{2}$

$\therefore \alpha = \tan^{-1}\left(\frac{1}{2}\right)$

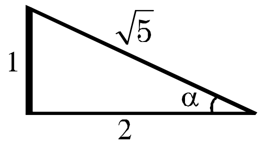
Also 

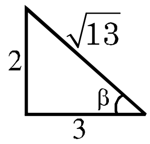
$\Rightarrow \cos 2\beta = \frac{5}{13}$

$\therefore \tan\beta = \sqrt{\frac{1-\cos 2\beta}{1+\cos 2\beta}} = \sqrt{\frac{1-\frac{5}{13}}{1+\frac{5}{13}}} = \sqrt{\frac{8}{18}} = \frac{2}{3}$

$\therefore \beta = \tan^{-1}\left(\frac{2}{3}\right)$

$\therefore \alpha + \beta = \tan^{-1}\left(\frac{1}{2}\right) + \tan^{-1}\left(\frac{2}{3}\right)$

Also 



$\Rightarrow \alpha = \sin^{-1}\frac{1}{\sqrt{5}} = \cos^{-1}\frac{2}{\sqrt{5}}$ and $\beta = \sin^{-1}\frac{2}{\sqrt{13}} = \cos^{-1}\frac{3}{\sqrt{13}}$

$\therefore \alpha + \beta = \sin^{-1}\frac{1}{\sqrt{5}} + \cos^{-1}\frac{3}{\sqrt{13}}$

$= \sin^{-1}\frac{2}{\sqrt{13}} + \text{cso}^{-1}\frac{2}{\sqrt{5}}$

Also $\tan^{-1}\frac{1}{4}+\tan^{-1}\frac{2}{9}+\cot^{-1}\left(\frac{3}{2}\right)=\tan^{-1}\left(\frac{\frac{1}{4}+\frac{2}{9}}{1-\frac{1}{4}\cdot\frac{2}{9}}\right)+\tan^{-1}\frac{2}{3}$

$=\tan^{-1}\left(\frac{17/36}{34/36}\right)+\tan^{-1}\left(\frac{2}{3}\right)$

$=\tan^{-1}\left(\frac{1}{2}\right)+\tan^{-1}\left(\frac{2}{3}\right)=\alpha+\beta$

9. Ans. (B,C,D)

Sol. Given parabola $y=x^2+ax+1$

$\Rightarrow y=\left(x+\frac{a}{2}\right)^2+1-\frac{a^2}{4}$

As no any point of the parabola lies below x axis, $1-\frac{a^2}{4}\geq 0$

$\therefore \frac{a^2}{4}\leq 1$

$\therefore\ a\in[-2,2]$

$\therefore$ No. of integral values of a = 5

The tangent to $y=x^2+ax+1$ at (0,1) is $T=0$

$\Rightarrow \frac{y+1}{2}=x\cdot 0+a\left(\frac{x+0}{2}\right)+1$

$\Rightarrow\ y+1=ax+2\Rightarrow y=ax+1$

This is tangent to $x^2+y^2=r^2$ also

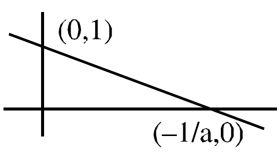
$\therefore\ 1^2=r^2\left(a^2+1\right)\Rightarrow r^2=\frac{1}{a^2+1}$

$\therefore$ for maximum value of a, $r^2=\frac{1}{4+1}$

$\therefore r=\frac{1}{\sqrt{5}}$

for maximum value of radius r, a^2 must be minimum i.e. $a^2=0$, for this
a = slope of tangent = 0

Also, area enclosed by the tangent y = ax + 1 with coordinate axes $=\frac{1}{2}\cdot 1\cdot\left|-\frac{1}{a}\right|=\frac{1}{2|a|}$

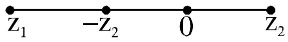


$\therefore$ Minimum area (when $|a| = 2$)

$= \dfrac{1}{2 \times 2} = \dfrac{1}{4}$

10. Ans. (A,B)

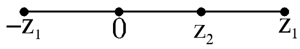
Sol. (1) $|z_1| + |z_2| = 4 \Rightarrow |z_1| + |-z_2| = |z_1 + (-z_2)|$



$\therefore$ $z_1 \& (-z_2)$ are collinear with origin such that $z_1 \& z_2$ both lie towards the same side of origin.

$\therefore$ z_1, z_2 and origin will also be collinear such that z_1 and z_2 will lie opposite to the side of origin $\therefore \arg\left(\dfrac{z_1}{z_2}\right) = \pi$

(2) $|z_1| + |z_2| = |z_1 + z_2|$

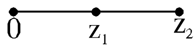


$\therefore$ z_1 & z_2 are collinear with origin such that z_1 & z_2 both lie towards the same side of origin

$\therefore \arg\left(\dfrac{z_1}{z_2}\right) = 0$

(3) $|z_1| - |z_2| = -4 \Rightarrow |z_1| - |z_2| = -|z_1 - z_2|$

$\therefore$ $|z_1| + |z_2 - z_1| = |z_2|$

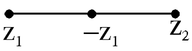


$\therefore$ z_1, z_2 and origin will

such that z_1 and z_2 will be towards the same side of origin $\therefore \arg\left(\dfrac{z_1}{z_2}\right) = 0$

(4) $|z_2| - |z_1| = |z_1 + z_2|$

$\Rightarrow$ $|z_1 + z_2| + |z_1| = |z_2|$

$\Rightarrow$ 

$\therefore z_1, z_2$ and origin will be collinear such that z_1 and z_2 will lie opposite to the side of origin $\therefore$

$\arg\left(\dfrac{z_1}{z_2}\right) = \pi$

11. Ans. (B,C,D)

Sol. Let E = Easy, D= Difficult, T = True / false type & M = MCQ

Total no. of questions = 300 + 200 + 500 + 400 = 1400

$P(E) = \dfrac{300 + 500}{1400} = \dfrac{8}{14} = \dfrac{4}{7}$ $\qquad$ $P(T) = \dfrac{300 + 200}{1400} = \dfrac{5}{14}$

$P(D)=\frac{200+400}{1400}=\frac{6}{14}=\frac{3}{7}$ $\qquad$ $P(M)=\frac{500+400}{1400}=\frac{9}{14}$

$P(E\cap T)=\frac{300}{1400}=\frac{3}{14},P(D\cap T)=\frac{200}{1400}=\frac{1}{7}$

$P(E\cap M)=\frac{500}{1400}=\frac{5}{14},P(D\cap M)=\frac{400}{1400}=\frac{2}{7}$

$\therefore\ P(E/M)=\frac{P(E\cap M)}{P(M)}=\frac{5/14}{9/14}=\frac{5}{9}$

$P(M/E)=\frac{P(E\cap M)}{P(E)}=\frac{5/14}{4/7}=\frac{5}{8}$

$P(D/T)=P\frac{(D\cap T)}{P(T)}=\frac{1/7}{5/14}=\frac{2}{5}$

$P(E\cap T)=\frac{300}{1400}=\frac{3}{14}$

12. Ans. (A,B)

Sol. (1) $\text{adj}(\text{adj}\,A)=|A|^{n-2}A$ (common property)

(2) $\text{adj}(AB)^T=\text{adj}(B^TA^T)=\text{adj}\,A^T\cdot\text{adj}\,B^T=(\text{adj}\,A)^T\cdot(\text{adj}\,B)^T$

(3) $|\text{adj}(\text{adj}\,A)|=\left||A|^{n-2}A\right|$

$=\left(|A|^{n-2}\right)^n|A|=|A|^{(n-1)^2}$

(4) Let $C=A^2B^2-B^2A^2$

Then $C^T=(A^2B^2-B^2A^2)^T=(A^2B^2)^T-(B^2A^2)^T$

$=(B^2)^T\cdot(A^2)^T-(A^2)^T(B^2)^T$

$=(B^T)^2(A^T)^2-(A^T)^2(B^T)^2$

$=B^2(-A)^2-(-A)^2(B)^2$

$=B^2A^2-A^2B^2$

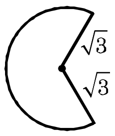
$=-C$

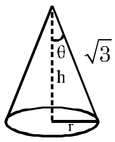
$\therefore$ C is skew symmetric matrix of odd order 'n'

$\therefore$ C is non-invertible.

SECTION-II

1. Ans. 2.09

Sol. 



$r = \sqrt{3}\sin\theta$

$h = \sqrt{3}\cos\theta$

$v = \frac{1}{3}\pi r^2 h = \frac{\pi}{3}\left(\sqrt{3}\sin\theta\right)^2 \cdot \sqrt{3}\cos\theta$

$v = \pi\sqrt{3}\sin^2\theta\cos\theta$ **...(1)**

Applying AM – GM inequality in $\frac{\sin^2\theta}{2}, \frac{\sin^2\theta}{2}$ & $\cos^2\theta,$ we have

$$\frac{2\left(\frac{\sin^2\theta}{2}\right)+\cos^2\theta}{3} \geq \sqrt[3]{\left(\frac{\sin^2\theta}{2}\right)^2 \cos^2\theta}$$

$\therefore\ \sin^2\theta \cdot \cos\theta \leq \sqrt{\frac{4}{27}} = \frac{2}{3\sqrt{3}}$

$\therefore\ V_{max} = \pi\sqrt{3}\sin^2\theta\cos\theta\,|_{max} = \pi\sqrt{3} \cdot \frac{2}{3\sqrt{3}}$

$= \frac{2\pi}{3} \approx 2.09$

2. Ans. 2.00

Sol. $\frac{x\ell n\, x + \ell n\, x}{x-1} > k \qquad \forall x > 1$

Let $f(x) = \left(\frac{x+1}{(x-1)}\ell n\, x\right)$

Then $f'(x) = \left(\frac{x+1}{(x-1)}\right)\cdot\frac{1}{x} + \ell n\, x \cdot \frac{(x-1)\cdot 1 - (x+1)\cdot 1}{(x-1)^2}$

$= \frac{(x+1)}{x(x-1)} - \frac{2\,\ell n\, x}{(x-1)^2}$

$= \frac{x^2 - 1 - 2x\,\ell n\, x}{x(x-1)^2} = \frac{g(x)}{x(x-1)^2}$

$\therefore g(x) = x^2 - 1 - 2x\,\ell n\, x$

$\therefore g'(x) = 2x - 0 - 2x \cdot \frac{1}{x} - 2 \cdot \ell nx \cdot 1 = 2\left[(x-1) - \ell n x\right]$

$\therefore\ g''(x) = 2\left[1 - \frac{1}{x}\right] = 2\left[\frac{x-1}{x}\right] > 0 \forall x > 1$

$\therefore g'(x)$ is strictly increasing $\forall x > 1$

$\therefore g'(x) > g'(1) \forall x > 1 \Rightarrow g'(x) > 0$

$\therefore$ g(x) is strictly increasing $\forall x > 1$

$\therefore g(x) > g(1) \forall x > 1$

$\therefore g(x) > 0 \forall x > 1$

$\therefore\ f'(x) = \frac{g(x)}{x(x-1)^2} > 0\ \forall x > 1$

$\therefore\ f(x)$ is strictly increasing $\forall x > 1$

$\therefore\ f(x) > f(1) \forall x > 1$

$\therefore$ Minimum value of f(x)

$= \lim_{x \to 1^+} f(x) = \lim_{x \to 1^+} \frac{(x+1)\ell n x}{(x-1)}$

$= \lim_{y \to 0^+} (2+y)\frac{\ell n(1+y)}{y} = 2$

$\therefore$ Minimum value of $f(x) = 2$

$\therefore\ f(x) > 2\ \ \forall x > 1$

$\therefore\ K_{max} = 2$

3. Ans. 34.00

Sol. $f'(x) = \frac{2x+1}{(x-1)^2 + 1} > 0 \forall x \in \left(\frac{-1}{2}, 1\right]$ & $f'(x) < 0 \forall x \in \left[-1, -\frac{1}{2}\right)$

$\therefore f(x)$ strictly increases in $\left(\frac{-1}{2}, 1\right]$ & f (x) strictly decreases in $\left[-1, -\frac{1}{2}\right)$

$\therefore\ f(x)_{min} = f\left(-\frac{1}{2}\right)$ and $f(x)$ will be maximum either at $x = 1$ or at $x = -1$

$\therefore\ f(x)=\int_0^x \frac{2(t-1)+3}{t^2-2t+2}dt$

$=\left[\ell n\left(t^2-2t+2\right)+3\tan^{-1}(t-1)\right]_0^x$

$=\ell n\left(x^2-2x+2\right)+3\tan^1(x-1)-\ell n2+\frac{3\pi}{4}$

$\therefore\ f\left(-\frac{1}{2}\right)=\ell n\left(\frac{13}{4}\right)+3\tan^{-1}\left(-\frac{3}{2}\right)-\ell n2+\frac{3\pi}{4}$

$=\ell n\left(\frac{13}{8}\right)+3\left(\tan^{-1}1-\tan^{-1}\frac{3}{2}\right)$

$=\ell n\left(\frac{13}{8}\right)-3\tan^{-1}\left(\frac{1}{5}\right)$

$\therefore\ f(1)=0+0+\frac{3\pi}{4}-\ell n\,2\ \&\ f(-1)=\ell n5-3\tan^{-1}2-\ell n2+\frac{3\pi}{4}$

$=\left(\frac{3\pi}{4}-\ell n2\right)-\left(3\tan^{-1}2-\ell n5\right)$

$=f(1)-(\text{a}+\text{ve quantity})$

$\therefore f(-1)<f(1)$

$\therefore\ f(x)$ maxima $=f(1)=\frac{3\pi}{4}-\ell n\,2$

$\therefore$ Range of $f(x)=\left[\ell n\left(\frac{13}{8}\right)-3\tan^{-1}\frac{1}{5},\frac{3\pi}{4}-\ell n2\right]$

$a=\ell n\frac{13}{8}-3\tan^{-1}\frac{1}{5}\ \&\ b=\frac{3\pi}{4}-\ell n\,2$

$\Rightarrow a+b=\ell n\frac{13}{8}-\ell n\,2+3\left(\tan^{-1}1-\tan^{-1}\frac{1}{5}\right)$

$=\ell n\frac{13}{16}+3\tan^{-1}\left(\frac{1-\frac{1}{5}}{1+\frac{1}{5}}\right)$

$=\ell n\frac{13}{16}+3\tan^{-1}\left(\frac{2}{3}\right)$

$=\ell n\left(\frac{m}{n}\right)+3\tan^{-1}\left(\frac{p}{q}\right)$

$\therefore\ m = 13 \qquad n = 16$

$P = 2 \qquad q = 3$

$\therefore\ m + n + p + q = 34$

4. Ans. 50.00 or 60.50

Sol. $(x^2 - 11)(y+1) = -4 \Rightarrow (x^2 - 11)\cdot(y+1) = (-2)\cdot 2$

$\therefore\ x^2 - 11 = -2, y + 1 = 1 \Rightarrow P(x_0, y_0) \equiv (3,1)$

It can easily be proved that no any other such point P may exist.

$\therefore\ y = \frac{-4}{x^2 - 11} - 1 \qquad \therefore\ \frac{dy}{dx} = \frac{4\cdot 2x}{(x^2 - 11)^2} - 0$

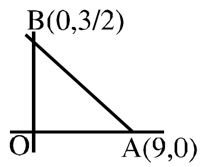
At $P(x_0, y_0) \equiv P(3,1)$

The slope of tangent, $m = \frac{4\cdot 2\cdot 3}{(3^2 - 11)^2} = 6$

$\therefore$ the slope of normal $= -\frac{1}{m} = -\frac{1}{6}$

$\therefore$ Equation of normal at P (3,1)

$y - 1 = -\frac{1}{6}(x - 3) \Rightarrow 6y - 6 + x - 3 = 0$



$\Rightarrow x + 6y = 9$

$\Rightarrow \frac{x}{9} + \frac{y}{3/2} = 1$

$\therefore$ Area of $\Delta OAB = \frac{1}{2}\cdot 9\cdot\frac{3}{2} = \frac{27}{4} = 6\cdot 75$

or Area of $\Delta OAB = \int_0^9 y\,dx = \int_0^9 \left(\frac{9 - x}{6}\right)dx$

$= \frac{1}{6}\left[\frac{(9-x)^2}{-2}\right]_0^9 = \frac{1}{6}\left(0 - \frac{-81}{2}\right)$

$= \frac{81}{12} = \frac{27}{4} = 6.75$

5. Ans. 22.00

Sol. $\frac{dy}{dx} + \frac{6y}{x} = 3x^2y^2$

$$\Rightarrow \frac{-1}{y^2}\frac{dy}{dx}+\left(\frac{-6}{x}\right)\cdot\frac{1}{y}=-3x^2$$

$$\frac{1}{y}=t \;\therefore \frac{-1}{y^2}\frac{dy}{dx}=\frac{dt}{dx}$$

$$\Rightarrow \frac{dt}{dx}-\frac{6}{x}\cdot t=-3x^2$$

$$\text{I.F.} = e^{\int -\frac{6}{x}dx}=\frac{1}{x^6}$$

$\therefore$ The solution of differential equation :

$$t\cdot\frac{1}{x^6}=\int -3x^2\cdot\frac{1}{x^6}dx$$

$$\Rightarrow \frac{1}{x^6y}=\frac{-3}{-3x^3}+C$$

Putting $x=1, y=\frac{1}{3}$

$\Rightarrow 3=1+C \quad \therefore\ C = 2$

$$\Rightarrow \frac{1}{x^6y}=\frac{1}{x^3}+2$$

$$\Rightarrow y=\frac{\left(\frac{1}{x^6}\right)}{\frac{1}{x^3}+2}=\frac{1}{x^3\left(1+2x^3\right)}$$

$$=\frac{1}{x^m\left(1+n\cdot x^P\right)} \Rightarrow 3=m, n=2, p=3$$

$\therefore\ m^2+n^2+p^2=22$

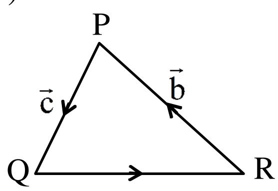
6. Ans. (108.00)

Sol. We have $\vec{a}+\vec{b}+\vec{c}=\vec{0}$

$\Rightarrow \vec{c}=-\vec{a}-\vec{b}$

Now, $\frac{\vec{a}.(-\vec{a}-2\vec{b})}{(-\vec{a}-\vec{b}).(\vec{a}-\vec{b})}=\frac{3}{7}$

$\Rightarrow \frac{9+2\vec{a}.\vec{b}}{9-16}=\frac{3}{7}$



$\Rightarrow \vec{a}.\vec{b}=-6$

$\Rightarrow |\vec{a}\times\vec{b}|^2=a^2b^2-(\vec{a}.\vec{b})^2$ = 9 × 16 – 36 = 108