(1001CJA102120060)

Test Pattern

ALLEN CAREER INSTITUTE KOTA (RAJASTHAN) path to success

CLASSROOM CONTACT PROGRAMME

JEE(Advanced)
FULL SYLLABUS

SAMPLE PAPER-2

ANSWER KEY PAPER-1

PART-1 : PHYSICS

SECTION-I	Q.	1	2	3	4	5	6	7	8	9	10
	A.	D	B	D	C	A	A,C	A,C	A,C	A,B,C,D	A,C
	Q.	11	12								
	A.	A,C	A,C								
SECTION-II	Q.	1	2	3	4	5	6				
	A.	12.50	2.00	3.00	0.25	2.46 to 2.48	2.40 to 2.60				

PART-2 : CHEMISTRY

SECTION-I	Q.	1	2	3	4	5	6	7	8	9	10
	A.	D	D	D	B	A,B,C	A,B,C	B,D	A,C,D	A,D	A,B,C
	Q.	11	12								
	A.	A,D	A,B,D								
SECTION-II	Q.	1	2	3	4	5	6				
	A.	51.20	3.00	4.00	7.95 - 7.96	7.00	723.00				

PART-3 : MATHEMATICS

SECTION-I	Q.	1	2	3	4	5	6	7	8	9	10
	A.	B	B	D	C	A,B	A,B,D	A,B,D	A,C	B,C	B,C,D
	Q.	11	12								
	A.	A,B,C	A,C								
SECTION-II	Q.	1	2	3	4	5	6				
	A.	4.00	7.00	4.00	9.00	1.00	3.00				

SAMPLE PAPER-2

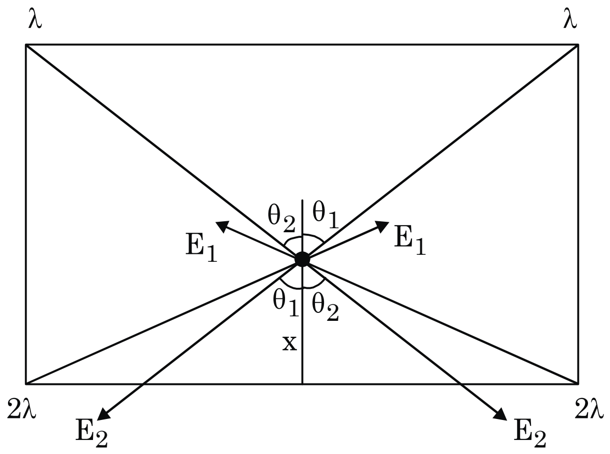
PAPER-1

PART-1 : PHYSICS SOLUTION

SECTION-I

1. Ans. (D)

Sol. Let P be the point where E_{net} due to wires = 0



(E_{net}) at P = $2E_1 \cos\theta_1 - E_2\cos\theta_2 = 0$

$$E_p = \frac{4K\lambda x}{(x^2+1)} - \frac{2K\lambda(1-x)}{(1-x)^2+1} = 0$$

$3x^3 + 5x - 5x^2 - 1 = 0$

$\Rightarrow$ x = 0.255 (only one real]

2. Ans. (B)

Sol. There will be change in flux at different points of disc and hence eddy current will flow. According to lenz law, disc will move in such a way that it oppose the motion of point charge.

3. Ans. (D)

4. Ans. (C)

Sol. At C : Let speed = V_1

$$mg\cos\theta = \frac{mV_1^2}{R} \quad ....(1)$$

$$mg[R - R\cos\theta] = \frac{1}{2}mV_1^2 - \frac{1}{2}mU^2 ....(2)$$

From (1) & (2), we get

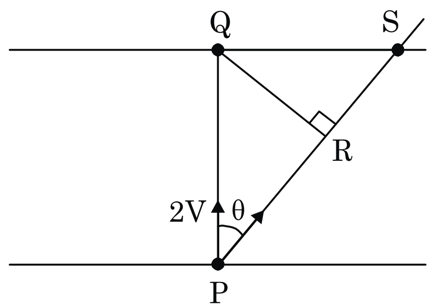
$$U = \sqrt{\frac{gR}{2}}$$

Let the speed at B = V

$$mgR = \frac{1}{2}mV^2 - \frac{1}{2}mU^2 \Rightarrow V = \sqrt{\frac{5gR}{2}}$$

5. Ans. (A)

Sol. $\tan\theta = \frac{V}{2V} = \frac{1}{2}$



At point R F_{app} = F

At P: $(F_{app})_{max} = \left(\frac{V_0}{V_0 - 2V}\right)F$

At S: $(F_{app})_{min} = \left(\frac{V_0}{V_0 + V}\right)F$

6. Ans. (A,C)

Sol. $k_s\ell = \frac{kq^2}{\ell^2}$ ...(i)

$$k_s(\ell + x) - \frac{kq^2}{(\ell+x)^2} = Ma$$

$$a = \left(\frac{3k_s}{M}\right)x \quad \text{[From (i)]}$$

$$T = 2\pi\sqrt{\frac{M}{3k_s}} = \frac{2\pi}{3}\sqrt{\frac{3M}{k_s}}$$

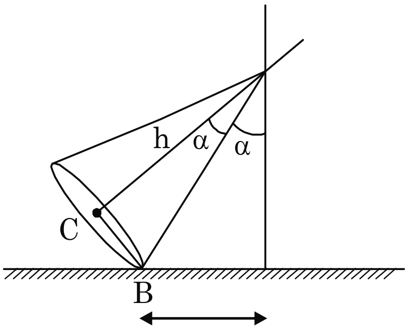
$\Delta U_{spring} = K_s\ell x$

$\Delta U_{electrostatic} = \frac{2Kq^2}{\ell^3}x$

7. Ans. (A,C)

Sol. $\omega_1 h \tan\alpha = \omega_2 R$

$\Rightarrow \dfrac{\omega_1}{\omega_2} = \dfrac{R/h}{\tan\alpha} = 1$



h sec α sin α = h tan α

8. Ans. (A,C)

Sol. The Ampere forces on the sides OP and O'P are directed along the same line, in opposite direction and have equal values, hence the net force as well as the net torque of these forces about the axis OO' is zero. The Ampere's force on the segment PP' and the corresponding moment of this force about the axis OO' is effective and is deflecting in nature. In equilibrium (in the dotted position), the deflecting torque must be equal to the restoring torque developed due to the weight of the shape. If the length of each side is ℓ, then

$$i\ell B(l\cos\theta) = (S\ell\rho)g\frac{\ell}{2}\sin\theta + (S\ell\rho)g\frac{\ell}{2}\sin\theta + (S\ell\rho)g\ell\sin\theta$$

So, $i\ell^2 B\cos\theta = 2S\rho g\ell^2\sin\theta$

Hence, $B = \dfrac{2S\rho g}{i}\tan\theta$

9. Ans. (A,B,C,D)

Sol. $P = -\beta\dfrac{\partial s}{\partial x} \Rightarrow \rho \propto P$

10. Ans. (A,C)

Sol. $\sqrt{2MK_p} = \sqrt{6KM}$ ....(i)

$K_p = K + E$ ....(ii)

From (i) & (ii)

$2K_p = 3E$

$\sqrt{4MK_d} = \sqrt{6MK_1}$

$K_d = K_1 + E$

$4K_d = 6(K_d - E) = 6E = 2K_d$

$K_d = 3E = 2K_p$

11. Ans. (A,C)

Sol. E_{net} between the plates

$= \dfrac{V}{d} = \dfrac{200}{0.01} = 2\times10^4\,V/m$

When terminal velocity is achieved

$qE_{net} = mg$

$n \times 1.6 \times 10^{-19} \times 2 \times 10^4$

$= \dfrac{4\pi}{3}(8 \times 10^{-7})^3 \times 900 \times 10$

$n \simeq 6$

12. Ans. (A,C)

Sol. $dR_3 = R_3\,\alpha dT$

$\Delta R^3 = \dfrac{R_3}{2}KT^2 = 12$

$R_3' = 3/2$

I_1 in 60 Ω and 312 Ω = $\dfrac{50}{372}$

I_2 in 100 Ω and 500 Ω = $\dfrac{50}{600}$

$V_C - V_D = 312\,I_1 - 500\,I_2$

$= 41.94 - 41.67 = 0.27\,V$

SECTION-II

1. Ans. 12.50

Sol. For I_{max} in inductor, $\dfrac{dI}{dt} = 0$

Hence q = CE

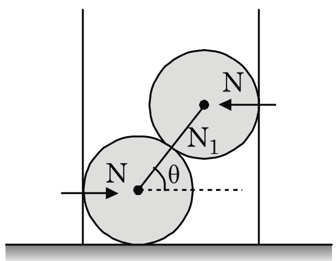
$W_{battery} = \dfrac{q^2}{2C} + \dfrac{1}{2}LI^2$

Energy stored inside inductor

$\dfrac{1}{2}LI^2 = \dfrac{1}{2}CE^2 = 12.50\,J$

2. Ans. 2.00

Sol.



$N_1 \cos\theta = N$ ....(i)

$N_1 \sin\theta = mg$ ....(ii)

From (i) and (ii)

$N \tan\theta = mg$

For toppling to start $\rightarrow 2Nr \sin\theta = MgR$

$$\Rightarrow \frac{m}{M} = \frac{R}{2r\cos\theta} = \frac{R}{2(R-r)} = 2$$

3. Ans. 3.00

Sol. Air resistance is neglected and the balls are considered as perfectly elastic. If the balls are dropped from height h, they reach the ground with speed $v = \sqrt{2gh}$. The bottom ball first hits the ground, and then collides with the top ball, which receives the largest possible energy if the lower ball is at rest after the two collisions.

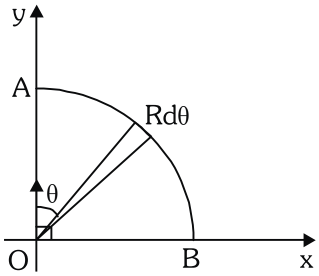
The bottom ball rebounds with speed v and collides with the top ball moving downwards at speed –v. Since the speed of the ball of mass m_2 is to be zero after the collision, the equations expressing the conservation of momentum and energy are

$$(m_2 - m_1)v = m_1 u \quad \text{and} \quad (m_1 + m_2)\frac{v^2}{2} = m_1\frac{u^2}{2}$$

The speed u of the top ball after the collision and the ratio of the masses can be calculated from these equations, giving u = 2v and $m_1/m_2 = 1/3$.

4. Ans. 0.25

Sol. $$dV = \int_0^{\pi/2} (rd\theta)(r\sin\theta)\omega\, 2\left(\frac{\mu_0}{4\pi}\right)\frac{m\cos\theta}{r^3}$$



$$\Delta V_{AB} = \frac{\mu_0 m\omega}{4\pi r}$$

5. Ans. 2.46 to 2.48

Sol. $ma\cos\theta + f = mg\sin\theta$

$ma\cos\theta + \mu[mg\cos\theta + ma\sin\theta] = mg\sin\theta$

$$a = g\left[\frac{\sin\theta - \mu\cos\theta}{\cos\theta + \mu\sin\theta}\right] = \frac{10\left[\frac{1}{2} - \frac{1}{2\sqrt{3}}\left(\frac{\sqrt{3}}{2}\right)\right]}{\left[\frac{\sqrt{3}}{2} + \frac{1}{4\sqrt{3}}\right]} = \frac{10}{4\left(\frac{6+1}{4\sqrt{3}}\right)}$$

$$= \frac{10\sqrt{3}}{7} = \frac{17.32}{7} = 2.47$$

6. Ans. 2.40 to 2.60

Sol. Bead will move towards left relatively.

$$a_{rel} = \frac{GM\ell}{r_0^3} \text{ and } x_0 = \frac{1}{2}a_{rel}t^2$$

$$\int_0^{\ell} \frac{GMm\,dx}{(r_0 + x)\ell} = ma$$

$$\frac{GM}{\ell}\left(\frac{1}{r_0+\ell} - \frac{1}{r_0}\right) = a$$

$$\frac{GM}{\ell}\frac{\ell}{(r_0)(r_0+\ell)} - \frac{GM}{(r_0+x)^2} = a_{rel}$$

$$\frac{GM}{r_0^2}\left[\left(1 - \frac{\ell}{r_0}\right) - \left(1 - \frac{2x}{r_0}\right)\right] = a_{rel}$$

$$\frac{GM\ell}{r_0^3} = a_{rel}$$

r_0 is very large and hence r_0 can taken constant.

PART–2 : CHEMISTRY SOLUTION

SECTION-I

1. Ans. (D)

Sol. $\Lambda_m^0(CH_3COOH) = \lambda_m^0(H^+) + \lambda_m^0(CH_3COO^-)$

$= 3.474 \times 10^{-2} + 1.351 + 10^{-2}$

$= 4.825 \times 10^{-2}\ ohm^{-1}m^2mol^{-1}$

$k = G \times G^* = \frac{1}{2000} \times 20 = 0.01\,ohm^{-1}m^{-1}$

$\Lambda_m = \frac{k}{C} = \frac{0.01ohm^{-1}m^{-1}}{0.1 \times 10^3 molm^{-3}}$

$= 1 \times 10^{-4}\ ohm^{-1}m^2mol^{-1}$

Now, $\alpha = \frac{\Lambda_m}{\Lambda_m^0} = \frac{1 \times 10^{-4}}{4.825 \times 10^{-2}}$

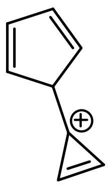
$\therefore\ [H^+] = C\alpha = 0.1 \times \frac{10^{-2}}{4.825} = 2.07 \times 10^{-4} M$

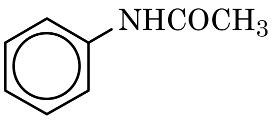
2. Ans. (D)

Sol. $\ell = 0$ to $(n - 1)$

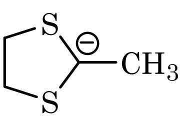
Each subshell has maximum electrons $2(2\ell + 1)$.

3. Ans. (D)

Sol.  Now both rings are aromatic, hence polarity is stablized.



Ph—$\bar{N}$—C(=O)—CH_3 is stabilized by additional resonance with –C(=O)–group .



this is more stable as ⊖: shows back bonding with sulphur.

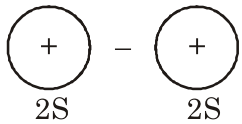
4. Ans. (B)

Sol. On increasing the temperature, due to increase in effective collision, rate of reaction increase and it is found that for most of the reaction, 10°C increase in temperature causes increase of rate by 2 to 3 times. It is true that collision frequency (A) also increases but it does not have major contribution for increased reaction. Major contribution comes from $(e^{-E_a/RT})$ factor due to increased temperature.

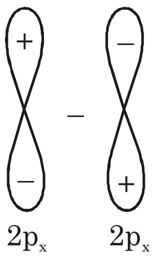
5. Ans. (A,B,C)

Sol. (A) $\sigma 2p_z \rightarrow$ Ungerade & LUMO of C_2

(B) $d\pi$–$d\pi$(ABMO) $\rightarrow$ Gerade, π–bond is formed

(C) 

$\rightarrow$ Ungerade & LUMO of Li_2

(D) 

$\rightarrow$ Ungerade & HOMO of B_2

6. Ans. (A,B,C)

7. Ans. (B,D)

Sol. (A) As per lock & key model, cavities on enzyme & structure of reactants are complimentary in shapes.

(B) Enzyme catalyst function only at optimum temperature.

(C) Vitamins are co–enzymes & Metal ions activations.

(D) Enzyme catalysed reactions are two step $\therefore$ Not elementary.

8. Ans. (A,C,D)

Sol. $Li[CuMe_2]$: Ox.state of Cu = +1

$\overset{+1}{K_3}[Co(\overset{-1}{CN})_5(\overset{-1}{H})]$: Ox. state of Co = +3

$In[Co(en)_3]Cl_3$, en is ethane-1,2-diamine not diammine.

9. Ans. (A,D)

Sol. D–mannose & D–galactose do not form same osazone & they have different configuration at C–2 & C–3.

10. Ans. (A,B,C)

Sol. Theory based.

11. Ans. (A,D)

Sol. Highly electropositive metals are extracted by electrolysis of their fused salts.

12. Ans. (A,B,D)

Sol. Theory based.

SECTION-II

1. Ans.(51.20)

Sol. Reaction Quotient $Q_P = \frac{\left(P_{NH_3}\right)^2}{\left(P_{H_2}\right)^3\left(P_{N_2}\right)}$ at 1 hour

$$Q_P = \frac{(200)^2}{(50)(2)^3} = 100$$

$$\Delta G = \Delta G^\circ + RT\ell nQ_P$$

$$\Rightarrow \Delta G = -(15.53\times 2) + \frac{\frac{25}{3}\times 300}{1000}\{\ell n100\}$$

$$\Delta G = -19.56\,kJ/mol$$

$$\Delta G = \Delta H - T\Delta S$$

$$-19.56 \times 1000 = (-4.2 \times 1000) - (300)\Delta S$$

$$\Delta S = 51.2\ JK^{-1}.$$

2. Ans. (3.00)

Sol. $NaBr + AgNO_3(aq.) \longrightarrow AgBr\downarrow + NaNO_3$

$NaBr + H_2SO_4 \longrightarrow Na_2SO_4 + Br_2\uparrow + H_2O$

$NaNO_2 + AgNO_3 \longrightarrow AgNO_2\downarrow + NaNO_3$

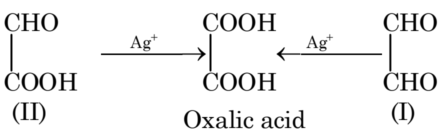
$NaNO_2 + conc.H_2SO_4 \longrightarrow NO_2\uparrow + SO_2\uparrow$

$NaI + AgNO_3 \longrightarrow AgI\downarrow + NaNO_3$

$NaI + conc.H_2SO_4 \longrightarrow I_2 + SO_2$

3. Ans. (4.00)

Sol. (i) Since (B) on oxidation with Tollen's reagent, gives oxalic acid indicates that (B) has either of the following structure (I) and (II):

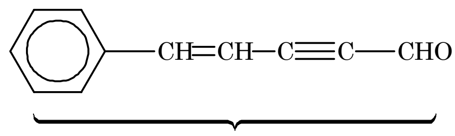


Structure of (B)

(ii) Since compound (A) also undergoes reduction with Lindlar's catalyst indicating the presence of at least one C≡C bond in (A).

(iii) (A) on reductive ozonolysis produces one mole of benzaldehyde and 2 moles of (B).

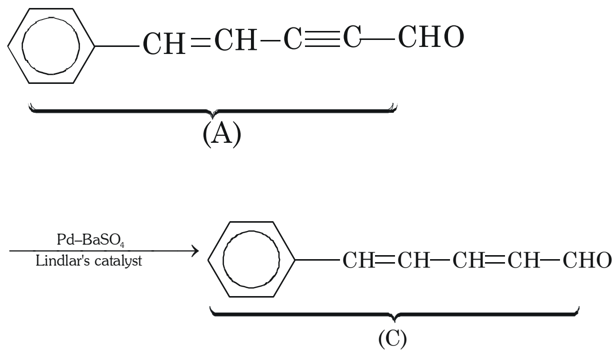
Hence probable structure of (B) is (II) and that for (A) is



Structure of (A)

(iv) As (A) does not undergo self aldol condensation, so structure of (A) is further verified.

(v)



So, total number of geometrical isomers of (C) are 4.

4. Ans. (7.95 – 7.96)

Sol. $\Delta x \times m\Delta V \geq \frac{h}{4\pi}$

$$\Rightarrow \lambda \times m\Delta V \geq \frac{h}{4\pi}$$

$$\Rightarrow \frac{h}{mv} \times m\Delta v \geq \frac{h}{4\pi}$$

$$\Rightarrow \frac{\Delta v}{v} \geq \frac{1}{4\pi}$$

$$\Rightarrow \frac{\Delta v}{v} \times 100 \geq \frac{1}{4\pi} \times 100 \approx 8$$

5. Ans.(7.00)

Sol. (i), (ii), (iii), (iv), (v), (viii), (ix) produce H_2 gas.

6. Ans.(723.00)

Sol. 7 stereoisomers are possible and 4 are optically active.

PART-3 : MATHEMATICS SOLUTION

SECTION - I

1. Ans. (B)

Sol. $a^3c + ac^3 + b^3c + bc^3 + a^3d + ad^3 + b^3d + bd^3$

$= a^3(c+d) + b^3(c+d) + c^3(a+b) + d^3(a+b)$

$= (c + d)(a^3 + b^3) + (a + b)(c^3 + d^3)$

$a + b = -5$ $\quad$ $c + d = +5$

$ab = -100$ $\quad$ $cd = 100$

$a^3 + b^3 = (a + b)(a^2 + b^2 - ab)$

$c^3 + d^3 = (c + d)(c^2 + d^2 - cd)$

$= (-5)(25 + 300)$

$= (5) \times (25 - 300)$

$= -5 \times 325$

$= 5 \times (-275)$

$= -1625$

$= -1375$

$= 5 \times (-1625) + (-5) \times (-1375)$

$= -8125 + 6875 = -1250$

2. Ans. (B)

Sol. $f(x) = -\sqrt{x^2}\sin x + x\sqrt{x^2}$

$f(x) = x|x| - |x| \sin x$

$f(-\infty) = \lim_{x\to-\infty} x^2\left(1-\frac{\sin x}{x}\right) = -\infty$

$f(\infty) = \lim_{x\to\infty} x^2\left(1-\frac{\sin x}{x}\right) = \infty$

Range of f(x) = R

$$f(x) = \begin{cases} -x^2 + x\sin x & x < 0 \\ x^2 - \sin x & x \ge 0 \end{cases}$$

$$f'(x)\begin{cases} -2x + \sin x + x\cos x & ; \quad x < 0 \\ 2x - \sin x - x\cos x & ; \quad x \ge 0 \end{cases}$$

$f'(x) \ge 0 \ \forall \ x \ \infty(-\infty, \infty)$

at $x = 0$ $f'(x) = 0$

f(x) is one-one and onto.

3. Ans. (D)

Sol. $A = \int_0^{\ln\sqrt{2}} (g(x) - f(x))dx$

$= \int_0^{\ln\sqrt{2}} \left(-\frac{2}{3}e^x + \frac{4}{3}e^{-x}\right)dx = 2 - \frac{4\sqrt{2}}{3}$

4. Ans. (C)

Sol. $P(a\cos\theta, b\sin\theta)$

$Q(0, b\,\mathrm{cosec}\theta)$

Equation of A′P is

$y = \frac{b\sin\theta}{a\cos\theta + a}(x + a)$

Put $x = 0$ to get $M\left(0, \frac{b\sin\theta}{\cos\theta + 1}\right)$

$OQ^2 - MQ^2$

$= b^2\,\mathrm{cosec}^2\theta - \left(b\,\mathrm{cosec}\theta - \frac{b\sin\theta}{\cos\theta + 1}\right)^2$

$= \frac{2b^2}{\cos\theta + 1} - \frac{b^2\sin^2\theta}{(\cos\theta + 1)} = b^2$

5. Ans. (A,B)

Sol. p(r) = Probability of success

$= {}^nC_r\left(\frac{1}{5}\right)^r\left(\frac{4}{5}\right)^{n-r}$

$1 - p(0) > \frac{3}{4}$

$1 - \left(\frac{4}{5}\right)^n > \frac{3}{4}$

$\left(\frac{4}{5}\right)^n < \frac{1}{4}$

$\left(\frac{4}{5}\right)^6 > \frac{1}{4}$ and $\left(\frac{4}{5}\right)^7 < \frac{1}{4}$

Least value is 7

6. Ans. (A,B,D)

Sol. $f(x) = \left(\sin x - \frac{1}{2}\right)^2 + \frac{7}{4}$

For min. $\sin x = \frac{1}{2}$.

$x = \frac{\pi}{6}, \frac{5\pi}{6}$

N = 1

7. Ans. (A,B,D)

Sol. Partially differentiating w.r.t. x taking y as a constant

$f'(x+y) = f'(x) + e^x (e^y - 1)$

$x = 0$

$f'(y) = f'(0) + e^y - 1$

$f'(y) = e^y + 1$

Integrating both sides

$f(y) = e^y + y + c$

putting $y = 0,\ 0 = 1 + 0 + c \Rightarrow c = -1$

$\therefore$ $f(x) = e^x + x - 1$

Now, verify the options.

8. Ans. (A,C)

Sol. $(\text{adj}\ (\text{adj}\ A)) = |A|^{n-2} A$

$= |A|\ A$

$\det(\text{adj}\ (\text{adj}\ A)) = |A|^3\ |A|$

$23^4 = |A|^4 \qquad \Rightarrow |A| = 23$

$\therefore \quad |A| = 3u + 11$

$\Rightarrow \quad 3u + 11 = 23 \qquad \therefore u = 4$

9. Ans. (B,C)

Sol. $|Z| = 1 \Rightarrow z = \cos\theta + i\sin\theta$

$|z^2 + z - 1|^2 = (\cos 2\theta + \cos\theta - 1)^2 + (\sin 2\theta + \sin\theta)^2$

$= 3 + 2\cos\theta - 2\cos 2\theta - 2\cos\theta$

$= 3 - 2\cos 2\theta$

$\Rightarrow$ $|z^2 + z - 1|$ will be maximum if $\cos 2\theta = -1$ and minimum when $\cos 2\theta = 1$

10. Ans. (B,C,D)

Sol. Add both equations

$\tan X + 2\tan Y + \tan Z = 0$

$\tan X\ \tan Y \tan Z + \tan Y = 0$

$\tan Y (\tan X \tan Z + 1) = 0$

$\tan Y \neq 0 \qquad \tan X\ \tan Z = -1$

$\tan X - \tan Z = \frac{-4}{\sqrt{3}}$

$$\tan X + \frac{1}{\tan X} = \frac{-4}{\sqrt{3}}$$

on solving $\tan X = -\sqrt{3}$ or $\tan X = \frac{-1}{\sqrt{3}}$

If $\tan X = -\sqrt{3}$; $\tan Y = \frac{1}{\sqrt{3}}$; $\tan Z = \frac{1}{\sqrt{3}}$

If $\tan X = \frac{-1}{\sqrt{3}}$; $\tan Y = \frac{-1}{\sqrt{3}}$ (not possible)

11. Ans. (A,B,C)

Sol. Required point is (3, 2, –1)

12. Ans. (A,C)

Sol. Use series expansion.

SECTION - II

1. Ans. (4.00)

Sol. $P = \log_4 p_1 + \log_4 p_2 + \log_4 p_3 + \log_4 p_4$

$= \log_4 (p_1 p_2 p_3 p_4)$

Also $p_1 + p_2 + p_3 + p_4 \geq 4(p_1 p_2 p_3 p_4)^{1/4}$

$\therefore \quad P \leq 4 \qquad \Rightarrow N = 4$

$\therefore \quad \log_2 N^2 = 4$

2. Ans. (7.00)

Sol. $\frac{1}{x_{n+1}} = \frac{x_n^2 + x_n + 1}{x_n^2 + x_n}$

$= 1 + \frac{1}{x_n} - \frac{1}{x_n + 1}$

$N = 2014$

3. Ans. (4.00)

Sol. $\frac{1}{\sin^2\theta} - \frac{1}{\cos^2\theta} - \frac{1}{\tan^2\theta} - \frac{1}{\cot^2\theta} - \frac{1}{\sec^2\theta} - \frac{1}{\text{cosec}^2\theta} = -3$

$$\frac{(\cos^2\theta - \sin^2\theta) - (\sin^4\theta + \cos^4\theta)}{\sin^2\theta\cos^2\theta} = -2$$

$\cos 2\theta - 1 + 2\sin^2\theta\cos^2\theta = -2\sin^2\theta\cos^2\theta$

$\cos^2 2\theta - \cos 2\theta = 0$

$\Rightarrow \quad \cos 2\theta = 0$ or $\cos 2\theta = 1$

$\Rightarrow 2\theta = (2n+1)\frac{\pi}{2}$ or $2\theta = 2n\pi$ (not possible)

$\theta = (2n+1)\frac{\pi}{4}$

4. Ans. (9.00)

Sol. Assume O to be origin and let circumradius be R and

$$\overrightarrow{OA}=\vec{a},\ \overrightarrow{OB}=\vec{b},\ \overrightarrow{OC}=\vec{c}$$

$$|\overrightarrow{AB}|^2=|\vec{a}-\vec{b}|^2=2R^2-2\vec{a}\cdot\vec{b}$$

$$|\overrightarrow{AC}|^2=|\vec{a}-\vec{c}|^2=2R^2-2\vec{a}\cdot\vec{c}$$

$$\overrightarrow{AO}\cdot\overrightarrow{BC}=-a(\vec{c}-\vec{b})=\vec{a}\cdot\vec{b}-\vec{a}\cdot\vec{c}$$

$$=\left(R^2-\frac{1}{2}|AB|^2\right)-\left(R^2-\frac{1}{2}|\overrightarrow{AC}|^2\right)$$

$$=\frac{1}{2}\left(|\overrightarrow{AC}|^2-|\overrightarrow{AB}|^2\right)$$

$$\Rightarrow\quad 2\cdot\overrightarrow{AO}\cdot\overrightarrow{BC}=|\overrightarrow{AC}|^2-|\overrightarrow{AB}|^2=9$$

5. Ans. (1.00)

Sol. Let $\pi x-\frac{\pi}{4}=\theta\in\left[\frac{-\pi}{4},\frac{7\pi}{4}\right]$

So, $\left(3-\sin\left(\frac{\pi}{2}+2\theta\right)\right)\sin\theta\geq\sin(\pi+3\theta)$

$\Rightarrow$ $(3-\cos2\theta)\sin\theta\geq-\sin3\theta$

$\Rightarrow$ $\sin\theta[3-4\sin2\theta+3-\cos2\theta]\geq0$

$\Rightarrow$ $\sin\theta(6-2(1-\cos2\theta)-\cos2\theta)\geq0$

$\Rightarrow$ $\sin\theta(4+\cos2\theta)\geq0$

$\Rightarrow$ $\sin\theta\geq0$

$\Rightarrow$ $\theta\in[0,\pi]\Rightarrow 0\leq\pi x-\frac{\pi}{4}\leq\pi$

$\Rightarrow$ $x\in\left[\frac{1}{4},\frac{5}{4}\right]$

$\Rightarrow$ $\beta-\alpha=1$

6. Ans. (3.00)

Sol. $$\lim_{x\to0}\frac{2\sin\frac{3x}{2}\cdot\cos\left(\frac{3x-2a}{2}\right)-3\left(2\sin\frac{x}{2}\cdot\cos\frac{3x+2a}{2}\right)}{x^3}$$

$$=\lim_{x\to0}2\cos\left(\frac{3x+2a}{2}\right)\left(\frac{-4\left(\frac{x}{2}\right)^3}{x^3}\right)$$

$=-\cos a$