ALLEN CAREER INSTITUTE KOTA (RAJASTHAN) path to success

CLASSROOM CONTACT PROGRAMME

JEE(Advanced)
FULL SYLLABUS

SAMPLE PAPER-1

ANSWER KEY PAPER-2

PART-1 : PHYSICS

SECTION-I	Q.	1	2	3	4	5	6	7	8	9	10
	A.	A,D	B,C	A,B,D	A,B,C	A,D	A,D	B	A	A	B
SECTION-II	Q.	1	2	3	4	5	6	7	8		
	A.	0.25	12.00	6.00	1.33	2.00	6.66 to 6.67	3.00	17.00		

PART-2 : CHEMISTRY

SECTION-I	Q.	1	2	3	4	5	6	7	8	9	10
	A.	B,C	B,C	B	A,C	A,B,D	A,B,C,D	B	C	B	C
SECTION-II	Q.	1	2	3	4	5	6	7	8		
	A.	5.66	6.00	9.10 TO 9.40	36.00	6.00	0.18 TO 0.20	4.00	5.00		

PART-3 : MATHEMATICS

SECTION-I	Q.	1	2	3	4	5	6	7	8	9	10
	A.	A,C	A,C,D	B,C	B,D	A,C	B,C	A	C	A	A
SECTION-II	Q.	1	2	3	4	5	6	7	8		
	A.	1.00	4.00	1.00	6.00	8.00	10.00	3.00	10.00		

SAMPLE PAPER-1

PAPER-2

PART-1 : PHYSICS — SOLUTION

SECTION-I

1. Ans. (A,D)

Sol. $\pi r^2\sqrt{2gy} = -\pi x^2 \dfrac{dy}{dx}$

$r^2\sqrt{2gy}\,dt = -\dfrac{y^{2/a}}{k^{2/a}}dy$

$$\boxed{k^{2/a}r^2\sqrt{2g}\int_0^t dt = +\int_4^{16} y^{\frac{2}{a}-\frac{1}{2}}dy}$$

(A) k = 1, a = 4

$\sqrt{5}\times10^{-4}\sqrt{20}\,t = 12$

t = 12000 sec

(B) k = 1, a = 2

$r^2\sqrt{2g}\,t = t\int_4^{16} y^{\frac{1}{2}}\,dy$

$r^2\sqrt{2g}\,t = \dfrac{2^4\left(16^{3/2}-4^{3/2}\right)}{3}$

$\sqrt{5}\times10^{-4}\sqrt{20}\,t = \dfrac{2}{3}(56)$

$t = \dfrac{2(56)\times10^4}{3\sqrt{5}\sqrt{20}}$

$= \dfrac{2}{3}(56)\times1000$

(C) $(4)\sqrt{5}\times10^{-4}\sqrt{20}t = \dfrac{2}{3}(56)$

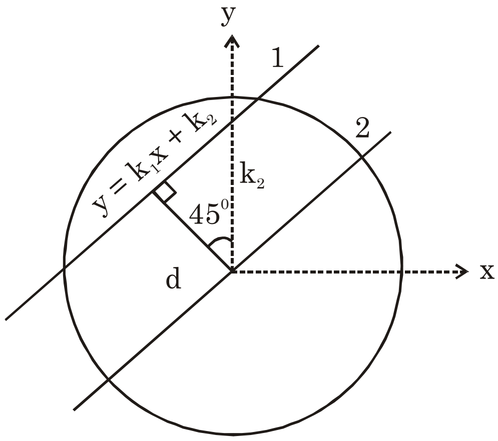
$t = \dfrac{2(56)\times1000}{3\times4}$

(D) $\sqrt{4}\sqrt{5}\times10^{-4}\sqrt{20}\,t = \dfrac{2}{3}(56)$

t = 6000 sec

2. Ans. (B,C)

Sol.



$I_1 = I_2 + md^2$

$I_1 = \dfrac{MR^2}{2} + M\left(\dfrac{K_2}{\sqrt{2}}\right)^2$

(A) $I_1 = \dfrac{MR^2}{2} + \dfrac{MR^2}{4}$

$= \dfrac{3MR^2}{4}$

(B) $I_1 = \dfrac{MR^2}{2} + \dfrac{MR^2}{2} = MR^2$

(C) $I_1 = \dfrac{MR^2}{2} + \dfrac{MR^2}{2} = MR^2$

(D) $I_1 = \dfrac{MR^2}{2} + \dfrac{MR^2}{4} = \dfrac{3MR^2}{4}$

3. Ans. (A,B,D)

Sol. Power leaving the system = P

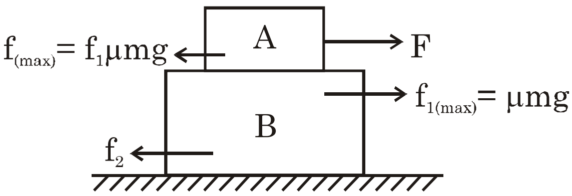
$P = \sigma(4\pi C^2)T_C^4$

Net power leaving B = P

$P = \sigma(4\pi b^2)T_B^4 - \sigma(4\pi b^2)T_C^4$

4. Ans. (A,B,C)

Sol.



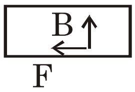
$f_{(max)} = 3\mu mg$

Block 'B' always remains at rest as net force on B is zero.

$F_{GROUND} = \sqrt{f^2 + \mu}$

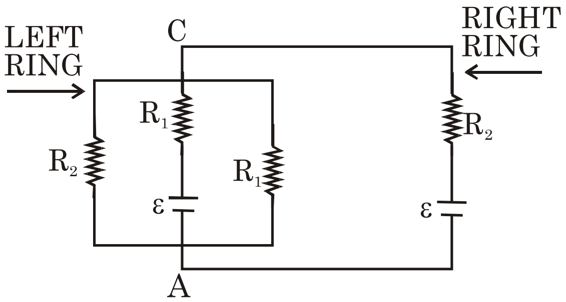
$= \sqrt{(\mu mg)^2 + (3mg)^2}$

$= \boxed{mg\sqrt{9+\mu}}$



5. Ans. (A,D)

Sol. Simplified diagram may be



So there will be current in each ring and so magnetic force on them will be nonzero.

6. Ans. (A,D)

Sol. $30 = \frac{\varepsilon_1}{20}(4)$

$\varepsilon_1 = 150$

$\varepsilon_2 = \frac{\varepsilon_1}{20} \times 6 = 45V$

7. Ans. (B)

Sol. I $P \propto V$

w^+

$\frac{I}{V} \propto V$

$T \propto V^2$

U = +ve

Q = 4

II. w^+

$\Delta U = 0$

$\theta = +$

III $\Delta U = 0$

w^-

$\theta = -$

IV w^-

$U = -\ V \propto T$

$\theta = -w$

I A,C,D

II A,B,D

III B,E

IV B,E

8. Ans. (A)

Sol. (A) $\frac{1}{2}(P_0 + 2P_0)4V_0 = 6P_0V_0$

(B) AB $P \propto V$

$T \propto V^2$

$\frac{T_A}{T_B} = \left(\frac{4V_0}{8V_0}\right)^2$

$T_B = 400$ K.

ΔUAB = nCUDT

$-\frac{3}{25}\frac{3}{2}\left(\frac{25}{3}\right) \times 300 = 450J$

(C) WCYCLIZ = _______

$-(4P_0)7V_0 + \frac{1}{2}(R_0 + 4R_0)3V_0 + P_0V_0$

$-\left(28 + \frac{15}{2} + 1\right)P_0V_0 = \frac{-39}{2}P_0V_0$

(D) UCD = $U_{CD} = nCv\Delta T = \frac{3}{25}\frac{3}{2}(-700)$

$T_B = 400$ L

$T_C = 800$ K

$T_B = 100$ K

CD = $V \propto T$

9. Ans. (A)

Sol. $F = \frac{mv^2}{r} = \frac{ke^2}{r^4}$ .... (1) $v^2 \propto \frac{1}{r^3}$

$mvr = \frac{nh}{2\pi}$ .... (2) $vr \propto n$

$v^2 \propto \frac{v^3}{n^3}$ $\boxed{v \propto n^3}$

$r \propto n^{-2}$

(A) $I = \frac{dq}{dt} = \frac{e}{T} = \frac{e}{2\pi r}v$

$I \propto \frac{v}{r} \propto \frac{n^3}{n^{-2}}$

(B) $B = \frac{\mu_0 I}{2\pi r} \propto \frac{n^5}{n^{-2}}$

(C) $L = mvr \propto n$

(D) $\mu = NIA \propto n^5 n^{-4}$

$\mu \propto n$

10. Ans. (B)

Sol. $U = U_0 \ln\left(\frac{r}{r_0}\right)$

$F = \frac{mv^2}{r} = \frac{U_0 r_0}{r}\frac{1}{r_0} = \frac{U_0}{r}$

$v \propto r^0$

$mvr = \frac{nh}{2\pi}$

$vr \propto n$

$r \propto n$

$v \propto n^0$

(A) $I \propto \frac{v}{r} = n^{-1}$

(B) $B \propto \frac{I}{r} \propto \frac{v}{r^2} \propto n^{-2}$

(C) $L = mvr \propto n$

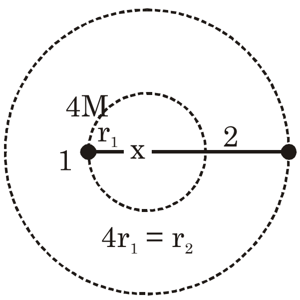
(D) $\mu = NIA \propto \frac{v}{r}r^2 \propto vr$

$\mu \propto n$

SECTION-II

1. Ans. 0.25

Sol.

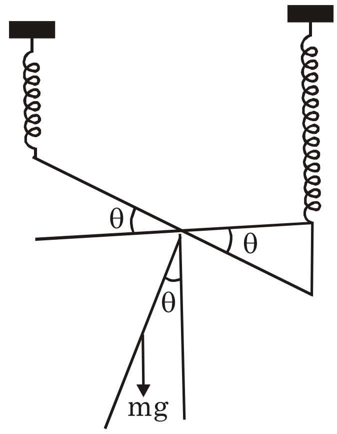


$\frac{K.E_{4m}}{K.E_m} = \frac{\frac{1}{2}4mv_1^2}{mv_2^2}$

$= \frac{4r_1^2w^2}{r_2^2w^2} = \frac{4r_1^2}{r^2} = \frac{1}{4}$

2. Ans. 12.00

Sol. Let the body be displaced through an angle θ about its mean position.



Net torque on the body at this position about C.

$\tau = -\left(2Ka\theta a\cos\theta + mga\sin\theta\right)$

$= -\left(2Ka^2 + mga\right)\theta$

(where θ is small, sin θ = θ and cos θ =1)

Angular acceleration

$\alpha = \frac{\tau}{I} = -\frac{-\left(2Ka^2 + mga\right)\theta}{\frac{m\times(2a)^2}{12} + \frac{m\times(2a)^2}{3}}$

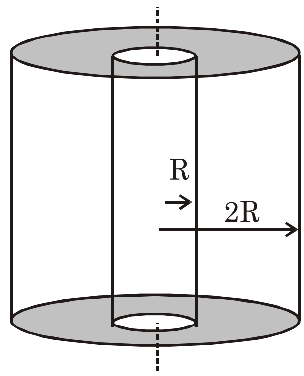
or $\alpha = -\frac{-\left(2Ka^2 + mga\right)}{\frac{5ma^2}{3}}$

$\Rightarrow \omega = \sqrt{\frac{\left(2Ka^2 + mga\right)}{5ma^2/3}} = \sqrt{\left(\frac{6K}{5m} + \frac{3g}{5a}\right)}$

$= \sqrt{\frac{6}{5}\times\frac{96}{6} + \frac{30}{5\times5}\times 104} = 12 \text{ rad/s}$

3. Ans. 6.00

Sol.



$$H = -K,\ 2\pi rl\frac{dT}{dr}$$

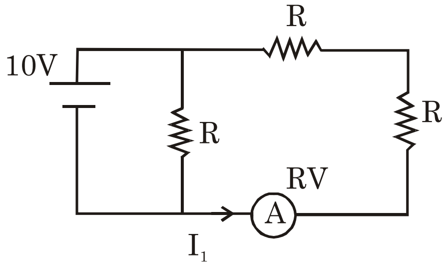
$$\int_{R_1}^{R_2}\frac{H\,dr}{2\pi rl} = -K\int_{T_1}^{T_2} dT$$

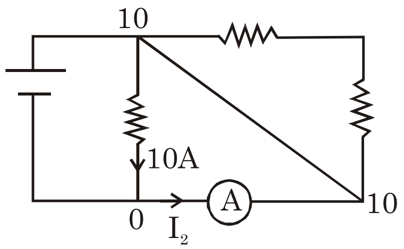
$$H = \frac{2\pi / k\left(T_1 - T_2\right)}{\ln\frac{R_2}{R_1}} H_i = H_f$$

$\therefore$ n = 6

4. Ans. 1.33

Sol.





$I_2 = 4I_1$

$$\frac{10}{RV} = \frac{4(10)}{(4+RV)}$$

4 + RV = 4RV

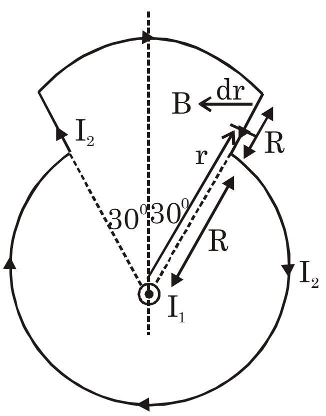
3RV = 4

RV = 1.33

5. Ans. 2.00

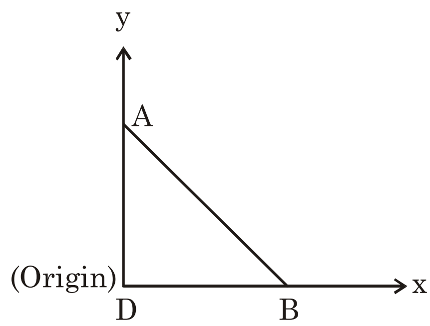
Sol. $\int d\tau = 2\int_R^{2R}\frac{\mu_0 I_1 I_2}{2\pi x}\times \sin 30^0\,dx$

$\tau = 20$



6. Ans. 6.66 to 6.67

Sol.



Potential on vertex of cube of side a is V_0

Let $V \propto \frac{1}{4\pi\varepsilon_0}\rho^x a^y$

Dimensionally x = 1, y = 2

If side is made half potential at vertex $= \frac{V_0}{4}$

Net potential at centre due to eight cubes of side $\frac{a}{2} = 8\times\frac{V_0}{4}$

$8\times\frac{V_0}{4} = 2V_0$

7. Ans. 3.00

Sol. $\ell = 16.2 \pm 0.1$ cm

$b = 10.1 \pm 0.1$ cm

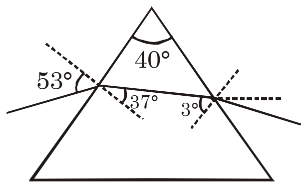
$\ell b = 163.62 \pm 2.6\ \text{cm}^2$

using proper rules

$\ell b = 164 \pm 3\ \text{cm}^2$

8. Ans. 17.00

Sol.



$1\sin 53 = \frac{4}{3}\sin r_1$

$\frac{4}{5} = \frac{4}{3}\sin r_1$

$\Rightarrow r_1 = 37° \Rightarrow r_2 = 3°$

$\frac{4}{3}\times\sin 3°$ = sine $\Rightarrow$ e = 4°

$\delta = i + e - A$

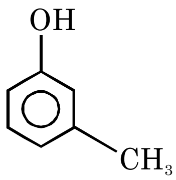
= 53 + 4 – 40

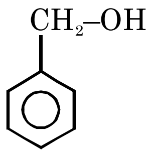
= 17°

PART–2 : CHEMISTRY SOLUTION

SECTION-I

1. **Ans. (B,C)**
2. **Ans.(B,C)**
3. **Ans. (B)**

Sol.  Functional group is phenol.

 Functional group is Alcohol.

4. **Ans. (A,C)**

Sol. $X^+ + H_2O \rightleftharpoons XOH + H^+$

$$K_H = \frac{k_w}{k_b} = \frac{10^{-14}}{4\times10^{-5}} = 2.5\times10^{-10}$$

$CH^2 \simeq k_H$

$(H^+) = Ch = \sqrt{c\cdot k_H}$

$10^{-5} = \sqrt{c\times2.5\times10^{-10}}$

C = 0.4 molar

$$0.4 = \frac{80}{M_{xy}}\times\frac{1}{2} \Rightarrow M_{xy} = \frac{40}{0.4} = 100$$

$$h = \sqrt{\frac{k_H}{C}} = \sqrt{\frac{2.5\times10^{-10}}{0.4}}$$

$h = 2.5\times10^{-5}$

$\%h = 2.5\times10^{-3}$

$$r_{x^+} + r_{y^-} = \sqrt{3}\frac{a}{2}$$

$$1.6 + 1.864 = r_{x^+} + r_{y^-} = \frac{\sqrt{3}}{2}a$$

a = 4Å

$$\text{density} = \frac{1\times\frac{100}{N_0}}{(4\times10^{-8})^3}\ \text{gm/cc}$$

= 2.6 gm/c.c.

5. **Ans. (A,B,D)**
6. **Ans. (A,B,C,D)**
7. **Ans. (B)**

Sol. P–(3); Q–(1),(5); R–(1),(2),(5); S–(3)

8. **Ans. (C)**
9. **Ans. (B)**
10. **Ans. (C)**

Sol.

(P) $E_{cell} = -\frac{0.0591}{2}\log\frac{C}{2C} = 8.89\times10^{-3}$

Spontaneous, cell works, conc. cell.

(Q) $E_{cell} = -0.0591\log\frac{1}{1} = 0$

Non spontaneous, cell will not work conc. cell.

(R) $E_{cell} = -\frac{0.0591}{2}\log\frac{0.01}{0.1^2} - 0.74 + 0.8 = 0.06$

Spontaneous, cell works.

(S) $E_{cell} = -\frac{0.0591}{1}\log\frac{Cl^-}{K_{sp}} - \frac{0.0591}{1}\log\frac{1}{[Ag^+]}$

$= \frac{0.0591}{2}\log\frac{0.1\times0.1}{K_{sp(AgCl)}} = +ve$

Spontaneous, cell works, conc. cell.

at anode,

$Ag(s) + Cl^-(aq) \rightarrow AgCl(s) + e^-$

SECTION-II

1. **Ans. (5.66)**

Sol.: [Range 5.5 to 5.7]

$$PI = \frac{2.2+9.1}{2} = \frac{11.3}{2} = 5.66$$

2. **Ans. (6.00)**

Sol.: 2, 3, 4, 6, 7, 8

With coordination number six and oxidation state +2, wo (n – 1)d orbitals are not available for d^2sp^3.

3. **Ans. (9.10 to 9.40)**

Sol.: $z = 1 + \frac{Pb}{RT}$

$$\frac{dz}{dp} = \frac{1}{10} = \frac{L}{RT}$$

$b = \frac{RT}{10} = \frac{22.4}{10} = 2.24$

$4 \times V \times N_0 = 2.24$

$V = \frac{2.24}{4 \times 6} \times 10^{-23} \ell$

$= \frac{2.24 \times 1000}{24} \times 10^{-23}\ cm^3$

$= 9.33 \times 10^{-22}\ cm^3$

4. Ans. (36.00)

Sol.: (a) ZrI_4 Oxidation state = 4

(b) $[Ni(CO)_4]$ Bond order = 3

(c) $Na_2\ [Zn(CN)_4] = 3$

5. Ans. (6.00)

Sol. All aldehyde hemeacetal α-hydroxy keto group, HCHO, can be oxidised by Tollen's regent.

(i), (ii), (iii), (v), (vi), (viii)

6. Ans. (0.18 to 0.20)

Sol.:

$HNO_3 + 2H_2SO_4 \rightleftharpoons NO_2^+ + H_3O^+ + 2HSO_4^-$

1	100			
0.7	99.7	0.3	0.3	0.6
	≅ 100			

Total moles of particles = 0.7 + 0.3 + 0.3 + 0.6

= 1.9

$\therefore \quad m = 1.9 \times \frac{1000}{9800} \simeq 0.19$

7. Ans. (4.00)

Sol.:

(A) $Ag_2CrO_4\downarrow$ (yellow) + $NH_3 \rightarrow$ $[Ag(NH_3)_2]^+ + CrO_4^{2-}$

(B) $Cu(OH)_2 \cdot CuSO_4\downarrow$(blue) + $8NH_3 \rightarrow$ $2[Cu(NH_3)_4]^{2+} + SO_4^{2-} + 2OH^-$

(C) $PbSO_3\downarrow$ (white) + $4OH^- \rightarrow$ $[Pb(OH)_4]^{2-} + SO_3^{2-}$

Soluble in sodium hydroxide not in NH_3.

(D) Soluble in sodium hydroxide not in NH_3.

(E) $Ni(OH)_2\downarrow$ (green) + $6NH_2 \rightarrow [Ni(NH_3)_6]^{2+}$ (deep blue coloration) + $2OH^-$

(F) $Zn_2(PO_4)_2\downarrow$ (white) + $12NH_3 \rightarrow$ $3[Zn(NH_3)_4]^{2+} + 2PO_4^{3-}$

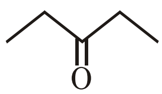
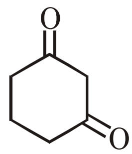
(G) Insoluble in NH_3, soluble appreciably in boiling concentrated H_2SO_4.

(H) $Bi(OH)_2NO_3$ is insoluble in NH_3.

(I) $Mn(OH)_2$ is insoluble in excess conc. solution of ammonia

8. Ans. (5.00)

Sol. Chloroform given by

$CH_3 - CH_2 - OH$, , ,

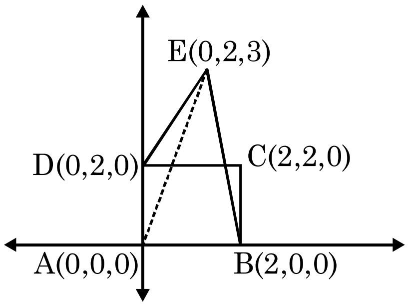
$Ph-\overset{O}{\overset{||}{C}}-CH_3$, $CCl_3-\overset{O}{\overset{||}{C}}-H$

PART-3 : MATHEMATICS — SOLUTION

SECTION-I

1. Ans. (A,C)

Sol. Plane ABE is $\begin{vmatrix} x & y & z \\ 0 & 2 & 3 \\ 2 & 0 & 0 \end{vmatrix} = 0$



$x(0) - y(-6) + z(-4) = 0$

$\Rightarrow\ 3y - 2z = 0$

2. Ans. (A,C,D)

Sol. z_1, z_2 lie on the circle whose centre is $(-2, -3)$ and radius is equal to.

$\because\ \arg(z_3 - z_1) = \arg(z_3 - z_2)$

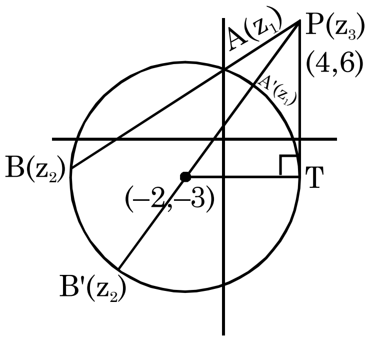
$\Rightarrow\ z_1, z_2$ and z_3 are collinear

$PA \cdot PB = PT^2$

$|z_3 - z_1|\ |z_3 - z_2| = OP^2 - OT^2$

$= 36 + 81 - 16$

$= 101$



$|z_1 - z_2|$ is maximum $\Rightarrow z_1$ and z_2 are extremities of diameter, $|z_1| = 4 - \sqrt{13}$ and $z_2 = 4 + \sqrt{13}$

$\therefore\ |z_1 z_2| = 16 - 13 = 3$

$|z_1| + |z_2| = 8$

3. Ans. (B,C)

Sol. $(A^{-1}BA)^{-1} = 2(\text{adj } A)$

$A^{-1}BA = 2 \text{ adj } A$

$BA = 2A(\text{adj } A) = 2|A|\ I_3$

$B = 2|A|A^{-1} = 10A^{-1}$

$\therefore\ \det(B) = 10^3 \times \frac{1}{5} = 200$

$\text{Tr.}(B) = 10\ \text{Tr.}(A^{-1}) = 10 \times 3 = 30$

4. Ans. (B,D)

Sol. As tangents are perpendicular to each other and its points of intersections $P\left(\frac{7}{5}, -\frac{4}{5}\right)$ lies on directrix. AB is focal chord.

Equation of AB is $x - y = 4$, as focus is (α, β)

so $\alpha - \beta = 4\ \Rightarrow\ \beta = \alpha - 4$

Now $\angle ASP = 90°$

$$\Rightarrow\ \frac{-2 - \alpha + 4}{2 - \alpha} \cdot \frac{-\frac{4}{5} - \alpha + 4}{\frac{7}{5} - \alpha}$$

$$\Rightarrow\ \frac{2 - \alpha}{2 - \alpha} \cdot \frac{\frac{16}{5} - \alpha}{\frac{7}{5} - \alpha} = -1$$

$$\Rightarrow\ \frac{16}{5} - \alpha = \alpha - \frac{7}{5};\ \ \alpha = \frac{23}{10}$$

$$\Rightarrow\ \text{Focus is } \left(\frac{23}{10}, -\frac{17}{10}\right)$$

Also, $\ell_1 = AS = \frac{3\sqrt{2}}{10},\ \ \ell_2 = BS = \frac{27\sqrt{2}}{10}$

$$\frac{1}{a} = \frac{1}{\ell_1} + \frac{1}{\ell_2}\ \ \Rightarrow a = \frac{27\sqrt{2}}{100}\ \ \Rightarrow 4a = \frac{27\sqrt{2}}{25}$$

5. Ans. (A,C)

Sol. $$I_0 = \int_0^{\pi/4} \underset{I}{\left(x-\frac{\pi}{4}\right)^{50}} \underset{II}{\sec^2 x\, dx}$$

$$I_0 = \left[\left(x-\frac{\pi}{4}\right)^{50} \cdot \tan x\right]_0^{\pi/4}$$

$$-50\int_0^{\pi/4}\left(x-\frac{\pi}{4}\right)^{49} \cdot \tan x\, dx$$

$$\therefore\ I_0 = -50\int_0^{\pi/4}\left(x-\frac{\pi}{4}\right)^{49} \tan x\, dx$$

$$\therefore\ I_1 = \frac{\int_0^{\pi/4}\left(x-\frac{\pi}{4}\right)^{50}\sec^2 x\, dx}{\int_0^{\pi/4}\left(x-\frac{\pi}{4}\right)^{49}\cdot \tan x\, dx} = -50$$

$$I_2 = \frac{1}{100}\int_0^{\pi/4} \underset{I}{\left(x-\frac{\pi}{4}\right)^{100}} \cdot \underset{II}{\sec^2 x\, dx}$$

$$-\int_0^{\pi/4} x^{99}\left(\frac{1-\tan x}{1+\tan x}\right)dx$$

Apply King prop.,

$I_2 = 0$

$\therefore$ $I_1 < I_2$

$I_1 \cdot I_2 = 0$

6. Ans. (B,C)

Sol. $\frac{\cos A}{a} = \frac{b^2+c^2-a^2}{2abc}, \frac{\cos B}{b} = \frac{c^2+a^2-b^2}{2abc}, \frac{\cos C}{c} = \frac{a^2+b^2-c^2}{2abc}$

$$\therefore \frac{\cos A}{a} + \frac{\cos B}{b} + \frac{\cos C}{c} = \frac{a^2+b^2+c^2}{2abc}$$

$$\&\ \frac{1}{r_1} + \frac{1}{r_2} + \frac{1}{r_3} = \frac{s-a}{\Delta} + \frac{s-b}{\Delta} + \frac{s-c}{\Delta}$$

$$= \frac{3s-2s}{\Delta} = \frac{s}{\Delta} = \frac{1}{r}$$

7. Ans. (A)

Sol.

$$\frac{d\,(g(x))}{d\,f(x)} = \begin{cases} \frac{-2}{1+(f(x))^2}, & f(x) \in (-1,1) \\ \frac{-2}{1+(f(x))^2}, & f(x) \in (-\infty,-1) \\ \frac{-2}{1+(f(x))^2}, & f(x) \in (1,\infty) \end{cases}$$

(P) $\frac{-2}{1+(f(x))^2} = -\frac{1}{13}$

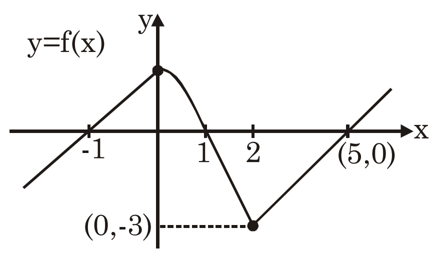
$\Rightarrow$ $1 + (f(x))^2 = 26$

$\Rightarrow$ $f(x) = \pm 5$

$f(x) = 5$ when $x = 10$

$f(x) = -5$ when $x = -6$

(Q)



f(x) has local maxima at x = 0 and local minima at x = 2.

(R) From the graph of y = f(x) we can see that f(x) = –k has 2 positive and one negative root if $-k \in (-3, 1)$

$\Rightarrow k \in (-1, 3)$

(S) $g'(x) = \frac{-2}{1+(f(x))^2} \times f'(x)$

$g'(x) < 0 \Rightarrow f'(x) > 0$

$\Rightarrow x \in (-\infty, 0) \cup (2, \infty)$

8. Ans. (C)

Sol.

(P) $\frac{d^2y}{dx^2} = 6x - 4$

$\Rightarrow \frac{dy}{dx} = 3x^2 - 4x + c$

at $x = 1$, $\frac{dy}{dx} = 0 \Rightarrow c = 1$

$\therefore \frac{dy}{dx} = 3x^2 - 4x + 1$

$\Rightarrow$ $y = x^3 - 2x^2 + x + d$

at $x = 1, y = 5$

$\Rightarrow$ $5 = 1 - 2 + 1 + d \Rightarrow d = 5$

$\therefore$ $f(x) = x^3 - 2x^2 + x + 5$

$f(0) = 5$

(Q) $\frac{dy}{dx} = 2ax + b$

$\left.\frac{dy}{dx}\right|_{(1,2)} = 2a + b$

$y = ax^2 + bx + \frac{7}{2}$ passes through (1, 2)

$\Rightarrow$ $2 = a + b + \frac{7}{2}$

$\Rightarrow$ $\boxed{a + b = \frac{-3}{2}}$ .....(1)

$y = x^2 + 6x + 10$

$\frac{dy}{dx} = 2x + 6$

Slope of normal at (–2, 2) = $\frac{-1}{2(-2)+6}$

$= -\frac{1}{2}$

$\therefore$ $\boxed{2a + b = -\frac{1}{2}}$ .....(2)

From (1) and (2)

$a = 1, b = \frac{-5}{2}$

$\therefore$ $\frac{a}{2} - b = \frac{1}{2} + \frac{5}{2} = 3$

(R) $5\alpha(3x^2) + 10\alpha(2x) + 1 + 2\frac{dy}{dx} = 0$

$\Rightarrow$ $\frac{dy}{dx} = \frac{-15\alpha^2x^2 - 20\alpha x - 1}{2}$

$\Rightarrow$ $\frac{dy}{dx}$ at A = $\frac{-1}{2}$ A : (0, 2)

Equation of normal at A is

$y - 2 = 2(x - 0)$

$\Rightarrow$ $y = 2x + 2$

Let normal meets the curve at B.

$\Rightarrow$ $5\alpha^2x^3 + 10\alpha x^2 + x + 4x + 4 - 4 = 0$

$\Rightarrow$ $5x(\alpha x + 1)^2 = 0$

$\Rightarrow$ $x = \frac{-1}{\alpha}$

$\therefore$ B : $\left(-\frac{1}{\alpha}, \frac{-2}{\alpha} + 2\right)$

Slope of tangent at B

$= \frac{-\frac{15\alpha^2}{\alpha^2} + \frac{20\alpha}{\alpha} - 1}{2} = 2$

$\therefore$ Equation of tangent is

$y + \frac{2}{\alpha} - 2 = 2\left(x + \frac{1}{\alpha}\right)$

$\Rightarrow$ $\boxed{y = 2x + 2}$

(S) $f(x) = \frac{px}{e^x} - \frac{x^2}{2} + x$

$f'(x) = -px\,e^{-x} + pe^{-x} - x + 1$

$f'(x) = pe^{-x}(-x + 1) + (-x + 1)$

$= (pe^{-x} + 1)(1 - x) \le 0, \ \forall\ x < 0$

$\Rightarrow$ $p \le -1$

9. Ans. (A)

Sol.

(P) $f(x) = \text{sgn}(\sin^2 x - \sin x - 1)$

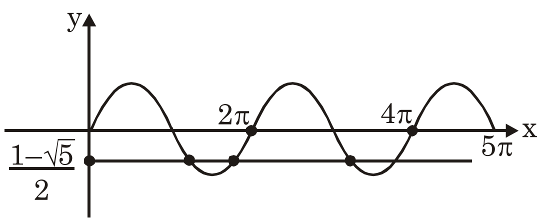
$\sin^2 x - \sin x - 1 = 0$

$\Rightarrow$ $\sin x = \frac{1 \pm \sqrt{5}}{2}$

$\Rightarrow$ $\sin x = \frac{1 - \sqrt{5}}{2}$

f(x) will be discontinuous when

$\sin x = \frac{1 - \sqrt{5}}{2}$



$\Rightarrow$ $x \in (0, 4\pi]$ or $x \in (0, 5\pi]$

(Q) $y = \left[\frac{f(x)}{g(x)}\right]$

$$y = \left[\frac{x^2 - x + k - 2}{x^2 - x + 1}\right]$$

$$y = \left[1 + \frac{k-3}{x^2 - x + 1}\right]$$

$$y = 1 + \left[\frac{k-3}{\left(x - \frac{1}{2}\right)^2 + \frac{3}{4}}\right]$$

for y to be continuous in R

$$0 \le k - 3 < \frac{3}{4} \Rightarrow k \in \left[3, \frac{15}{4}\right)$$

$$\Rightarrow a = 3,\ b = \frac{15}{4}$$

$$b - \frac{5a}{4} = \frac{15}{4} - \frac{15}{4} = 0$$

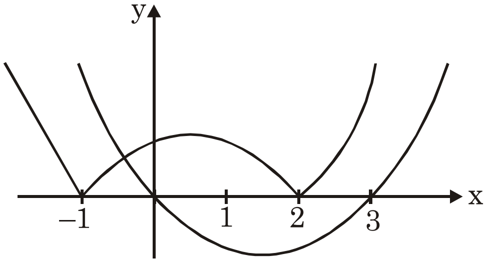
(R) $f(x) = \begin{cases} \cos^{-1} x, & \frac{1}{2} \le |x| \le 1 \\ \frac{2|x|}{3} + \frac{\pi}{3} - \frac{1}{3}, & |x| < \frac{1}{2} \end{cases}$

$$f(x) = \begin{cases} \cos^{-1} x, & x \in \left[-1, -\frac{1}{2}\right] \cup \left[\frac{1}{2}, 1\right] \\ -\frac{2x}{3} + \frac{\pi}{3} - \frac{1}{3}, & x \in \left(-\frac{1}{2}, 0\right) \\ \frac{2x}{3} + \frac{\pi}{3} - \frac{1}{3}, & x \in \left[0, \frac{1}{2}\right) \end{cases}$$

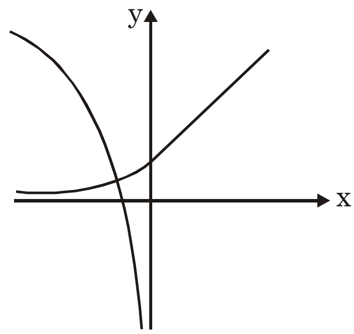
$\cos^{-1} x$ is non differentiable at x = –1, 1.

f(x) is non-differentiable at x = –1, $-\frac{1}{2}$, 0, $\frac{1}{2}$, 1.

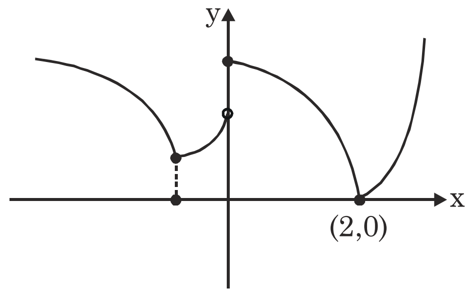
(S) max [| $x^2 - x - 2$ | , $x^2 - 3x$]



max (ln(–x), e^x)



y = f(x)



From the above graph of y = f(x) we can see that f(x) is non differentiable at 3 points.

10. Ans. (A)

Sol.

(P) If a = 1

$$f(x) = \lim_{t\to\infty} \frac{(1+\sin \pi x)^t - 1}{(1+\sin \pi x)^t + 1}$$

If x ∈ (0, 1) ∪ (2, 3) ∪ (4, 5)

f(x) = 1

If x ∈ (1, 2) ∪ (3, 4) ∪ (5, 6)

f(x) = –1

If x = 2, 3, 4, 5

f(x) = 0

∴ f(x) is discontinuous at x = 1, 2, 3, 4, 5

(Q) If a = 3

$$f(x) = \lim_{t\to\infty} \frac{(3+\sin \pi x)^t - 1}{(3+\sin \pi x)^t + 1}$$

f(x) = 1 ∀ x ∈ R

∴ f(x) is continuous ∀ x ∈ R

(R) If a = 0.5

$$f(x) = \lim_{x\to\infty} \frac{(0.5+\sin \pi x)^t - 1}{(0.5+\sin \pi x)^t + 1}$$

Limit does not exist at

$$x = \frac{1}{6}, \frac{5}{6}, \frac{13}{6}, \frac{17}{6}, \frac{25}{6}, \frac{29}{6}$$

(S) If a = 0

$$f(x)=\lim_{t\to\infty}\frac{(\sin\pi x)^t-1}{(\sin\pi x)^t+1}$$

(i) If $x=\frac{1}{2},\frac{5}{2},\frac{9}{2}$

f(x) = 0

(ii) If $x\in\left(0,\frac{1}{2}\right)\cup\left(\frac{1}{2},\frac{3}{2}\right)\cup\left(\frac{3}{2},\frac{5}{2}\right)\cup\left(\frac{5}{2},\frac{7}{2}\right)$

$\cup\left(\frac{7}{2},\frac{9}{2}\right)\cup\left(\frac{9}{2},\frac{11}{2}\right)\cup\left(\frac{11}{2},6\right)$

f(x) = –1

(iii) If $x=\frac{3}{2},\ \frac{7}{2},\ \frac{11}{2}$

f(x) is not defined

$\therefore$ f(x) is discontinuous at

$$x=\frac{1}{2},\frac{3}{2},\frac{5}{2},\frac{7}{2},\frac{9}{2},\frac{11}{2}$$

SECTION-II

1. Ans. (1.00)

Sol. Given $2(a_1+a_2+......+a_n)=b_1+b_2+......+b_n$

$\Rightarrow\ 2\times\frac{n}{2}(2c+(n-2)2)=c\left(\frac{2^n-1}{2-1}\right)$

$\Rightarrow\ 2n^2-2n=c(2^n-1-2n)$

$\Rightarrow\ c=\frac{2n^2-2n}{2^n-1-2n}\in\mathbb{N}$

So, $2n^2-2n\ge 2^n-1-2n$

$\Rightarrow\ 2n^2+1\ge 2^n\Rightarrow n<7$

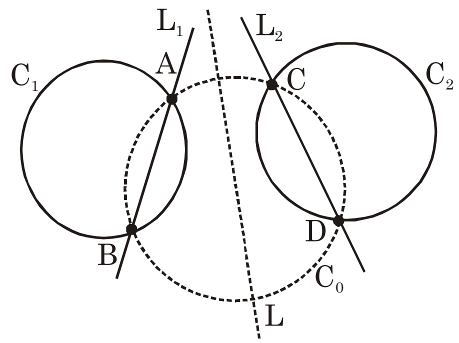
$\Rightarrow$ n can be 1,2,3,....,

Checking c against these values of n we get c = 12 (when n = 3)

Hence number of such c = 1

2. Ans. (4.00)

Sol.: We represent the given situation through the following diagram :



• C_0 is the circle passing through A, B, C and D.

• L is the radical axis of C_1 and C_2 given by.

$S_1-S_2=0$

$\Rightarrow\ 2x-2y-6=0$

We use the fact that the radical axes of three circles taken two at a time are concurrent. Thus, L, L_1 and L_2 are concurrent :

$$\begin{vmatrix}2&-2&-6\\1&2&3\\2&3&\lambda\end{vmatrix}=0\ \Rightarrow\ \lambda=4$$

3. Ans. (1.00)

Sol. $\sin y\frac{dy}{dx}-2\cos y\cos x=-\sin^2 x\cos x$

Put $-\cos y=z$

$\Rightarrow\ \sin y\frac{dy}{dx}=\frac{dz}{dx}$

$\frac{dz}{dx}+(2\cos x)z=-\sin^2 x\cos x$

I.F. $=e^{\int 2\cos dx}=e^{2\sin x}$

$z\cdot e^{2\sin x}=-\int e^{2\sin x}\cos x\sin^2 x\,dx$

$\therefore\ z=-\cos y=\frac{-\sin^2 x}{2}-\frac{1}{4}+\frac{\sin x}{2}+ce^{-2\sin x}$

y = 0 at x = 0

$\Rightarrow\ -1=0-\frac{1}{4}+0+c$

$\Rightarrow\ c=-\frac{3}{4}$

$\therefore\ \cos y = \frac{1}{2}\sin^2 x - \frac{1}{2}\sin x + \frac{1}{4} + \frac{3}{4}e^{-2\sin x}$

$\therefore\ A + B + C + D = 1$

4. Ans. (6.00)

Sol.: Let $P(E_k) = \lambda K(K+1)$

$$\sum_{k=1}^{n} P(E_k) = 1$$

$$\Rightarrow\ \lambda\sum_{k=1}^{n} k(k+1) = 1$$

$$\Rightarrow\ \lambda = \frac{3}{n(n+1)(n+2)}$$

Let P(E) denote the probability that a coin selected at random is biased.

$$P(E_1/E) = \frac{P(E_1)\times P(E/E_1)}{\sum_{k=1}^{n} P(E_k)\times P(E/E_k)}$$

$$P(E_1/E) = \frac{2\lambda\times\frac{1}{n}}{\sum_{k=1}^{n}\lambda\cdot k\cdot(k+1)\cdot\frac{k}{n}}$$

$$= \frac{2}{\sum_{k=1}^{n} k^2(k+1)}$$

$$= \frac{24}{n(n+1)(n+2)(3n+1)}$$

$$\therefore\ F(n) = \frac{24}{n(n+1)(n+2)(3n+1)}$$

$$\sum_{r=1}^{k} F(r)\cdot(3r+1)$$

$$= \sum_{r=1}^{k}\frac{24}{r(r+1)(r+2)}$$

$$= 12\sum_{r=1}^{k}\left(\frac{1}{r(r+1)} - \frac{1}{(r+1)(r+2)}\right)$$

$$= 12\left(\frac{1}{1.2} - \frac{1}{(k+1)(k+2)}\right)$$

$$\therefore\ \lim_{k\to\infty}\sum_{r=1}^{k} F(r)(3r+1) = \frac{12}{2} = 6$$

5. Ans. (8.00)

Sol.: $\frac{1}{a}, \frac{1}{a_2}, \frac{1}{a_3}, \ldots\ldots, \frac{1}{a_9}$ are in A.P.

$$d = \frac{1}{4} - \frac{1}{5} = \frac{1}{20}$$

Series is $\frac{1}{20}, \frac{2}{20}, \frac{3}{20}, \ldots\ldots, \frac{9}{20}$

$$D = \begin{vmatrix} 20 & \frac{20}{2} & \frac{20}{3} \\ \frac{20}{4} & \frac{20}{5} & \frac{20}{6} \\ \frac{20}{7} & \frac{20}{8} & \frac{20}{9} \end{vmatrix}$$

$D = (20)^3\ |A|$

$$\Rightarrow\ \frac{D}{|A|} = 8000$$

6. Ans. (10.00)

Sol.: $|A|^{16} = 4^8.5^{16}$

$|A| = 10$

$x + y + z = 10$

7. Ans. (3.00)

Sol.: $\frac{1}{a_1a_n} + \frac{1}{a_2a_{n-1}} + \ldots.. + \frac{1}{a_na_1}$

$$= \frac{2}{a_1 + a_n}\left[\frac{1}{a_1} + \frac{1}{a_2} + \ldots.. + \frac{1}{a_n}\right]$$

$$\Rightarrow\ 1 - \tan\theta = \frac{1}{1+\sin 2\theta}(1+\tan\theta)$$

$$\Rightarrow\ (\cos\theta - \sin\theta) = \frac{\cos\theta + \sin\theta}{(\cos\theta + \sin\theta)^2}$$

$\Rightarrow\ \cos 2\theta = 1\ \Rightarrow\ \theta = k\pi$

8. Ans. (10.00)

Sol. $S_{100} = \frac{0}{\left({}^{100}C_0\right)^5} + \frac{1}{\left({}^{100}C_1\right)^5} + \ldots$ (i)

Also $S_{100} = \frac{100}{\left({}^{100}C_0\right)^5} + \frac{99}{\left({}^{100}C_1\right)^5} + \ldots.$ (ii)

$\therefore$ On adding (1) and (2), we get

$2S_{100} = 100t_{100}$

$$\Rightarrow \frac{t_{100}}{S_{100}} = \frac{2}{100}$$