(1001CJA102120057)

Test Pattern

ALLEN CAREER INSTITUTE KOTA (RAJASTHAN) path to success

CLASSROOM CONTACT PROGRAMME

JEE(Advanced)
FULL SYLLABUS

SAMPLE PAPER-1

ANSWER KEY — PAPER-1

PART-1 : PHYSICS

SECTION-I	Q.	1	2	3	4	5	6	7	8	9	10
	A.	A,B,D	A,B,C,D	A,C,D	A,B,C	A,C,D	A,B,C,D	A	A	A	B
SECTION-II	Q.	1	2	3	4	5	6	7	8		
	A.	4.80	2.82 to 2.83	60.00	10.38 to 10.40	1.84	0.00	6.00	6.00		

PART-2 : CHEMISTRY

SECTION-I	Q.	1	2	3	4	5	6	7	8	9	10
	A.	A,B,C	B,C,D	B,C,D	B,D	A,C,D	A,B,C	B	C	B	D
SECTION-II	Q.	1	2	3	4	5	6	7	8		
	A.	23.72	5.00	1.00	13.00	6.00	28.00 to 34.00	6.00	56.00		

PART-3 : MATHEMATICS

SECTION-I	Q.	1	2	3	4	5	6	7	8	9	10
	A.	B,C,D	B,C	A,C	A,C,D	A,B,D	A,C,D	A	C	C	B
SECTION-II	Q.	1	2	3	4	5	6	7	8		
	A.	27.00	6.82 or 6.83	13.66 or 13.67	7.00	80.00	2.00	16.00	7.00		

SAMPLE PAPER-1

PAPER-1

PART-1 : PHYSICS SOLUTION

SECTION-I

1. Ans. (A,B,D)

Sol. Activity of A = λN_A

Activity of B = $2\lambda N_B$

$\lambda N_A = 2\lambda N_B$

$N_A = 2N_B$

2. Ans. (A,B,C,D)

Sol. $w = \frac{\lambda D}{d}$

$n = \left(\frac{xd}{\lambda D}\right)$

$\frac{dn}{dt} = \left(\frac{xv}{\lambda D}\right)$

3. Ans. (A,C,D)

Sol. net force on magnet is zero in uniform magnetic field.

4. Ans. (A,B,C)

Sol. $a = -bx = -w^2x$ $\qquad w = \sqrt{b}$

$V_{max} = aw$ $\qquad A = \frac{u}{\sqrt{b}}$

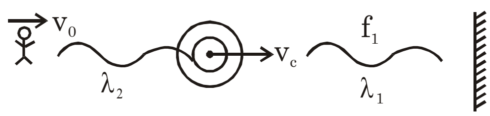
5. Ans. (A,C,D)

Sol. $H_{max} = \frac{2T\cos\theta}{r\rho g} = 1cm$

$H \propto \cos\theta_C$

6. Ans. (A,B,C,D)

Sol.



$\lambda_1 = (v - v_c)T = \left(\frac{v - v_c}{f}\right)$

$\lambda_2 = (v + v_c)T = \left(\frac{v + v_c}{f}\right)$

$f_1 = \frac{v}{x_1} = \left(\frac{v}{v + v_c}\right)f$

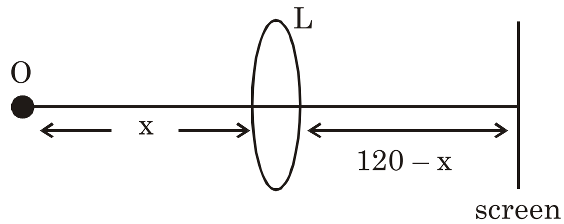
$\Delta f_1 = f_{echo} - f_{street}$

$= \left(\frac{v}{v - v_c}\right)f.\left(\frac{v + v_1}{v}\right) - f\left(\frac{v + v_0}{v + v_c}\right)$

$= \frac{2v_c(v + v_0)f}{(v^2 - v_c^2)}$

7. Ans. (A)

Sol.



$m_1 : m_2 = 1 : 9$

$m_1 . m_2 = 1$

$\therefore \quad m_1 = \frac{1}{3} \qquad m_2 = 3$

$m = \frac{v}{u}$

$\frac{1}{3} = \frac{120 - x_1}{x_1}$

$4x_1 = 360$

$x_1 = 90$

$x_2 = 120 - 90 = 30$

$D = 120$

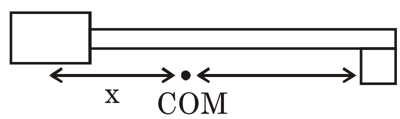
$d = x_1 - x_2 = 60$

$f = \frac{D^2 - d^2}{4D} = 22.5$ cm

8. Ans. (A)

9. Ans. (A)

Sol.



$x = \frac{M.L.}{3M + M} = \frac{L}{4}$

from conservation of angular momentum about COM

$Mv_0\left(\frac{3L}{4}\right) = Iw$

$I = 3M\left(\frac{L}{4}\right)^2 + M\left(\frac{3L}{4}\right)^2 = \frac{3ML^2}{16} + \frac{9ML^2}{16}$

$= \frac{3}{4}ML^2$

$\frac{3Mv_0L}{4} = \frac{3}{4}ML^2.w$

$\boxed{w = \frac{v_0}{L}}$

10. Ans. (B)

Sol. From cons of linear momentum

$Mv_0 = 4M.v_c$

$v_c = \frac{v_0}{4}$

Velocity of $3M = v_c - \frac{L}{4}w$

$= \frac{v_0}{4} - \frac{L}{4} \times \frac{v_0}{L} = 0$

SECTION-II

1. Ans. 4.80

Sol. $\frac{1}{t} = \frac{1}{t_1} + \frac{1}{t_2}$

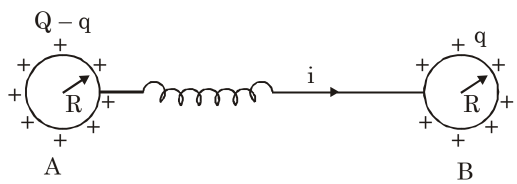
$t = \frac{t_1t_2}{t_1 + t_2} = \frac{24 \times 6}{24 + 6} = 4.80$ Hr.

2. Ans. 2.82 to 2.83

Sol. Let q = charge on B at time 't'

$V_A - V_B = L\frac{di}{dt}$

$\Rightarrow K\frac{Q-q}{R} - K\frac{q}{R} = L\frac{d^2q}{dt^2}$



[Spheres are at large distance. Hence charge on one does not affect the potential of other]

$\Rightarrow \frac{d^2q}{dt^2} = -\frac{2K}{LR}\left[q - \frac{Q}{2}\right]$ ...... (i)

Let $q - \frac{Q}{2} = x$ which means $\frac{d^2q}{dt^2} = \frac{d^2x}{dt^2}$

In terms of x, the differential equation (i) becomes

$\frac{d^2x}{dt^2} = \frac{2K}{LR}x$

Solution to this equation is

$x = x_0\sin(\omega t + \delta)$ $\left[\omega = \sqrt{\frac{2K}{LR}}\right]$

$\Rightarrow q - \frac{Q}{2} = x_0\sin(\omega t + \delta)$

$\Rightarrow q = \frac{Q}{2} + x_0\sin(\omega t + \delta)$

Current

$i = \frac{dq}{dt} = x_0\omega\cos(\omega t + \delta)$

Just after closing the switch (at $t = 0^+$) the current is zero

$\therefore \delta = \frac{\pi}{2}$

$\therefore q = \frac{Q}{2} + x_0\cos\omega t$

Also, at t = 0, q = 0

$\therefore x_0 = -Q/2$

$\therefore q = \frac{Q}{2}[1 - \cos\omega t]$

$q = \frac{Q}{2}$ means $\cos\omega t = 0$

$\Rightarrow \omega t = \frac{\pi}{2}$

$\Rightarrow \sqrt{\frac{2K}{LR}}t = \frac{\pi}{2}$

$\Rightarrow t = \frac{\pi}{2}\sqrt{\frac{LR}{2K}} = \frac{\pi}{2\sqrt{2}} = \sqrt{\frac{LR}{K}}$

$\therefore a = 2\sqrt{2}$

$= 2 \times 1.414$

$= 2.828$

$\simeq 2.83$

3. Ans. 60.00

Sol. Initially $V_R = V_{C_1} = 2.5 \times 4 = 10V$

$$Q_1 = CV = 3 \times 10 = 30\ \mu C$$

$$U_1 = \frac{1}{2} \times 3 \times 10^2 = 150\ \mu J$$

Finally $V_{C_1} = V_{C_2} = \frac{Q_1}{C_1 + C_2} = \frac{30}{5} = 6\ V$

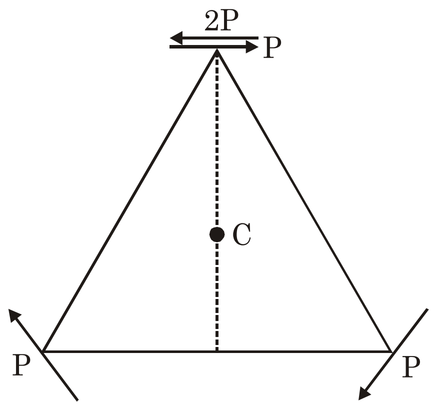
$$U_2 = \frac{1}{2} \times (C_1 + C_2) \times V^2$$

$$= \frac{1}{2} \times 5 \times 36 = 90\ \mu J$$

$\therefore$ Heat liberated = 150 – 90 = 60 μJ

4. Ans. 10.38 to 10.40

Sol.



From superposition

$$E = \left(\frac{2KP}{r^3}\right) \text{ opposite to } 2\vec{P}$$

$$= \frac{2KP}{\left(\frac{L}{\sqrt{3}}\right)^3} \qquad r = \frac{2}{3} \times \frac{\sqrt{3}L}{2} = \frac{L}{\sqrt{3}}$$

$$= 6\sqrt{3}\left(\frac{KP}{L^3}\right)$$

$\therefore$ a = 6 × 1.732 = 10.392

5. Ans. 1.84

Sol. Length of 1 main scale division

= 1/10 = 0.1 cm

No. of vernier scale divisions = 10

Main scale reading = 1.8 cm

Coinciding vernier scale division = 4th

Least Count

= Length of smallest main scale division/ No. of vernier scale divisions

= 0.1 /10

= 0.01 cm

(vernier calliper has a least count 0.01 cm. So, measurement is accurate only upto three significant figures.)

length = Main scale reading + coinciding vernier scale division × LC

= 1.8 + 4 × 0.01

= 1.8 + 0.04

= 1.84 cm

6. Ans. 0.00

Sol. Heat lost by water = Heat given by ice

(3 + 1) × 1 × (20 – θ)

= 1 × 0.5 × 40 + 1 × 80 + 1 × 1 × θ

80 – 4θ = 20 + 80 + θ

5θ = 20

Not possible ⇒ θ = 0°

7. Ans. 6.00

Sol. After every collision velocity becomes $\frac{1}{\sqrt{2}}$ times.

$$\frac{1}{8} = \left(\frac{1}{\sqrt{2}}\right)^n \Rightarrow n = 6$$

8. Ans. 6.00

Sol. Magnitude of viscous force, $F = \eta A = \frac{dv}{dr}$

⇒ viscous force per unit area $\sigma = \frac{F}{A} = \eta\frac{dv}{dr}$

$$v = v_0\left(1 - \frac{r^2}{R^2}\right)$$

$$\Rightarrow \frac{dv}{dr} = -\frac{2V_0 r}{R^2}$$

$$\Rightarrow \sigma = \eta\frac{2v_0 r}{R^2} \qquad \text{....... (i)}$$

Volume rate of flow, Q

consider an annular element at r from axis, width dr.

$dA = 2\pi r dr$; $dQ = v.dA = v_0\left(1 - \frac{r^2}{R^2}\right)2\pi r dr$

$$Q = \int dQ = 2\pi v_0\left[\frac{r^2}{2} - \frac{r^4}{4R^2}\right]_0^R = \frac{\pi}{2}R^2 v_0$$

$$\Rightarrow v_0 = \frac{2Q}{\pi R^2}$$

$\therefore$ (i) ⇒ $\sigma = \eta\frac{4Q}{\pi R^4}r$, R = 0.1 m

At r = 0.04 m,

$$\sigma = (0.75)4 \times \frac{\pi}{2} \times 10^{-2} \times \frac{0.04}{\pi \times 10^{-4}}$$

$= 6\ Nm^{-2}$

PART–2 : CHEMISTRY SOLUTION

SECTION-I

1. Ans. (A,B,C)

Sol. (A) Only first four spectral lines belonging to Balmer series in hydrogen spectrum lie in visible region.

(B) If a light of frequency ν falls on a metal surface having work function $h\nu_0$, photoelectric effect will take place only if $\nu \geq \nu_0$, since ν_0 is the minimum frequency required for photoelectric effect.

(C) The number of photoelectrons ejected from a metal surface in photoelectric effect depends upon the intensity of incident radiations.

2. Ans. (B,C,D)

Sol. B : Magnesite is ore of Mg

C : Anode mud will have Au only

D : Self reduction occurs during roasting.

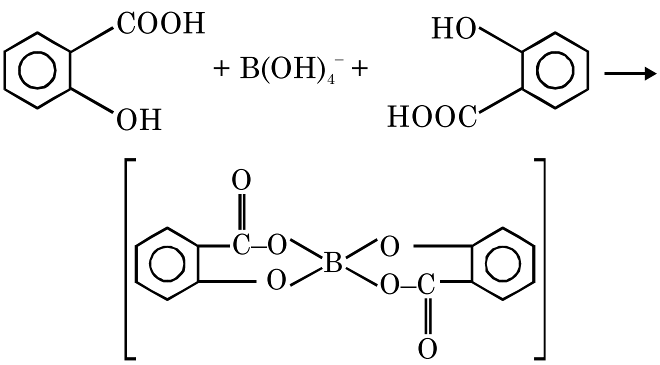
3. Ans. (B,C,D)

4. Ans. (B,D)

5. Ans. (A,C,D)

Sol. $Na_2B_4O_7 \cdot 10H_2O + aq \rightleftharpoons 4H_2BO_3 + 2NaOH;$

$H_2BO_3 + OH^- \rightleftharpoons B(OH)_4^-$



– Optically resolvable due to asymmetric structure

– Two six membered rings.

6. Ans. (A,B,C)

Sol. Option (D) incorrect because

• Rate of SN^1 depend on stability of carbocation formed.

7. Ans. (B)

Sol.

	A +	B →	C +	D
t=0	1	1	0	0
t=t	0.25	0.25	0.75	0.75

$$\frac{dx}{dt} = k(a-x)^{1/2}\,(b-x)^{1/2}$$

Here a = b

$$\frac{dx}{dt} = k(a-x)$$

$\therefore$ $t = \frac{1}{k}\ln\frac{a}{a-x}$

$\therefore$ We'll treat it as first order, single reactant.

$\therefore$ $t = 2 \times t_{1/2} = 2 \times 0.693 / 2.31 \times 10^{-3} = 600$ s.

8. Ans. (C)

Sol. $\frac{d}{dT}(\ln k) = \frac{E_a}{RT^2}$

$$0 + \frac{2500}{T^2} + \frac{3}{T} = \frac{E_a}{RT^2}$$

$\therefore$ $E_a = (2500 + 3T)R$

9. Ans. (B)

Sol. $CuSO_4 + KI \rightarrow Cu_2I_2\downarrow + I_2$
(white)

$I_2 + Na_2S_2O_3 \rightarrow Na_2S_4O_6 + NaI$
reduced oxidised

10. Ans. (D)

Sol. Bottles 1, 2, 3, 4 have $Pb(NO_3)_2$, HCl, Na_3CO_3 and $CuSO_4$.

When mixing :

Bottle 1 + Bottle 2 → $PbCl_2$

Bottle 1 + Bottle 3 → $PbCO_2$

Bottle 1 + Bottle 4 → $PbSO_4$

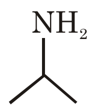
Bottle 2 + Bottle 3 → CO_2

Bottle 2 + Bottle 4 → No visible reaction

Bottle 3 + Bottle 4 → $CuCO_3$

SECTION-II

1. Ans. (23.72)

Sol.: X is 

$$\% \text{ Nitrogen} = \frac{14}{59} \times 100 = 23.72\%$$

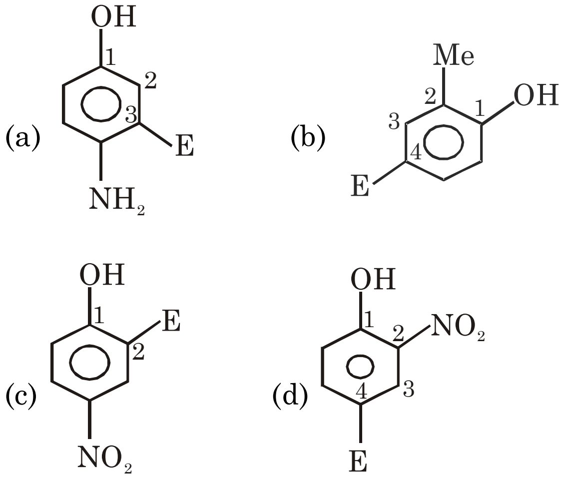
2. Ans. (5.00)

Sol.: 1, 4, 5, 6, 8 can be oxidised.

3. Ans. (1.00)

4. Ans. (13.00)

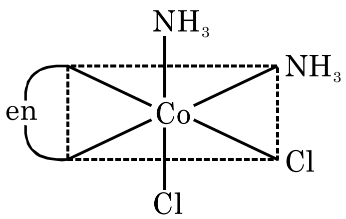
Sol.: 3 + 4 + 2 + 4 = 13



5. Ans. (6.00)

Sol.: (a) Cis-isomer is op. active

(b)



is op. active

(c) optically inactive.

(d) op. inactive

(e) always optically active

(f) square planar, optically inactive

(g) EDTA complexes are optically active

(h) 3 didentate ligands are always optically active

(i) same as (h)

6. Ans. (28.00 to 34.00)

Sol.: $\frac{(In^-)}{(HIn^-)} = \frac{25}{75} = \frac{1}{3}$

$pH = 9.6 + \log \frac{(In^-)}{(HIn^-)}$

$= 9.6 + \log \frac{1}{3}$

$pH = 9.12 = x$

$NaOH + HCl \rightarrow NaCl + H_2O$

1 1

$Na_2CO_3 + HCl \rightarrow NaHCO_3 + NaCl$

1.m.mole	x	0	0
1– x	0	x	–

$pH = pka_2 + \log \frac{(CO_3^{-2})}{(HCO_3^-)}$

$9.12 = 11 + \log \frac{1-x}{x}$

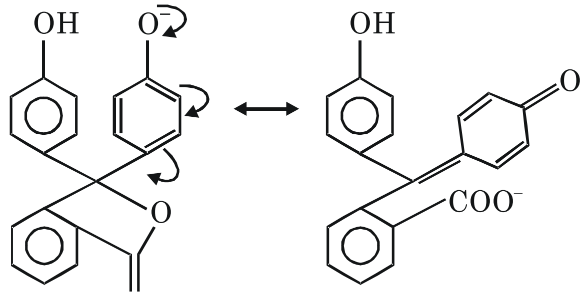
$1.88 = \log \frac{x}{1-x}$

$75.85 = \frac{x}{1-x}$

$x = \frac{75.85}{76.85} = 0.99$

$\therefore\ 0.99 = 0.1 \times V_{HCl}$

$V_{HCl\ Total} = 9.9 + 10 = 19.9\ ml = y$



Colored form

$\therefore\ z = 0$

$\therefore\ x + y + z = 9.12 + 19.9 + 0$

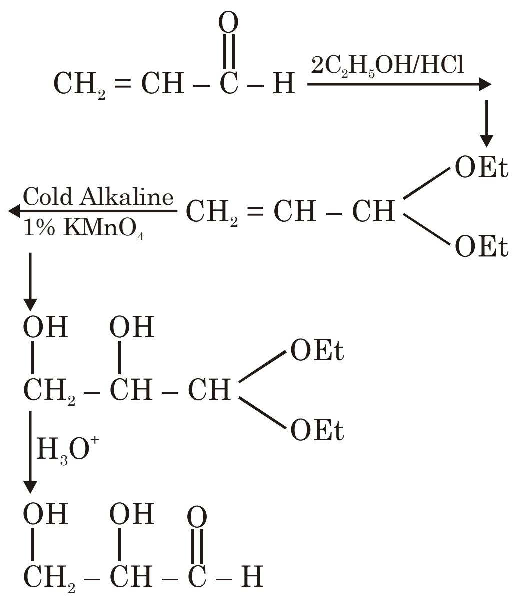
$= 29.02$

7. Ans. (6.00)

Sol.: a, b, c, f, h, i

8. Ans. (56.00)

Sol.:



PART-3 : MATHEMATICS SOLUTION

SECTION-I

1. Ans. (B,C,D)

Sol. $\Delta = \begin{vmatrix} p & q & r \\ q & r & p \\ r & p & q \end{vmatrix} = 0$ (for Non trivial solution)

$-(p + q + r)(p^2 + q^2 + r^2 - pq - qr - rp) = 0$

$p + q + r = 0$

Represents line x = y = z

$p^2 + q^2 + r^2 - pq - qr - rp = 0$

$\frac{1}{2}(p - q)^2 + (q - r)^2 + (r - p)^2 = 0$

p, q, r must be equal p = q = r

$\Rightarrow$ identical planes x + y + z = 0

2. Ans. (B,C)

Sol. $-2 < \log_{\frac{1}{\sqrt{3}}}\left(\frac{|z|^2 - |z| + 1}{2 + |z|}\right) < 2$

$\left(\frac{1}{\sqrt{3}}\right)^2 < \frac{|z|^2 - |z| + 1}{2 + |z|} < \left(\frac{1}{\sqrt{3}}\right)^{-2}$

$\Rightarrow \frac{1}{3} < \frac{|z|^2 - |z| + 1}{2 + |z|} < 3$

Let $|z| = t \geq 0$

$\frac{1}{3} < \frac{t^2 - t + 1}{2 + t}$

$2 + t < 3t^2 - 3t + 3$

$\Rightarrow 3t^2 - 4t + 1 > 0$

$\Rightarrow (t - 1)(3t - 1) > 0$

$t < \frac{1}{3}$ or $t > 1$ ...(i)

$\frac{t^2 - t + 1}{2 + t} < 3$

$\Rightarrow t^2 - 4t - 5 < 0$

$\Rightarrow (t + 1)(t - 5) < 0$

$\Rightarrow -1 < t < 5$ ...(ii)

From (i) and (ii)

$t \in \left[0, \frac{1}{3}\right) \cup (1, 5) \Rightarrow 0 \leq |z| < \frac{1}{3}$ or $1 < |z| < 5$

3. Ans. (A,C)

Sol. $f : R \to R$ $f(x) = (x^2 + \sin x)(x - 1)$

$f(1^+) = f(1^-) = f(1) = 0$

$fg(x) : f(x).g(x)$ $fg : R \to R$

let $fg(x) = h(x) = f(x).g(x)$ $h : R \to R$

option (c) $h'(x) = f'(x)g(x) + f(x)g'(x)$

$h'(1) = f'(1)g(1) + 0,$

(as $f(1) = 0$, $g'(x)$ exists}

$\Rightarrow$ if g(x) is differentiable then h(x) is also differentiable (true)

option (A) If g(x) is continuous at x = 1 then $g(1^+) = g(1^-) = g(1)$

$h'(1^+) = \lim_{h \to 0^+} \frac{h(1 + h) - h(1)}{h}$

$h'(1^+) = \lim_{h \to 0^+} \frac{f(1 + h)g(1 + h) - 0}{h} = f'(1)g(1)$

$h'(1^-) = \lim_{h \to 0^+} \frac{f(1 - h)g(1 - h) - 0}{-h} = f'(1)g(1)$

So h(x) = $f(x).g(x)$ is differentiable at x = 1 (True)

option (B) (D) $h'(1^+) = \lim_{h \to 0^+} \frac{h(1 + h) - h(1)}{-h}$

$h'(1^+) = \lim_{h \to 0^+} \frac{f(1 + h)g(1 + h)}{h} = f'(1)g(1^+)$

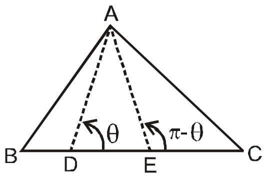
$h'(1^-) = \lim_{h \to 0^+} \frac{f(1 - h)g(1 - h)}{-h} = f'(1).g(1^-)$

$\Rightarrow g(1^+) = g(1^-)$

So we cannot comment on the continuity and differentiability of the function.

4. Ans. (A,C, D)

Sol.



if we apply m-n Rule in Δ ABE, we get

(1+1) cot θ = 1.cot B – 1.cot θ

2 cotθ = cot B – cotθ

3 cotθ = cot B

tan θ = 3 tan B ....(1)

Similarly, if we apply m-n Rule in Δ ACD, we get

(1+1) cot (π – θ) = 1.cotθ – 1.cotC.

cotC = 3 cotθ ⇒ tan θ = 3 tanC ....(2)

form (1) and (2) we can say that

tan B = tan C ⇒ B=C

$\because$ A + B + C = π

$\therefore$ A = π – (B + C)

= π – 2B $\because$ B = C

$\therefore$ tan A = – tan2B

$$= -\left(\frac{2\tan B}{1-\tan^2 B}\right) = -\frac{\frac{2\tan\theta}{3}}{1-\frac{\tan^2\theta}{9}}$$

$$\Rightarrow \tan A = \frac{6\tan\theta}{\tan^2\theta - 9}$$

5. Ans. (A,B,D)

Sol. Characteristics equation for A :

det (A – λI$_3$) = 0

$$\begin{vmatrix} 1-\lambda & 2 & 2 \\ 2 & 1-\lambda & 2 \\ 2 & 2 & 1-\lambda \end{vmatrix} = 0$$

$$\Rightarrow \lambda^3 - 3\lambda^2 - 9\lambda - 5I = 0$$

Characteristics equation

⇒ $A^3 - 3A^2 - 9A - 5I = 0$

To determine inverse :

$(A^3 - 3A^2 - 9A - 5I)A^{-1} = 0$

$A^2 - 3A - 9I - 5A^{-1} = 0$

⇒ $5A^{-1} = A^2 - 3A - 9I$

$$A^{-1} = \frac{1}{5}[A^2 - 3A - 9I]$$

$$A^2 = \begin{bmatrix} 9 & 8 & 8 \\ 8 & 9 & 8 \\ 8 & 8 & 9 \end{bmatrix}$$

[For the matrix to be invertible det $A^2 \neq 0$]

det $A^2 \neq 0$

⇒ A^2 is invertible.

6. Ans. (A,C,D)

Sol. (A) Applying $\int_a^b f(a+b-x)dx$

$$I = \int_a^{\pi-a} (\pi - x)f(\sin(\pi - x))dx = \int_a^{\pi-a} (\pi - x)f(\sin x)dx$$

$$= \pi\int_a^{\pi-a} f(\sin x)dx - \int_a^{\pi-a} xf(\sin x)dx$$

$$I = \pi\int_a^{\pi-a} f(\sin x)dx - I$$

$$\Rightarrow \quad 2I = \pi\int_a^{\pi-a} f(\sin x)dx$$

$$I = \frac{\pi}{2}\int_a^{\pi-a} f(\sin x)dx$$

(B) $\int_{-a}^{a} f^2(x)dx = 2\int_0^a f^2(x)dx$ is true only when f(x) is either even or odd.

(C) $\cos^2 x$ is periodic with fundamental period π

$$\int_0^{n\pi} f(\cos^2 x)dx = n\int_0^{\pi} f(\cos^2 x)dx$$

(D) Put $x + c = t \Rightarrow dx = dt$

$x = 0,\ t = c$

$x = b - c \quad t = b$

$$\int_0^{b-c} f(x+c)dx = \int_c^b f(t)dt = \int_c^b f(x)dx$$

Paragraph for Question 7 and 8

7. **Ans.(A)**

8. **Ans.(C)**

Sol. $f(x) = x^3(x-2)^2(x-1)$

$f'(x) = x^3(x-2)^2 + 2x^3(x-2)(x-1) + 3x^2(x-2)^2(x-1)$

$= x^2(x-2)(x^2 - 2x + 2x^2 - 2x + 3x^2 - 9x + 6)$

$= x^2(x-2)(6x^2 - 13x + 6)$

$f'(x) = x^2(x-2)(2x-3)(3x-2)$

$\Rightarrow$ maxima at $x = \frac{3}{2}$ & minima at $x = \frac{2}{3}, 2$

$\sum x_i^2 = \frac{9}{4}$ (For maxima)

$f(2) = 0$

$f\left(\frac{2}{3}\right) = \frac{8}{27} \times \frac{16}{9} \times \left(-\frac{1}{3}\right) = -\frac{128}{729}$

9. **Ans. (C)**

10. **Ans. (B)**

Sol. Given function f(x) can be rewritten as,

$$f(x) = \begin{cases} 0, & x < -1 \\ 1+x, & -1 \le x \le 0 \\ 1-x, & 0 < x \le 1 \\ 0, & x > 1 \end{cases}$$

$$\Rightarrow \quad f(x-1) = \begin{cases} 0, & x-1 < -1 \\ 1+(x-1), & -1 \le x-1 \le 0 \\ 1-(x-1), & 0 < x-1 \le 1 \\ 0, & x-1 > 1 \end{cases}$$

$$\text{or} \quad f(x-1) = \begin{cases} 0, & x < 0 \\ x, & 0 \le x \le 1 \\ 2-x, & 1 < x \le 2 \\ 0, & x > 2 \end{cases}$$

$$\text{Also} \quad f(x+1) = \begin{cases} 0, & x+1 < -1 \\ 1+(x+1), & -1 \le x+1 \le 0 \\ 1-(x+1), & 0 < x+1 \le 1 \\ 0, & x+1 > 1 \end{cases}$$

$$\text{or} \quad f(x+1) = \begin{cases} 0, & x < -2 \\ 2+x, & -2 \le x < -1 \\ -x, & -1 < x \le 0 \\ 0, & x > 0 \end{cases}$$

Now, $g(x) = f(x-1) + f(x+1)$

$$= \begin{cases} 0, & x < -2 \\ 2+x, & -2 < x \le -1 \\ -x, & -1 < x \le 0 \\ x, & 0 < x \le 1 \\ 2-x, & 1 < x \le 2 \\ 0, & x > 2 \end{cases}$$

It is easy to check that g(x) is continuous for all $x \in R$ and non-differentiable at $x = -2, -1, 0, 1, 2$

SECTION-II

1. **Ans. (27.00)**

Sol.: Let O, $A(\vec{a}), B(\vec{b}), C(\vec{c})$ be the vertices of given tetrahedron

$V = \frac{1}{6}[\vec{a}\ \vec{b}\ \vec{c}]$

The centroid are $\frac{\vec{a}+\vec{b}}{3}, \frac{\vec{b}+\vec{c}}{3}, \frac{\vec{c}+\vec{a}}{3}, \frac{\vec{a}+\vec{b}+\vec{c}}{3}$

$$V' = \frac{1}{6}\left[\frac{\vec{c}-\vec{a}}{3}\ \frac{\vec{c}-\vec{b}}{3}\ \frac{\vec{c}}{3}\right] = \frac{1}{6\times 27}[\vec{a}\ \vec{b}\ \vec{c}] = \frac{1}{27}V$$

$\therefore \quad k = 27$

2. **Ans. (6.82 or 6.83)**

Sol.: $\because$ Equation of circle

$(x-2)^2 + (y+2)^2 + \lambda(x+y) = 0 \quad \ldots(i)$

$\because$ Centre lies on the x-axis

$\therefore$ $\lambda = -4$ put in (i)

$\therefore$ equation of circle is $x^2 + y^2 - 8x + 8 = 0$

(α, β) lies on it

$\Rightarrow \quad \beta^2 = -\alpha^2 + 8\alpha - 8 \ge 0$

$\therefore$ greatest value of 'α' is $4 + 2\sqrt{2}$.

3. Ans. (8.00)

Sol. $\frac{3^{y_1}+3^{y_2}+3^{y_3}}{3} \geq \left[3^{(y_1+y_2+y_3)}\right]^{\frac{1}{3}}$

$\Rightarrow \quad 3^{y_1}+3^{y_2}+3^{y_3} \geq 3^4$

$\Rightarrow \quad \log_3(3^{y_1}+3^{y_2}+3^{y_3}) \geq 4$

$\Rightarrow \quad m = 4$

Also, $\frac{x_1+x_2+x_3}{3} \geq \sqrt[3]{x_1x_2x_3}$

$\Rightarrow \quad x_1x_2x_3 \leq 27$

$\Rightarrow \quad \log_3 x_1 + \log_3 x_2 + \log_3 x_3 \leq 3$

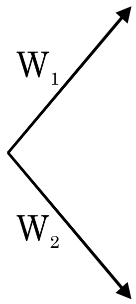
$\Rightarrow \quad M = 3$

Thus, $\log_2(m^3) + \log_3(M^2) = 6 + 2$

$= 8$

4. Ans. (7.00)

Sol.:



W_1: $\frac{1}{2} \xrightarrow{A} \frac{\frac{7!}{2!}+\frac{7!}{2!}-6!}{\frac{8!}{2!\,2!}}$

$= \frac{4(7!-6!)}{8!} = \frac{24.6!}{8!} = \frac{3}{7}$

W_2: $\frac{1}{2} \xrightarrow{A} \frac{7!}{\frac{8!}{2!}} = \frac{2\times 7!}{8!} = \frac{1}{4}$

Required probability $= \frac{\frac{1}{2}\times\frac{3}{7}}{\frac{1}{2}\times\frac{3}{7}+\frac{1}{2}\times\frac{1}{4}}$

$= \frac{12}{19} = \frac{p}{q}$

$\therefore \quad q - p = 7$

5. Ans. (80.00)

Sol.: $\vec{a}_1 = (a, b),\ \vec{a}_2 = \left(a-2, b-\sqrt{5}\right),\ \vec{a}_3 = \left(a+2, b-\sqrt{5}\right)$

Area of Δ, whose vertices are $\vec{a}_1, \vec{a}_2$ and $\vec{a}_3$, is

$$\Delta = \frac{1}{2}\begin{vmatrix} 1 & a & b \\ 1 & a-2 & b-\sqrt{5} \\ 1 & a+2 & b-\sqrt{5} \end{vmatrix}$$

$$\Rightarrow \Delta^2 = \frac{1}{4}\begin{vmatrix} 1 & a & b \\ 1 & a-2 & b-\sqrt{5} \\ 1 & a+2 & b-\sqrt{5} \end{vmatrix}\begin{vmatrix} 1 & a & b \\ 1 & a-2 & b-\sqrt{5} \\ 1 & a+2 & b-\sqrt{5} \end{vmatrix} \text{ and}$$

$|\vec{a}_1 - \vec{a}_2| = 3,\ |\vec{a}_1 - \vec{a}_3| = 3,\ |\vec{a}_2 - \vec{a}_3| = 4$

$\Rightarrow \quad \Delta = \sqrt{5\times2\times2\times1} = 2\sqrt{5}$

$$\left(2\sqrt{5}\right)^2 = \frac{1}{4}\begin{vmatrix} 1+\vec{a}_1\cdot\vec{a}_1 & 1+\vec{a}_1\cdot\vec{a}_2 & 1+\vec{a}_1\cdot\vec{a}_3 \\ 1+\vec{a}_2\cdot\vec{a}_1 & 1+\vec{a}_2\cdot\vec{a}_2 & 1+\vec{a}_2\cdot\vec{a}_3 \\ 1+\vec{a}_3\cdot\vec{a}_1 & 1+\vec{a}_3\cdot\vec{a}_2 & 1+\vec{a}_3\cdot\vec{a}_3 \end{vmatrix}$$

$\therefore$ Required answer = 4 × 5 × 4 = 80

6. Ans. (2.00)

Sol.: AB – BP = 2I, B = adj (B)

Let $B = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$,

$\therefore \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$

$\therefore$ b = c = 0 and a = d

$\therefore$ B = $\begin{bmatrix} a & 0 \\ 0 & a \end{bmatrix}$ = aI

aA – aP = 2I

aA = aP + 2I

Let $P = \begin{bmatrix} \alpha & \beta \\ \gamma & \delta \end{bmatrix}$ where $\begin{matrix} \alpha\delta - \beta\gamma = 3 \\ \alpha + \delta = 4 \end{matrix}$

$\therefore aA = \begin{bmatrix} a\alpha + 2 & a\beta \\ a\gamma & a\delta + 2 \end{bmatrix}$

$\Rightarrow a^2|A| = a^2(\alpha\delta - \beta\gamma) + 2a(\alpha + \delta) + 4$

$\Rightarrow a^2|A| = 3a^2 + 8a + 4$

$|A| = 15 \qquad T_r(B) < 0 \Rightarrow a < 0$

$\therefore 12a^2 - 8a - 4 = 0$

$a = -\frac{1}{3}, 1 \Rightarrow a = 1\ (\because a < 0)$

$\therefore \quad Tr(B) = 2a = -2/3$

7. **Ans. (16.00)**

Sol.: $a_1 = 15$, $a_2 = 15 + d$, $a_3 = 15 + 2d$ ....

$a_{10} = 15 + 9d$

$$\sum_{r=1}^{10} a_r^2 = 15^2 + (15 + d)^2 + ... + (15 + 9d)^2$$

$= 1185$

$15^2 \times 10 + d^2 (1^2 + 2^2 + ... + 9^2)$

$+ 30d (1 + 2 + ... + 9) = 1185$

$$2250 + d^2 \times \frac{9 \times 10 \times 19}{6} + 30d \times 45 = 1185$$

$15 \times 19\ d^2 + 30 \times 45d + 1065 = 0$

$19d^2 + 90d + 71 = 0$

$(19d + 71)(d + 1) = 0$

$\therefore$ $d = -1, \frac{-71}{19}$ (as a_2 is integer) (rejected)

$\therefore$ $S(n) - S(n-1) \geq 0$

$\Rightarrow T_n \geq 0$

$\therefore$ $n = 16$

8. **Ans. 7.00**

$S^2 = (1 + 3x + 5x^2 + ... + 21x^{10})$

$(1 + 3x + 5x^2 + ... + 21x^{10})$

Coeff. of $x^{18} = 21.17 + 19.19 + 17.21 = 1075$