SAMPLE QUESTION PAPER (SOLUTIONS)

CLASS: XII

Session: 2021-22

Mathematics (Code-041)

SECTION – A

1. Put $x^2 = t \Rightarrow 2x\,dx = dt \Rightarrow x dx = \frac{1}{2}\,dt$, **[½]**

$\therefore \int \frac{x}{1-9x^4}dx = \frac{1}{2}\int\frac{dt}{1-9t^2} = \frac{1}{2}\cdot\frac{1}{9}\int\frac{dt}{\left(\frac{1}{3}\right)^2 - t^2}$ **[½]**

$= \frac{1}{18}\cdot\frac{1}{2\times\frac{1}{3}}\log\left|\frac{\frac{1}{3}+t}{\frac{1}{3}-t}\right| + C = \frac{1}{12}\log\left|\frac{1+3t}{1-3t}\right| + C = \frac{1}{12}\log\left|\frac{1+3x^2}{1-3x^2}\right| + C$ **[1]**

OR

$\int e^x\left(\frac{\sin 4x - 4}{1-\cos 4x}\right)dx = \int e^x\left(\frac{2\sin 2x\cos 2x - 4}{2\sin^2 2x}\right)dx$

$= \int e^x\left(\frac{2\sin 2x\cos 2x}{2\sin^2 2x} - \frac{4}{2\sin^2 2x}\right)dx = \int e^x(\cot 2x - 2\,\mathrm{cosec}^2\, 2x)\,dx$ **[½]**

$= \int e^x(f(x) + f'(x))\,dx$ [where $f(x) = \cot 2x \Rightarrow f'(x) = -2\,\mathrm{cosec}^2\,2x$] **[½]**

$= e^x f(x) + C = e^x \cot 2x + C$ **[1]**

2. Order is 2. **[1]**

Squaring both the sides $\left(\frac{dy}{dx}\right)^3 = \left(\frac{d^2y}{dx^2} + x\right)^2$

Hence degree is 2. **[1]**

3. Let $\vec{a} = \hat{i} - 2\hat{j} + 3\hat{k}$ and $\vec{b} = 3\hat{i} - 2\hat{j} + \hat{k}$

$\therefore \cos\theta = \frac{\vec{a}.\vec{b}}{|\vec{a}||\vec{b}|}$ **[½]**

$= \frac{(\hat{i} - 2\hat{j} + 3\hat{k})\,.\,(3\hat{i} - 2\hat{j} + \hat{k})}{\sqrt{(1)^2 + (-2)^2 + (3)^2}\times\sqrt{(3)^2 + (-2)^2 + (1)^2}}$ **[½]**

$= \frac{3+4+3}{\sqrt{14}.\sqrt{14}} = \frac{10}{14} = \frac{5}{7}$ **[½]**

Hence, $\cos\theta = \frac{5}{7}$

$\Rightarrow \sin\theta = \sqrt{1-\cos^2\theta} = \sqrt{1-\frac{25}{49}}$

Hence; $\sin\theta = \sqrt{\frac{24}{49}} = \frac{2}{7}\sqrt{6}$ **[½]**

4. $\overrightarrow{PQ}$ = P.V. of Q – P.V. of P = $(3\hat{i}-5\hat{j}-4\hat{k})-(\hat{i}-2\hat{j}+2\hat{k}) = 2\hat{i}-3\hat{j}-6\hat{k}$ **[½]**

$|\overrightarrow{PQ}| = \sqrt{(2)^2+(-3)^2+(-6)^2} = \sqrt{4+9+36} = 7$ **[½]**

Unit vector in the direction of $\overrightarrow{PQ} = \frac{1}{7}(2\hat{i}-3\hat{j}-6\hat{k}) = \frac{2}{7}\hat{i}-\frac{3}{7}\hat{j}-\frac{6}{7}\hat{k}$ **[½]**

Direction cosines of the ray from P to Q are $\left(\frac{2}{7},\frac{-3}{7},\frac{-6}{7}\right)$. **[½]**

5. Let X denotes no. of heads in simultaneous toss of two coins, then the possible values of X are 0, 1 or 2. **[½]**

X	P(X)
0	$\frac{1}{4}$
1	$\frac{2}{4}$
2	$\frac{1}{4}$

[1½]

6. Let E_1 : select first purse and E_2 : select second purse

A : silver coin is drawn

$\Rightarrow$ $P(E_1) = \frac{1}{2}$, $P(E_2) = \frac{1}{2}$, $P(A/E_1) = \frac{3}{9}$, $P(A/E_2) = \frac{4}{7}$ **[½]**

$\therefore$ $P(A) = P(E_1)\,P(A/E_1) + P(E_2).P(A/E_2)$ (by total probability theorem)

$= \left(\frac{1}{2}\times\frac{3}{9}\right)+\left(\frac{1}{2}\times\frac{4}{7}\right)$ **[1]**

or $P(A) = \frac{1}{6}+\frac{2}{7} = \frac{19}{42}$ **[½]**

SECTION – B

7. Let $I = \int \frac{x^3}{(x-1)(x^2+1)}dx$

$$\frac{x^3}{(x-1)(x^2+1)} = 1+\frac{x^2-x+1}{(x-1)(x^2+1)}$$

Let $\frac{x^2-x+1}{(x-1)(x^2+1)} = \frac{A}{(x-1)}+\frac{Bx+C}{x^2+1}$ **[½]**

$x^2 - x + 1 = A(x^2+1) + (x-1)(Bx+C)$ .....(i)

Using (i); we have :

When $x = 1$, $A = \frac{1}{2}$

and when x = 0, $1 = A - C \Rightarrow C = A - 1 = -\frac{1}{2}$

On equating Coff. of x^2; $1 = A + B \Rightarrow B = 1 - A = \frac{1}{2}$ **[1]**

$\therefore \quad \frac{x^2 - x + 1}{(x-1)(x^2+1)} = \frac{1}{2}\left(\frac{1}{x-1}\right) + \frac{1}{2}\left(\frac{x-1}{x^2+1}\right)$ **[½]**

$I = \int\left[1 + \frac{1}{2}\left(\frac{1}{x-1}\right) + \frac{1}{2}\left(\frac{x-1}{x^2+1}\right)\right]dx$ **[1]**

$= \int\left[1 + \frac{1}{2}\left(\frac{1}{x-1}\right) + \frac{1}{2}\times\frac{1}{2}\left(\frac{2x}{x^2+1}\right) - \frac{1}{2}\left(\frac{1}{x^2+1}\right)\right]dx$

$= x + \frac{1}{2}\log(x-1) + \frac{1}{4}\log(x^2+1) - \frac{1}{2}\tan^{-1}x + C$

$= x + \frac{1}{4}\log(x-1)^2(x^2+1) - \frac{1}{2}\tan^{-1}x + C$ **[1]**

8. $x\frac{dy}{dx} = y - x\tan(y/x)$

$\Rightarrow \quad \frac{dy}{dx} = \left(\frac{y}{x}\right) - \tan\left(\frac{y}{x}\right)$ **[½]**

The given equation is homogenous differential equation.

Put $\frac{y}{x} = v$ i.e. $y = vx \Rightarrow dy/dx = v + x\frac{dv}{dx}$ **[½]**

$\therefore \quad v + x\frac{dv}{dx} = v - \tan v$

$\Rightarrow \quad x\frac{dv}{dx} = -\tan v$

$\Rightarrow \quad \frac{dv}{\tan v} = -\frac{dx}{x}$ **[½]**

Integrating both sides; we get :

$\int\frac{dv}{\tan v} = \int\frac{-dx}{x}$

$\Rightarrow \quad \int\cot v\, dv = -\int dx/x$

$\Rightarrow \quad \log|\sin v| = -\log|x| + \log C$

$\Rightarrow \quad \log|\sin v| = \log\left|\frac{C}{x}\right|$ **[½]**

$\Rightarrow \quad \sin\left(\frac{y}{x}\right) = C/x$

or $\quad x\sin\left(\frac{y}{x}\right) = C$ **[1]**

OR

The given differential equation is $\frac{dy}{dx}+2y\tan x=\sin x$

This is a linear equation of the form $\frac{dy}{dx}+Py=Q$

where $P=2\tan x$ and $Q=\sin x$

Now, I.F. = $e^{\int Pdx}=e^{\int 2\tan x dx}=e^{2\log|\sec x|}=e^{\log|\sec^2 x|}=\sec^2 x.$ **[1]**

The general solution of the given differential equation is,

$$y(\text{I.F.})=\int(Q\times \text{I.F.})dx+C$$

$\Rightarrow y(\sec^2 x)=\int(\sin x.\sec^2 x)dx+C$ **[½]**

$\Rightarrow y\sec^2 x=\int(\sec x.\tan x)dx+C$

$\Rightarrow y\sec^2 x=\sec x+C$ ....(1)

Now, $y=0,\ x=\frac{\pi}{3}$

Therefore, $0\times\sec^2\frac{\pi}{3}=\sec\frac{\pi}{3}+C$ **[½]**

$\Rightarrow$ $0 = 2 + C$

$\Rightarrow$ $C = -2$

Substituting $C = -2$ in equation (1), we get :

$y\sec^2x = \sec x - 2$

$\Rightarrow$ $y = \cos x - 2\cos^2 x$ **[1]**

Hence, the required solution of the given differential equation is $y = \cos x - 2\cos^2 x$

9. $\vec{a}=\hat{i}+\hat{j}+\hat{k}$, $\vec{b}=2\hat{i}+4\hat{j}-5\hat{k}$ and $\vec{c}=\lambda\hat{i}+2\hat{j}+3\hat{k}$

$\Rightarrow$ $\vec{b}+\vec{c}=\hat{i}(2+\lambda)+6\hat{j}-2\hat{k}=\vec{d}$ (let)

Unit vector along $\vec{d}$ = $\frac{\hat{i}(2+\lambda)+6\hat{j}-2\hat{k}}{\sqrt{(2+\lambda)^2+6^2+(-2)^2}}=\hat{d}$ **[1]**

$\Rightarrow$ $(\hat{i}+\hat{j}+\hat{k}).\left(\frac{\hat{i}(2+\lambda)+6\hat{j}-2\hat{k}}{\sqrt{(2+\lambda)^2+6^2+(-2)^2}}\right)=1$ $\because$ $\vec{a}\ .\ \hat{d}=1$ [½]

$\Rightarrow$ $(2+\lambda)+6-2=\sqrt{(2+\lambda)^2+36+4}$ or $6+\lambda=\sqrt{4+4\lambda+\lambda^2+40}$

$\Rightarrow$ $36+\lambda^2+12\lambda=44+4\lambda+\lambda^2$ or $8\lambda=8$

$\Rightarrow$ $\lambda = 1$ [½]

$\therefore$ Unit vector along $\vec{b}+\vec{c}$

$$=\frac{\hat{i}(2+\lambda)+6\hat{j}-2\hat{k}}{\sqrt{(2+\lambda)^2+6^2+(-2)^2}}=\frac{3\hat{i}+6\hat{j}-2\hat{k}}{\sqrt{9+36+4}}=\frac{3\hat{i}+6\hat{j}-2\hat{k}}{7}$$ **[1]**

10. We known that the shortest distance between the lines $\vec{r}=\vec{a}_1+\lambda\vec{b}_1$ and $\vec{r}=\vec{a}_2+\mu\vec{b}_2$

is given by $d=\left|\dfrac{(\vec{a}_2-\vec{a}_1).(\vec{b}_1\times\vec{b}_2)}{|\vec{b}_1\times\vec{b}_2|}\right|$ **[½]**

Comparing the given equation with the equations $\vec{r}=\vec{a}_1+\lambda\vec{b}_1$ and $r=\vec{a}_2+\mu\vec{b}_2$

respectively, $\vec{a}_1=4\hat{i}-\hat{j}$, $\vec{a}_2=\hat{i}-\hat{j}+2\hat{k}$, $\vec{b}_1=\hat{i}+2\hat{j}-3\hat{k}$ and $\vec{b}_2=2\hat{i}+4\hat{j}-5\hat{k}$

Now, $\vec{a}_2-\vec{a}_1=-3\hat{i}+0\hat{j}+2\hat{k}$ and $\vec{b}_1\times\vec{b}_2=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 1 & 2 & -3\\ 2 & 4 & -5\end{vmatrix}=2\hat{i}-\hat{j}+0\hat{k}$ **[½]**

$\therefore$ $(\vec{a}_2-\vec{a}_1).(\vec{b}_1\times\vec{b}_2)=(-3\hat{i}+0\hat{j}+2\hat{k}).(2\hat{i}-\hat{j}+0\hat{k})=-6$ and $|\vec{b}_1\times\vec{b}_2|=\sqrt{4+1+0}=\sqrt{5}$

[1]

$\therefore$ Shortest distance $d=\left|\dfrac{(\vec{a}_2-\vec{a}_1).(\vec{b}_1\times\vec{b}_2)}{|\vec{b}_1\times\vec{b}_2|}\right|=\left|\dfrac{-6}{\sqrt{5}}\right|=\dfrac{6}{\sqrt{5}}$ units. **[1]**

OR

The equation of the given line is $\vec{r}=2\hat{i}-\hat{j}+2\hat{k}+\lambda\left(3\hat{i}+4\hat{j}+2\hat{k}\right)$ ....(1)

The equation of the given plane is $\vec{r}.\left(\hat{i}-\hat{j}+\hat{k}\right)=5$ ....(2)

Substituting the value of $\vec{r}$ from equation (1) in equation (2), we obtain

$\left[2\hat{i}-\hat{j}+2\hat{k}+\lambda\left(3\hat{i}+4\hat{j}+2\hat{k}\right)\right].\left(\hat{i}-\hat{j}+\hat{k}\right)=5$ **[1]**

$\Rightarrow$ $\left[(3\lambda+2)\hat{i}+(4\lambda-1)\hat{j}+(2\lambda+2)\hat{k}\right].\left(\hat{i}-\hat{j}+\hat{k}\right)=5$

$\Rightarrow$ $(3\lambda+2)-(4\lambda-1)+(2\lambda+2)=5$

$\Rightarrow$ $\lambda=0$ **[½]**

Substituting this value in equation (1), we obtain the equation of the line as $\vec{r}=\left(2\hat{i}-\hat{j}+2\hat{k}\right)$

The point of intersection of the line and the plane is (2,–1,2) **[½]**

The distance d between the points (2, –1, 2) and (–1, –5, –10), is

$d=\sqrt{(-1-2)^2+(-5+1)^2+(-10-2)^2}=\sqrt{9+16+144}=\sqrt{169}=13$ units **[1]**

SECTION – C

11. We note that $x^3 - x \geq 0$ on $[-1, 0]$; $x^3 - x \leq 0$ on $[0, 1]$ and $x^3 - x \geq 0$ on $[1, 2]$.

So by property of definite integrals, we get

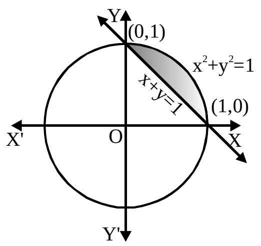
$$\int_{-1}^{2} |x^3 - x|\,dx = \int_{-1}^{0} (x^3 - x)\,dx + \int_{0}^{1} -(x^3 - x)\,dx + \int_{1}^{2} (x^3 - x)\,dx$$

$$= \int_{-1}^{0} (x^3 - x)\,dx + \int_{0}^{1} (x - x^3)\,dx + \int_{1}^{2} (x^3 - x)\,dx \quad \textbf{[1]}$$

$$= \left[\frac{x^4}{4} - \frac{x^2}{2}\right]_{-1}^{0} + \left[\frac{x^2}{2} - \frac{x^4}{4}\right]_{0}^{1} + \left[\frac{x^4}{4} - \frac{x^2}{2}\right]_{1}^{2} \quad \textbf{[1]}$$

$$= -\left(\frac{1}{4} - \frac{1}{2}\right) + \left(\frac{1}{2} - \frac{1}{4}\right) + (4 - 2) - \left(\frac{1}{4} - \frac{1}{2}\right) \quad \textbf{[1]}$$

$$= -\frac{1}{4} + \frac{1}{2} + \frac{1}{2} - \frac{1}{4} + 2 - \frac{1}{4} + \frac{1}{2} = \frac{3}{2} - \frac{3}{4} + 2 = \frac{11}{4} \quad \textbf{[1]}$$

12. 

Required area lies in the first quadrant within the circle $x^2 + y^2 = 1$ and above the line $x + y = 1$.
Let us first sketch the region whose area is to be found out.
This region is the intersection of the following regions $\{(x, y) ; x^2 + y^2 \leq 1\}$ **[1]**
and $\{(x, y) : x + y \geq 1\}$

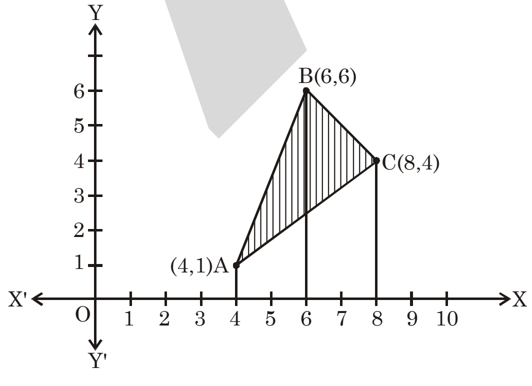
Required Area $= \int_{0}^{1} \sqrt{1 - x^2}\,dx - \int_{0}^{1} (1 - x)\,dx$ **[1]**

$$= \left[\frac{x}{2}\sqrt{1 - x^2} + \frac{1}{2}\sin^{-1} x\right]_{0}^{1} - \left[x - \frac{x^2}{2}\right]_{0}^{1} \quad \textbf{[1]}$$

$$= \left(\frac{\pi}{4} - 0\right) - \left(\frac{1}{2} - 0\right)$$

$$= \left(\frac{\pi}{4} - \frac{1}{2}\right) \text{ square units.} \quad \textbf{[1]}$$

OR



[1]

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Equation of line AB $\Rightarrow$ $y-1=\frac{5}{2}(x-4)$

$\Rightarrow$ $5x-2y-18=0$

$\Rightarrow$ $y=\frac{1}{2}(5x-18)$

Equation of line BC $\Rightarrow$ $y-6=(-1)(x-6)$

$\Rightarrow$ $y=12-x$

Equation of line AC $\Rightarrow$ $y-1=\frac{3}{4}(x-4)$

$\Rightarrow$ $y=\frac{1}{4}(3x-8)$

Area of $\Delta=\left[\int_4^6(\text{line AB})dx+\int_6^8(\text{line BC})dx\right]-\int_4^8(\text{line AC})\,dx$ **[1]**

$=\left[\frac{1}{2}\int_4^6(5x-18)dx+\int_6^8(12-x)dx\right]-\frac{1}{4}\int_4^8(3x-8)dx$ **[½]**

$=\left[\frac{1}{2}\left(\frac{5x^2}{2}-18x\right)_4^6+\left(12x-\frac{x^2}{2}\right)_6^8\right]-\frac{1}{4}\left(\frac{3x^2}{2}-8x\right)_4^8$ **[1]**

$=\frac{1}{2}\left(5\times\frac{6^2}{2}-18\times6-5\times\frac{4^2}{2}+18\times4\right)+\left(12\times8-\frac{8^2}{2}-12\times6+\frac{6^2}{2}\right)$

$-\frac{1}{4}\left[3\times\frac{8^2}{2}-8\times8-3\times\frac{4^2}{2}+8\times4\right]$

$=\frac{1}{2}[90-108-40+72]+[96-32-72+18]-\frac{1}{4}[96-64-24+32]$

$=\left(\frac{1}{2}\times14\right)+10-\left(\frac{1}{4}\times40\right)$ = 7 + 10 – 10 = 7 sq. units **[½]**

13. The given points whose position vectors are given are (1, 1, –2), (2, –1, 1), (1, 2, 1)

The equation of a plane passing through above three non-collinear points is

$$\begin{vmatrix}x-1 & y-1 & z+2\\ 2-1 & -1-1 & 1+2\\ 1-1 & 2-1 & 1+2\end{vmatrix}=0$$

$$\Rightarrow\quad\begin{vmatrix}x-1 & y-1 & z+2\\ 1 & -2 & 3\\ 0 & 1 & 3\end{vmatrix}=0$$

$\Rightarrow$ $(x-1)(-9)-(y-1)(3)+(z+2)(1)=0$

$\Rightarrow$ $9x+3y-z=14$ .....(1) **[1]**

and vector equation is $\vec{r}.(9\hat{i}+3\hat{j}-\hat{k})=14$ .....(2) **[½]**

The equation of a plane parallel to the plane (1) is $9x+3y-z=\lambda$ **[½]**

Since, this plane passes through the point (2, 3, 7);

$\Rightarrow \quad (9 \times 2) + (3 \times 3) - 7 = \lambda \Rightarrow \lambda = 20$

Hence, the plane is $9x + 3y - z = 20$ .....(2) **[1]**

Now, the distance between two parallel planes :

$9x + 3y - z = 14$ and $9x + 3y - z = 20$ is

$$= \left|\frac{20-14}{\sqrt{9^2+3^2+(-1)^2}}\right| = \frac{6}{\sqrt{91}} \text{ units}$$ **[1]**

CASE BASED / DATA BASED

14.(i) Let E_1 = Covid person

E_2 – Not covid person

A = Covid PCR Test result is positive

$P(E_1)$ = Probability that person selected has covid = 0.1% = 0.001

$P(E_2)$ = Probability that person selected does not have covid

$1 - P(E_1) = 1 - 0.001 = 0.999$

$P(A/E_1)$ = Probability that the test judges covid +ve, if person actually has covid

= 90% = 0.9

$P(A/E_2)$ = Probability that the test judges covid +ve if the person does not have covid

= 1% = 0.01

$$P(E_1/A) = \frac{P(E_1).P(A/E_1)}{P(E_1)\ P(A/E_1) + P(E_2).P(A/E_2)}$$

$$= \frac{0.001 \times 0.9}{0.001 \times 0.9 + 0.999 \times 0.01}$$

$$= \frac{90}{90+999}$$ **[2]**

$$= \frac{90}{1089}$$

$P(E_1/A) = 0.083$

(ii) P(Person selected will be diagnosed as covid +ve)

$= P(E_1) \times P(A/E_1) + P(E_2) \times P(A/E_2)$

$= 0.001 \times 0.9 + 0.999 \times 0.01$

$$= \frac{90}{100000} + \frac{999}{100000}$$

$$= \frac{1089}{100000} = 0.01089$$ **[2]**