HIGHLY RECOMMENDED
RAPID REVISION MOCK PAPERS

ALLEN®
CAREER INSTITUTE
KOTA (RAJASTHAN)
path to success

MADHYA PRADESH

INDORE | UJJAIN | BHOPAL | GWALIOR

ALLEN CELEBRATING 33 YEARS OF TRUST

Founded on 18 April 1988

191000+
Students Studying
Online & Classroom Courses
(2021-2022)

Solution of BOARD MOCK TEST PAPER

Class-XII

(ACADEMIC SESSION 2021-2022)

TARGET

MP BOARD EXAMINATION

SUBJECTS

MATHEMATICS

125+
Total Classroom
Campus

2200000+
Trusted & Chosen
by Students across all Modes
(since 1988)

40
Total Study
Centers

YOUR SMILE IS OUR HAPPINESS
ALLEN

600000 +
DLP STUDENTS HAVE TRUSTED ALLEN
(Since 1997)

148+
Total Test
Centers

ANSWER KEY

Mathematics

1. $\int \frac{dx}{\sqrt{4-9x^2}} = \frac{1}{3}\int \frac{dx}{\sqrt{\left(\frac{2}{3}\right)^2 - x^2}}$

Using the formula $\int \frac{dx}{\sqrt{a^2-x^2}} = \sin^{-1}\frac{x}{a} + c$

We have

$I = \int \frac{dx}{\sqrt{4-9x^2}}$

$= \frac{1}{3}\sin^{-1}\frac{x}{\frac{2}{3}} + C$

$= = \frac{1}{3}\sin^{-1}\frac{3x}{2} + C$

OR

$\int \frac{1-\sin 2x}{x+\cos^2 x}dx$

Let $x + \cos^2 x = t$

$\Rightarrow \frac{d}{dx}(x+\cos^2 x) = \frac{d}{dx}(t)$

$\Rightarrow 1 + 2\cos x(-\sin x) = \frac{dt}{dx}$

$\Rightarrow (1-\sin 2x)dx = dt$

$\therefore \quad I = \int \frac{1-\sin 2x}{x+\cos^2 x}dx = \int \frac{dt}{t} = \log t + C$

$= \log(x + \cos^2 x) + C$

2. The order and degree of differential equation $\left(\frac{d^2y}{dx^2}\right)^3 + 4\left(\frac{dy}{dx}\right)^7 + 5 = 0$ is 2 and 3 respectively

3. $\vec{a} + \vec{b} =$

$\left(2\hat{i} + 4\hat{j} - 5\hat{k}\right) + \left(\hat{i} + 2\hat{j} + 3\hat{k}\right) = 3\hat{i} + 6\hat{j} - 2\hat{k}$

$\Rightarrow \left|\vec{a} + \vec{b}\right| = \sqrt{(3)^2 + (-2)^2 + (6)^2}$

$= \sqrt{49} = 7$

4. Let a = 3, b = –2, c = 6

then $r = \sqrt{a^2+b^2+c^2} = \sqrt{(3)^2+(-2)^2+(6)^2}$

$= \sqrt{9+4+36} = \sqrt{49} = 7$

Hence direction cosines are

$l = \frac{a}{r} = \frac{3}{7}, \ m = \frac{b}{r} = \frac{-2}{7}, \ n = \frac{c}{r} = \frac{6}{7}$

5. P(exactly one of A or B)

$= P(A) \times P\left(\overline{B}\right) + P\left(\overline{A}\right) \times P(B)$

$= \frac{1}{3} \times \frac{3}{4} + \frac{2}{3} \times \frac{1}{4} = \frac{1}{4} + \frac{1}{6} = \frac{3+2}{12} = \frac{5}{12}$

$\Rightarrow P(A \text{ or } B) = \frac{5}{12}$

6. If two odice are thrown simultaneously then there are total 36 cases in which following are favourable

Sum 4 (1, 3), (2, 2), (3, 1)

8 (2, 6), (3, 5), (4, 4), (5, 3), (6, 2)

12 (6, 6)

Total 9 cases favourable out of 36

Hence probability $= \frac{9}{36} = \frac{1}{4}$

7. $I = \int x\tan^{-1}x\,dx$ using integration by parts taking $\tan^{-1}x$ as I function we have

$I = \tan^{-1}x.\frac{x^2}{2} - \int \frac{1}{1+x^2} \times \frac{x^2}{2}dx$

$= \frac{x^2}{2}.\tan^{-1}x - \frac{1}{2}\int \frac{1+x^2-1}{1+x^2}dx$

$= \frac{x^2}{2}.\tan^{-1}x - \frac{1}{2}\int 1 - \frac{1}{1+x^2}dx$

$\frac{x^2}{2}.\tan^{-1}x - \frac{1}{2}x + \frac{1}{2}\tan^{-1}x + C$

8. Find the general solution of $2x^2dy = (x^2 + y^2)\,dx$

$$\Rightarrow 2\frac{dy}{dx} = \frac{x^2+y^2}{x^2}$$

$$\Rightarrow 2\frac{dy}{dx} = 1+\left(\frac{y}{x}\right)^2 \quad (1)$$

Let $\frac{y}{x} = v \Rightarrow y = vx$

$$\Rightarrow \frac{dy}{dx} = v.1 + x\frac{dy}{dx} \quad (2)$$

Substituting value of (2) in (1), we get -

$$2\left(v + x\frac{dv}{dx}\right) = 1 + v^2$$

$$\Rightarrow 2v + 2x\frac{dv}{dx} = 1 + v^2$$

$$\Rightarrow 2x\frac{dv}{dx} = v^2 - 2v + 1$$

$$\Rightarrow 2\frac{dv}{dx} = \frac{(v-1)^2}{x}$$

$$\Rightarrow \frac{2}{(v-1)^2}dv = \frac{dx}{x}$$

Integrating both the sides,

$$2\int\frac{1}{(v-1)^2}dv = \int\frac{dx}{x}$$

$$2\frac{(v-1)^{-2+1}}{-2+1} = \log x + c$$

$$\frac{-2}{v-1} = \log x + c$$

$$\frac{-2}{\frac{y}{x}-1} = \log x + c$$

$-2x = (y - x)(\log x + c)$

$\Rightarrow 2x = (x - y)(\log x + c)$

OR

$$(x-1)\frac{dy}{dx} = 2xy \Rightarrow \frac{dy}{y} = \frac{2x}{x-1}dx$$

$$\Rightarrow \frac{dy}{y} = \left(2 + \frac{2}{x-1}\right)dx$$

Integrating both sides, we get

$\log y = 2x + 2\log(x - 1) + c$

When $x = 2, y = 1$

$\Rightarrow \log 1 = 2\times 2 + 2\log(2-1) + c$

$\Rightarrow 0 = 4 + 0 + c$

$\Rightarrow c = -4$

Hence particular solution is

$$\log y - \log(x-1)^2 = 2x - 4$$

$$\log\left(\frac{y}{(x-1)^2}\right) = 2x - 4$$

$$\Rightarrow \frac{y}{(x-1)^2} = e^{(2x-4)}$$

$$\Rightarrow y = (x-1)^2 e^{2(x-2)}$$

9. We know that

$$\sin B = \left|\frac{\overline{BA}\times\overline{BC}}{|\overline{BA}|\times|\overline{BC}|}\right|$$

$$|\overline{BA}| = \sqrt{(4)^2 + (-1)^2 + (3)^2} = \sqrt{26}$$

$$|\overline{BC}| = \sqrt{(2)^2 + (3)^2 + (-1)^2} = \sqrt{14}$$

$$\overline{BA}\times\overline{BC} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 4 & -1 & 3 \\ 2 & 3 & -1 \end{vmatrix} = -8\hat{i} + 10\hat{j} + 14\hat{k}$$

$$|\overline{BA}\times\overline{BC}| = \sqrt{64+100+196} = \sqrt{360} = 6\sqrt{10}$$

$$\text{Hence, } \sin B = \frac{6\sqrt{10}}{\sqrt{26}\sqrt{14}} = \frac{3\sqrt{10}}{\sqrt{91}}$$

10. Converting given cartesian equation of lines into vector form, we have

$$\frac{x-3}{3} = \frac{y-8}{-1} = \frac{z-3}{1} = \lambda$$

$$\Rightarrow \vec{r} = \left(3\hat{i} + 8\hat{j} + 3\hat{k}\right) + \lambda\left(3\hat{i} - \hat{j} + \hat{k}\right)$$

and

$$\frac{x+3}{-3}=\frac{y+7}{2}=\frac{z-6}{4}=\mu$$

$$\Rightarrow \vec{r}=\left(-3\hat{i}-7\hat{j}+6\hat{k}\right)+\mu\left(-3\hat{i}+2\hat{j}+4\hat{k}\right)$$

Hence

$\overline{a_1}=3\hat{i}+8\hat{j}+3\hat{k}$

$\overline{b_1}=3\hat{i}-\hat{j}+\hat{k}$

$\overline{a_2}=-3\hat{i}-7\hat{j}+6\hat{k}$

$\overline{b_2}=-3\hat{i}+2\hat{j}+4\hat{k}$

$$\text{S.d.}=\left|\frac{\left(\overline{a_2}-\overline{a_1}\right)\left(\overline{b_1}\times\overline{b_2}\right)}{\left|\overline{b_1}\times\overline{b_2}\right|}\right|$$

$$\text{Here } \overline{b_1}\times\overline{b_2}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 3 & -1 & 1\\ -3 & 2 & 4\end{vmatrix}$$

$=-6\hat{i}-15\hat{j}+3\hat{k}$

$\left|\overline{b_1}\times\overline{b_2}\right|=\sqrt{36+225+9}=\sqrt{270}=3\sqrt{30}$

$\overline{a_2}-\overline{a_1}=-6\hat{i}-15\hat{j}+3\hat{k}$

$\left(\overline{a_2}-\overline{a_1}\right).\left(\overline{b_1}\times\overline{b_2}\right)=36+225+9=270$

$$\text{Hence, S.D.}=\left|\frac{270}{\sqrt{270}}\right|=\sqrt{270}=3\sqrt{30}$$

OR

We know that equation of any plane passing through intersection of plane 2x - y = 0 and y - 3z = 0 is

$\left(2x-y\right)+\lambda(y-3z)=0$

$\Rightarrow 2x+\left(\lambda-1\right)y-3\lambda z=0$

But it is perpendicular to 4x + 5y - 3z - 8 = 0

$\Rightarrow 4\times2+5\left(\lambda-1\right)+\left(-3\right)\left(-3\lambda\right)=0$

$\Rightarrow 8+5\lambda-5+9\lambda=0$

$\Rightarrow 14\lambda=-3\Rightarrow\lambda=\frac{-3}{14}$

Hence required equation of plane is

$\left(2x-y\right)-\frac{3}{14}\left(y-3z\right)=0$

$\Rightarrow 28x-14y-3y+9z=0$

$\Rightarrow 28x-17y+9z=0$

11. $I=\int_0^1 \tan^{-1}\frac{2x}{1-x^2}dx$

Let $x=\tan\theta$

$dx=\sec^2\theta d\theta$

When x = 0 $\Rightarrow \tan\theta=0\ \Rightarrow\theta=0$

$x=1\ \Rightarrow\tan\theta=1\ \Rightarrow\theta=\frac{\pi}{4}$

$\frac{2x}{1-x^2}=\frac{2\tan\theta}{1-\tan^2\theta}=\tan2\theta$

$\therefore\ I=\int_0^1 \tan^{-1}\frac{2x}{1-x^2}dx=\int_0^{\frac{\pi}{4}}2\theta\sec^2\theta d\theta$

$=2\left[\theta\tan\theta-\int 1.\tan\theta d\theta\right]_0^{\frac{\pi}{4}}$

$=2\left[\theta\tan\theta-\log\left(\sec\theta\right)\right]_0^{\frac{\pi}{4}}$

$$=2\left\{\left[\frac{\pi}{4}.\tan\frac{\pi}{4}-\log\left(\sec\frac{\pi}{4}\right)\right]-\left[0.\tan0-\log(\sec0)\right]\right\}$$

$$=2\left\{\frac{\pi}{4}-\log\sqrt{2}\right\}=\frac{\pi}{2}-2\log\sqrt{2}=\frac{\pi}{2}-\log2$$

12. $y^2=4ax$ and $x^2=4ay$ have the point of intersection

$\Rightarrow y^4=16a^2x^2\Rightarrow y^4=16a^2.4ay$

$\Rightarrow y\left(y^3-\left(4a\right)^3\right)=0$

$\Rightarrow y=0$ and $y=4a$

$\Rightarrow x=0$ and $x=4a$

Area between the curves is

$$\int_0^{4a}\left(\sqrt{4ax}-\frac{x^2}{4a}\right)dx=\left[2\sqrt{a}\,\frac{x^{\frac{3}{2}}}{\frac{3}{2}}-\frac{x^3}{3\times 4a}\right]_0^{4a}$$

$$=\frac{2\sqrt{a}.2}{3}(4a)^{\frac{3}{2}}-\frac{(4a)^3}{12a}$$

$$=\frac{32}{3}a^2-\frac{16}{3}a^2=\frac{16}{3}a^2$$

OR

Point of intersection of curve $x^2 = 4y$ and line x =4y - 2 is obtained by

$x^2 = x + 2$

$\Rightarrow x^2 - x + 2 = 0 \Rightarrow (x-2)(x+1) = 0$

$\Rightarrow$ x = -1 or x = 2

Hence required area is

$$\int_{-1}^{2}\left(\frac{x+2}{4}\right)-\left(\frac{x^2}{4}\right)dx$$

$$\frac{1}{4}\left[\frac{x^2}{2}+2x-\frac{x^3}{3}\right]_{-1}^{2}$$

$$\frac{1}{4}\left[\left\{\frac{4}{2}+4-\frac{8}{3}\right\}-\left\{\frac{1}{2}-2+\frac{1}{3}\right\}\right]$$

$$\frac{1}{4}\left[\left(6-\frac{8}{3}\right)-\left(\frac{-7}{6}\right)\right]$$

$$\frac{1}{4}\left[\frac{10}{3}+\frac{7}{6}\right]=\frac{1}{4}\left[\frac{20+7}{6}\right]=\frac{27}{24}=1\frac{1}{8}$$

13. Let the equation of required plane be

ax + by + cz +d = 0

Since it passes through A(4, 5, -1), B (3, -9, 4), c(1, 0, 1) we have

4a + 5b - c + d = 0, 3a - 9b + 4c + d = 0

a + c + d = 0, d = -a - c

$\Rightarrow$ 4a + 5b - c -a - c = 0,

3a - 9b + 4c - a - c = 0

$\Rightarrow$ 3a + 5b -2c = 0,

2a - 9b + 3c = 0

$$\Rightarrow \frac{a}{15-18}=\frac{b}{-4-9}=\frac{c}{-27-10}=k$$

$\Rightarrow$ a = -3k, b = -13k, c = -37 k

$\Rightarrow$ d = 3k + 37k = 40k

Hence equation of plane is

(-3k)x +(-13k)y +(-37k)z + 40k = 0

$\Rightarrow$ 3x + 13y + 37z - 40 = 0

14. (i) P(Both student selected) = P(A and B)

= P(A) × P(B)

$$=\frac{3}{5}\times\frac{4}{5}=\frac{12}{25}$$

(ii) P(any one of them)

= P (A but not B) + (B but not A)

$$= P(A)\times P\left(\overline{B}\right)+P(B)\times P\left(\overline{A}\right)$$

$$=\frac{3}{5}\times\frac{1}{5}+\frac{4}{5}\times\frac{2}{5}$$

$$=\frac{3}{25}+\frac{8}{25}=\frac{11}{25}$$