

# माध्यमिक शिक्षा बोर्ड राजस्थान, अजमेर

विषय – गणित

कक्षा-XII

## SOLUTIONS

(खण्ड-अ)

1.

(i) (B)

$$g \circ f(x) = g[f(x)]$$

$$\therefore g[f(1)] = g(2) = 3$$

$$g[f(3)] = g(5) = 1$$

$$\text{तथा } g[f(4)] = g(1) = 3$$

$$\text{अतः } g \circ f \text{ का परिसर} = \{1, 3\}$$

(ii) (A)

$$\sin^{-1} \left( \sin \frac{2\pi}{3} \right)$$

$$= \sin^{-1} \left[ \sin \left( \pi - \frac{\pi}{3} \right) \right] \because \left[ -\frac{\pi}{2}, \frac{\pi}{2} \right]$$

$$= \frac{\pi}{3}$$

(iii) (C)

$$\text{दिया है } A = \begin{bmatrix} \alpha & \beta \\ \gamma & -\alpha \end{bmatrix}; A^2 = I$$

$$\text{अब } \begin{bmatrix} \alpha & \beta \\ \gamma & -\alpha \end{bmatrix} \begin{bmatrix} \alpha & \beta \\ \gamma & -\alpha \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} \alpha^2 + \beta\gamma & 0 \\ 0 & \alpha^2 + \beta\gamma \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\Rightarrow \alpha^2 + \beta\gamma = 1 \text{ or } 1 - \alpha^2 - \beta\gamma = 0$$

(iv) (A)

$$\text{दिया है, } \begin{vmatrix} 1+a & 1 & 1 \\ 1 & 1+a & 1 \\ 1 & 1 & 1+a \end{vmatrix} = 0$$

$R_1$  के सापेक्ष विस्तार

$$(1+a)[(1+a)^2 - (1)] - 1((1+a) - 1) + 1(1 - 1 - a) = 0$$

$$(1+a)(1+a^2 + 2a - 1) - a - a = 0$$

$$a^2 + 2a + a^3 + 2a^2 - 2a = 0$$

$$a^3 + 3a^2 = 0$$

$$a^2(a+3) = 0$$

$$\Rightarrow a = 0 \text{ or } a = -3.$$

(v) (A)

$$y = \sqrt{\sin x + y}$$

$$y^2 = \sin x + y$$

[दोनों ओर वर्ग करने पर]

$$2y \frac{dy}{dx} = \cos x + \frac{dy}{dx}$$

$$\frac{dy}{dx}(2y - 1) = \cos x$$

$$\boxed{\frac{dy}{dx} = \frac{\cos x}{2y - 1}}$$

(vi) (B)

$$\int_0^a \frac{1}{9x^2 + 1} dx = \frac{\pi}{12}$$

$$\int_0^a \frac{dx}{(3x)^2 + 1} = \frac{\pi}{12}$$

$$\frac{1}{3} [\tan^{-1}(3x)]_0^a = \frac{\pi}{12}$$

$$\frac{1}{3} [\tan^{-1} 3a - \tan^{-1} 0] = \frac{\pi}{12}$$

$$\tan^{-1} 3a = \frac{\pi}{4}$$

$$3a = \tan \frac{\pi}{4}$$

$$3a = 1$$

$$\boxed{a = \frac{1}{3}}$$

(vii) (C)

दिया गया है  $\cos x \frac{dy}{dx} + y \sin x = 1$

$$\Rightarrow \frac{dy}{dx} + y \tan x = \sec x$$

समाकलन स्थिरांक (I.F.) =  $e^{\int \tan x dx}$   
 $= e^{\log \sec x} = \sec x$

(viii) (A)

माना  $\vec{a} = \hat{i} + \hat{j} + \hat{k}$

$$\Rightarrow \hat{a} = \frac{\vec{a}}{|\vec{a}|} = \frac{1}{\sqrt{3}}(\hat{i} + \hat{j} + \hat{k})$$

(ix) (B)

दो निशानों की अभिष्ट प्रायिकता

$$\begin{aligned}
 &= P(A) P(B) P(C') + P(A) P(B') P(C) + P(A') P(B) P(C) \\
 &= (0.4 \times 0.3 \times 0.8) + (0.4 \times 0.7 \times 0.2) + (0.6 \times 0.3 \times 0.2) \\
 &= 0.096 + 0.056 + 0.036 \\
 &= 0.188
 \end{aligned}$$

(x) (A)

दिया है,  $A = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$

$$A^2 = A \times A = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 4 \\ 0 & 1 \end{bmatrix} \quad \dots\dots (1)$$

Also  $A^3 = A^2 \times A = \begin{bmatrix} 1 & 4 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 6 \\ 0 & 1 \end{bmatrix} \quad \dots\dots (2)$

(1) और (2) से  $A^n = \begin{bmatrix} 1 & 2n \\ 0 & 1 \end{bmatrix}$

(xi) (A)

माना  $z = \cos^{-1}(2x^2 - 1)$

$$x = \cos\theta \quad \dots\dots(1)$$

$$z = \cos^{-1}(2\cos^2\theta - 1)$$

$$z = \cos^{-1}(\cos 2\theta)$$

$$z = 2\theta$$

$$z = 2\cos^{-1} x$$

$$\frac{dz}{dx} = -\frac{2}{\sqrt{1-x^2}} \quad \dots\dots(1)$$

माना  $y = \cos^{-1} x$

$$\frac{dy}{dx} = \frac{-1}{\sqrt{1-x^2}} \quad \dots\dots(2)$$

$$\frac{dz}{dy} = \frac{\frac{dz}{dx}}{\frac{dy}{dx}} = \frac{\frac{-2}{\sqrt{1-x^2}}}{\frac{-1}{\sqrt{1-x^2}}} = 2$$

$$\boxed{\frac{dz}{dy} = 2}$$

(xii) (C)

माना  $\vec{a}$  तथा  $\vec{b}$  के मध्य कोण  $\theta$  है।

$$\begin{aligned}
 \Rightarrow \cos\theta &= \frac{\vec{a} \cdot \vec{b}}{|\vec{a}| |\vec{b}|} \\
 &= \frac{(2\hat{i} + \hat{j} - 3\hat{k}) \cdot (3\hat{i} - 2\hat{j} - 2\hat{k})}{\sqrt{14} \sqrt{17}} \\
 &= \frac{10}{\sqrt{238}} \\
 \Rightarrow \theta &= \cos^{-1}\left(\frac{10}{\sqrt{238}}\right)
 \end{aligned}$$

2.

(i) दिया है  $f$  तथा  $g$  दो व्युत्क्रम फलन इस प्रकार से है कि  $g \circ f$  परिभाषित है।

$$\Rightarrow (g \circ f)^{-1} = f^{-1} \circ g^{-1}$$

(ii)  $\tan^{-1}x + \tan^{-1}y = \frac{\pi}{4}$ 

$$\tan^{-1} \left[ \frac{x+y}{1-xy} \right] = \frac{\pi}{4}$$

$$\frac{x+y}{1-xy} = 1 \Rightarrow x+y = 1-xy \Rightarrow \boxed{x+y+xy=1}$$

(iii)  $A^T = A$ (iv)  $\sqrt{x} + \sqrt{y} = 1$  at  $\left(\frac{1}{4}, \frac{1}{4}\right)$ 

$$\frac{1}{2\sqrt{x}} + \frac{1}{2\sqrt{y}} \frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = -\frac{\sqrt{y}}{\sqrt{x}}$$

$$\text{at } \left(\frac{1}{4}, \frac{1}{4}\right)$$

$$\frac{dy}{dx} = -1$$

(v)  $\int_0^a \frac{1}{1+4x^2} dx = \frac{\pi}{8}$ 

$$\int_0^a \frac{1}{1+(2x)^2} dx = \frac{\pi}{8}$$

$$\frac{1}{2} [\tan^{-1}(2x)]_0^a = \frac{\pi}{8}$$

$$\frac{1}{2} [\tan^{-1} 2a - \tan^{-1} 0] = \frac{\pi}{8}$$

$$\tan^{-1} 2a = \frac{\pi}{4}$$

$$2a = \tan \frac{\pi}{4}$$

$$2a = 1$$

$$\boxed{a = \frac{1}{2}}$$

(vi) माना  $\vec{a} = 6\hat{i} + 2\hat{j} - 3\hat{k}$

$$\Rightarrow \hat{a} = \frac{\vec{a}}{|\vec{a}|} = \frac{6\hat{i} + 2\hat{j} - 3\hat{k}}{\sqrt{36 + 4 + 9}}$$

$$= \frac{6}{7}\hat{i} + \frac{2}{7}\hat{j} - \frac{3}{7}\hat{k}$$

$\therefore$  आवश्यक दिक् कोज्याएं  $\frac{6}{7}, \frac{2}{7}$  and  $-\frac{3}{7}$

3.

(i) दिया है  $f: \mathbb{R} \rightarrow \mathbb{R}, g: \mathbb{R} \rightarrow \mathbb{R}$

तथा  $f(x) = 2x - 3,$

$$g(x) = x^3 + 5$$

यदि  $f$  और  $g$  आच्छादक है, तो हम  $f^{-1}$  और  $g^{-1}$  ज्ञात कर सकते हैं।

अब ;  $y = f(x) = 2x - 3$

$$\Rightarrow x = \frac{y+3}{2}$$

$$\text{or } f^{-1}(y) = \frac{y+3}{2} \Rightarrow f^{-1}(x) = \frac{x+3}{2}$$

$$y = g(x) = x^3 + 5$$

$$x = (y - 5)^{1/3}$$

$$\text{or } g^{-1}(y) = (y - 5)^{1/3}$$

$$g^{-1}(x) = (x - 5)^{1/3}$$

$$\therefore (f \circ g)^{-1}(x) = g^{-1} \circ f^{-1}(x)$$

$$= g^{-1}[f^{-1}(x)]$$

$$= g^{-1}\left[\frac{x+3}{2}\right]$$

$$= \left(\frac{x+3}{2} - 5\right)^{1/3} = \left(\frac{x-7}{2}\right)^{1/3}$$

(ii)  $\cos^{-1}\left(\cos\frac{5\pi}{3}\right) + \sin^{-1}\left(\sin\frac{2\pi}{3}\right)$

$$= \cos^{-1}\cos\left(\pi + \frac{2\pi}{3}\right) + \sin^{-1}\sin\left(\pi - \frac{\pi}{3}\right)$$

$$= \pi - \frac{2\pi}{3} + \frac{\pi}{3}$$

$$= \frac{\pi}{3} + \frac{\pi}{3} = \frac{2\pi}{3}$$

(iii) दिया है  $A = \begin{bmatrix} 3 & -3 \\ -3 & 3 \end{bmatrix}$

$$A = \begin{bmatrix} 3 & -3 \\ -3 & 3 \end{bmatrix} \begin{bmatrix} 3 & -3 \\ -3 & 3 \end{bmatrix} = \begin{bmatrix} 18 & -18 \\ -18 & 18 \end{bmatrix}$$

और  $A^2 = \lambda A$

$$\begin{bmatrix} 18 & -18 \\ -18 & 18 \end{bmatrix} = \lambda \begin{bmatrix} 3 & -3 \\ -3 & 3 \end{bmatrix}$$

$\therefore 3\lambda = 18$  or  $\lambda = 6$

(iv) दिया है  $|A| = 5$

$$|B| = 3$$

$$|3AB| = 27|A| |B|$$

$$= 27 \times 5 \times 3$$

$$= 27 \times 15$$

$$= 405$$

(गुणधर्म से  $|kA| = k^n|A|$ )

जहाँ  $n \times n$  आव्यूह का क्रम है।

(v)  $y = \tan^{-1}x$

$$\tan y = x$$

$x$  के सापेक्ष अवकलन करने पर

$$\sec^2 y \frac{dy}{dx} = 1$$

$$\frac{dy}{dx} = \cos^2 y$$

$$\frac{d^2 y}{dx^2} = -2 \cos y \sin y \cdot \frac{dy}{dx}$$

$$= -\sin 2y \cos^2 y$$

(vi)  $\int \frac{\cos x - \sin x}{\sqrt{1 + \sin 2x}} dx$

$$= \int \frac{\cos x - \sin x}{(\cos x + \sin x)^2} dx$$

$$\cos x + \sin x = t$$

$$(-\sin x + \cos x) dx = dt$$

$$= \int \frac{\cos x - \sin x}{(\cos x + \sin x)} dx$$

$$= \int \frac{dt}{t}$$

$$= \log(t) + C$$

$$= \log(\cos x + \sin x) + C$$

(vii) दिया गया है  $x\sqrt{x+y^2}dx + y\sqrt{1+x^2}dy = 0$

चर का पृथक्करण करने पर

$$\frac{y}{\sqrt{1+y^2}}dy = -\frac{x}{\sqrt{1+x^2}}dx$$

दोनों तरफ समाकलन करने पर

$$\frac{y}{\sqrt{1+y^2}}dx = -\frac{x}{\sqrt{1+x^2}}dx$$

रखने पर  $1+y^2 = t$  &  $1+x^2 = z$

$$2ydy = dt \quad \& \quad 2xdx = dz$$

$$\frac{1}{2} \int \frac{dt}{\sqrt{t}} = \frac{1}{2} \int \frac{dz}{\sqrt{z}} \Rightarrow \sqrt{t} = \sqrt{z}$$

$$\Rightarrow \sqrt{1+y^2} = \sqrt{1+x^2} + C$$

(viii) दिया गया है ;  $\vec{a}$  तथा  $\vec{b}$  समान्तर चतुर्भुज की संगत भुजाएं है।

$$\Rightarrow \vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -1 & 3 \\ 2 & -7 & 1 \end{vmatrix}$$

$$= 20\hat{i} - 5\hat{j} - 5\hat{k}$$

$$\therefore |\vec{a} \times \vec{b}| = \sqrt{400 + 25 + 25} = \sqrt{450}$$

अतः समान्तर चतुर्भुज का क्षेत्रफल =  $|\vec{a} \times \vec{b}|$

$$= \sqrt{450} = 15\sqrt{2} \text{ sq.units}$$

(ix) P(द्वितीय गेंद नीली है)

= P(प्रथम गेंद लाल, द्वितीय गेंद नीली) + P(प्रथम गेंद नीली, द्वितीय गेंद भी नीली)

$$= \left(\frac{4}{9} \times \frac{5}{8}\right) + \left(\frac{5}{9} \times \frac{4}{8}\right)$$

$$= \frac{20}{72} + \frac{20}{72} = \frac{40}{72} = \frac{5}{9}$$

(x)  $A = \begin{bmatrix} 2 & -1 \\ 3 & 4 \end{bmatrix}$

$$A^{-1} = \frac{\text{adj}(A)}{|A|}, |A| \neq 0$$

$$|A| = (4 \times 2) - (3 \times -1)$$

$$= 8 + 3 = 11$$

$$|A| \neq 0$$

$$\text{adj}(A) = \begin{bmatrix} 4 & 1 \\ -3 & 2 \end{bmatrix}$$

$$A^{-1} = \frac{1}{11} \begin{bmatrix} 4 & 1 \\ -3 & 2 \end{bmatrix}$$

(xi) दिया गया है  $x + y = \tan^{-1}y$

$x$  के सापेक्ष अवकलन करने पर

$$1 + y' = \frac{1}{1+y^2} y'$$

$$(1 + y')(1 + y^2) = y'$$

$$1 + y^2 + y' + y^2 y' = y'$$

$$\Rightarrow y^2 y' + y^2 + 1 = 0$$

(xii) दिया गया है ;  $\vec{a}$  तथा  $\vec{b}$  समान्तर है।

$$\Rightarrow \frac{5}{3} = \frac{\lambda}{9} \quad \text{or} \quad \lambda = 15$$

(खण्ड-ब)

4.  $f(x) = \frac{4x+3}{3x+4}$

(i) एकैकी फलन के लिए : हम जानते  $f(x_1) = f(x_2) \forall x_1, x_2 \in \mathbb{R} - \left\{-\frac{4}{3}\right\}$

$$\Rightarrow \frac{4x_1+3}{3x_1+4} = \frac{4x_2+3}{3x_2+4}$$

$$\Rightarrow 12x_1x_2 + 16x_1 + 9x_2 + 12$$

$$= 12x_1x_2 + 9x_1 + 16x_2 + 12$$

$$\Rightarrow 7x_1 - 7x_2 = 0 \Rightarrow x_1 = x_2$$

फलन एकैकी है।

(ii) आच्छादक फलन के लिए : माना  $y = f(x)$

$$y = \frac{4x+3}{3x+4}$$

$$\Rightarrow 3xy + 4y = 4x + 3$$

$$\Rightarrow x(3y - 4) = 3 - 4y$$

$$\Rightarrow x = \frac{3-4y}{3y-4} \in \mathbb{R} - \left\{-\frac{4}{3}\right\}, \forall y \in \mathbb{R} - \left\{\frac{4}{3}\right\}$$

यदि  $3y - 4 \neq 0$

$$y \neq \frac{4}{3}$$

सहप्रान्त का प्रत्येक अवयव में अपने प्रांत में पूर्व प्रतिबिम्ब रखता है।

अतः परिसर = सहप्रान्त

फलन आच्छादक है।

अतः  $f(x)$  बहुएकैकी फलन है।

$\therefore$  एकैकी और आच्छादक दोनों है।

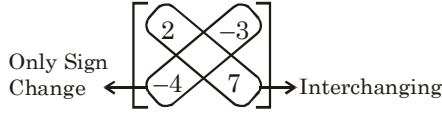


5. दिया गया है,  $A = \begin{bmatrix} 2 & -3 \\ -4 & 7 \end{bmatrix}$

$|A| = 14 - (12) = 2 \neq 0$

अतः A व्युत्क्रमणीय है।

$A^{-1} = \frac{\text{adj.}(A)}{|A|}$



$\text{adj}A = \begin{bmatrix} 7 & 3 \\ 4 & 2 \end{bmatrix}$

$A^{-1} = \frac{\text{adj.}(A)}{|A|}$

$= \frac{1}{2} \begin{bmatrix} 7 & 3 \\ 4 & 2 \end{bmatrix}$

$= \begin{bmatrix} \frac{7}{2} & \frac{3}{2} \\ 2 & 1 \end{bmatrix}$

अब RHS = 9I - A

$= 9 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 2 & -3 \\ -4 & 7 \end{bmatrix} = \begin{bmatrix} 9 & 0 \\ 0 & 9 \end{bmatrix} - \begin{bmatrix} 2 & -3 \\ -4 & 7 \end{bmatrix}$

$= \begin{bmatrix} 7 & 3 \\ 4 & 2 \end{bmatrix} = 2A^{-1} = \text{LHS} = 2 \begin{bmatrix} \frac{7}{2} & \frac{3}{2} \\ 2 & 1 \end{bmatrix}$

इति सिद्धम्

6.  $\text{LHS} = \begin{vmatrix} a^2 + 2a & 2a + 1 & 1 \\ 2a + 1 & a + 2 & 1 \\ 3 & 3 & 1 \end{vmatrix}$

$R_1 \rightarrow R_1 - R_2$  and  $R_2 \rightarrow R_2 - R_3$

$\Delta = \begin{vmatrix} a^2 - 1 & a - 1 & 0 \\ 2(a - 1) & a - 1 & 0 \\ 3 & 3 & 1 \end{vmatrix}$

$= (a - 1)^2 \begin{vmatrix} a + 1 & 1 & 0 \\ 2 & 1 & 0 \\ 3 & 3 & 1 \end{vmatrix}$

विस्तार

$(a - 1)^2 \cdot (a - 1) = (a - 1)^3$ .

7. दिया है  $y = \sin(\sin x)$  ... (1)

$$\frac{dy}{dx} = \cos(\sin x) \cdot \cos x \quad \dots (2)$$

$$\frac{d^2y}{dx^2} = \cos(\sin x) \frac{d}{dx}(\cos x) + \cos x \frac{d}{dx} \cos(\sin x)$$

$$\frac{d^2y}{dx^2} = -\sin x \cos(\sin x) + \cos x \cdot \{-\sin(\sin x)\} \cdot \cos x \quad \dots (3)$$

$\sin(\sin x) = y$  from (1) and

$$\cos(\sin x) = \frac{1}{\cos x} \left( \frac{dy}{dx} \right) \text{ from (2) in (3)}$$

$$\frac{d^2y}{dx^2} = \frac{-\sin x}{\cos x} \cdot \frac{dy}{dx} - y \cos^2 x$$

$$\frac{d^2y}{dx^2} + \tan x \frac{dy}{dx} + y \cos^2 x = 0$$

8.  $\int \frac{x-5}{(x-3)^3} e^x dx$

$$= \int e^x \left[ \frac{(x-3)-2}{(x-3)^3} \right] dx$$

$$= \int e^x \left[ \frac{(x-3)}{(x-3)^3} - \frac{2}{(x-3)^3} \right] dx$$

$$= \int e^x \left[ \frac{1}{(x-3)^2} + \left\{ \frac{-2}{(x-3)^3} \right\} \right] dx$$

$$= e^x / (x-3)^2 + C \quad \left[ \because \int e^x [f(x) + f'(x)] dx = e^x f(x) + C \right]$$

9. माना  $E_1$  : पहले पर्स को चुनना तथा  $E_2$  : दूसरे पर्स को चुनना

$A$  : चांदी का सिक्का निकालना

$$\Rightarrow P(E_1) = \frac{1}{2}, P(E_2) = \frac{1}{2}, P(A/E_1) = \frac{3}{9}, P(A/E_2) = \frac{4}{7}$$

$$\therefore P(A) = P(E_1) P(A/E_1) + P(E_2) \cdot P(A/E_2) \quad (\text{संपूर्ण प्रायिकता प्रमेय द्वारा})$$

$$= \frac{1}{2} \times \frac{3}{9} + \frac{1}{2} \times \frac{4}{7}$$

$$\text{या } P(A) = \frac{1}{6} + \frac{2}{7} = \frac{19}{42}$$

10. माना  $D = \begin{bmatrix} a & b \\ c & d \\ e & f \end{bmatrix}$

$$\therefore A + B - D = \begin{bmatrix} 1 & 3 \\ 3 & 2 \\ 2 & 5 \end{bmatrix} + \begin{bmatrix} -1 & -2 \\ 0 & 5 \\ 3 & 1 \end{bmatrix} - \begin{bmatrix} a & b \\ c & d \\ e & f \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1-1-a & 3-2-b \\ 3+0-c & 2+5-d \\ 2+3-e & 5+1-f \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \Rightarrow -a=0 \Rightarrow a=0, \quad 1-b=0 \Rightarrow b=1,$$

$$3-c=0 \Rightarrow c=3, \quad 7-d=0 \Rightarrow d=7,$$

$$5-e=0 \Rightarrow e=5, \quad 6-f=0 \Rightarrow f=6$$

$$\therefore D = \begin{bmatrix} 0 & 1 \\ 3 & 7 \\ 5 & 6 \end{bmatrix}$$

11.  $x = a(\cos 2t + 2t \sin 2t)$

$$\frac{dx}{dt} = a[-2 \sin 2t + 2(2t \cos 2t + \sin 2t)]$$

$$\frac{dx}{dt} = 4at \cos 2t \quad \dots\dots\dots(1)$$

$$y = a(\sin 2t - 2t \cos 2t)$$

$$\frac{dy}{dt} = a[2 \cos 2t - 2(-2t \sin 2t + \cos 2t)]$$

$$\frac{dy}{dt} = 4at \sin 2t \quad \dots\dots\dots(2)$$

By chain rule

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{4at \sin 2t}{4at \cos 2t}$$

$$\frac{dy}{dx} = \tan 2t$$

$$\frac{d^2y}{dx^2} = 2 \sec^2 2t \times \frac{dt}{dx}$$

$$\frac{d^2y}{dx^2} = 2 \sec^2 2t \times \frac{1}{4at \cos 2t}$$

$$\frac{d^2y}{dx^2} = \frac{1}{2at} \times \sec^3 2t$$

12. रैखिक समीकरण को आव्यूह रूप में लिखने पर 
$$\begin{bmatrix} 1 & 1 & 1 \\ 2 & 1 & -1 \\ 3 & 2 & k \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \\ 4 \end{bmatrix}$$

माना  $A = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 1 & -1 \\ 3 & 2 & k \end{bmatrix}$

रैखिक समीकरण का एक अद्वितीय हल होगा, यदि

$$|A| \neq 0$$

$$\begin{vmatrix} 1 & 1 & 1 \\ 2 & 1 & -1 \\ 3 & 2 & k \end{vmatrix} \neq 0$$

$$\Rightarrow 1(k+2) - 1(2k+3) + 1(4-3) \neq 0$$

$$\Rightarrow k \neq 0$$

13.  $I = \int_{\pi/6}^{\pi/3} \frac{dx}{1 + \sqrt{\tan x}}$

$$I = \int_{\pi/6}^{\pi/3} \frac{\sqrt{\cos x}}{\sqrt{\sin x} + \sqrt{\cos x}} dx \quad \dots\dots(1)$$

$$I = \int_{\pi/6}^{\pi/3} \frac{\sqrt{\sin x}}{\sqrt{\cos x} + \sqrt{\sin x}} dx \quad \dots\dots(2) \quad \left[ \text{Using } \int_a^b f(x) = \int_a^b f(a+b-x) \right]$$

(1) & (2) समीकरण को जोड़ने पर

$$2I = \int_{\pi/6}^{\pi/3} dx \Rightarrow 2I = \frac{\pi}{6} \Rightarrow I = \frac{\pi}{12}$$

14.  $\log\left(\frac{dy}{dx}\right) = 3x + 4y$

$$\Rightarrow \frac{dy}{dx} = e^{3x+4y} \Rightarrow \frac{dy}{dx} = e^{3x} \cdot e^{4y}$$

$$\Rightarrow \int e^{-4y} dy = \int e^{3x} dx \Rightarrow \frac{e^{-4y}}{-4} = \frac{e^{3x}}{3} + C \quad \dots(i)$$

दिया गया है कि  $y = 0$  जब  $x = 0$

$$C = \frac{-1}{4} - \frac{1}{3} = \frac{-7}{12}$$

$$\Rightarrow \frac{e^{-4y}}{-4} = \frac{e^{3x}}{3} - \frac{7}{12} \quad (\text{समीकरण (i) से})$$

$$\text{या } 3e^{-4y} + 4e^{3x} = 7$$

15.  $\vec{s} = (2\hat{i} + 4\hat{j} - 5\hat{k}) + (\lambda\hat{i} + 2\hat{j} + 3\hat{k})$

$\vec{s} = \hat{i}(2 + \lambda) + 6\hat{j} - 2\hat{k}$

मात्रक सदिश  $\hat{s} = \frac{\vec{s}}{|\vec{s}|} = \frac{(\lambda + 2)\hat{i} + 6\hat{j} - 2\hat{k}}{\sqrt{(\lambda + 2)^2 + 6^2 + 2^2}}$

दिया है :  $(\hat{i} + \hat{j} + \hat{k}) \cdot \hat{s} = 1$

$\therefore \frac{(\lambda + 2) + 6 - 2}{\sqrt{(\lambda + 2)^2 + 40}} = 1$

या  $(\lambda + 6) = \sqrt{(\lambda + 2)^2 + 40}$

$\lambda^2 + 12\lambda + 36 = \lambda^2 + 4\lambda + 44$  (दोनों ओर का वर्ग करने पर)

$\Rightarrow 8\lambda = 8 \Rightarrow \lambda = 1$

16. Let  $E_1$  : शत प्रतिशत उपस्थिति रखने वाले विद्यार्थी

$E_2$  : अनियमित विद्यार्थी

$E$  : A ग्रेड वाले विद्यार्थी

$P(E_1) = \frac{30}{100}$  and  $P(E_2) = \frac{70}{100}$

$P(E/E_1) = \frac{70}{100}$ ;  $P(E/E_2) = \frac{10}{100}$

आवश्यक प्रायिकता =  $P(E_1/E) = \frac{P(E_1) \times P(E/E_1)}{P(E_1) \times P(E/E_1) + P(E_2) \times P(E/E_2)}$  (बेज प्रमेय द्वारा)

$$= \frac{\frac{30}{100} \times \frac{70}{100}}{\left(\frac{30}{100} \times \frac{70}{100}\right) + \left(\frac{70}{100} \times \frac{10}{100}\right)}$$

$$= \frac{2100}{2100 + 700} = \frac{21}{28} = \frac{3}{4}$$

(खण्ड-स)

17.  $\tan^{-1} \left[ \frac{\sqrt{1+x} - \sqrt{1-x}}{\sqrt{1+x} + \sqrt{1-x}} \right] = \frac{\pi}{4} - \frac{1}{2} \cos^{-1} x, \frac{-1}{\sqrt{2}} \leq x \leq 1$

In LHS

put  $x = \cos 2\theta$

$$\tan^{-1} \left[ \frac{\sqrt{1 + \cos 2\theta} - \sqrt{1 - \cos 2\theta}}{\sqrt{1 + \cos 2\theta} + \sqrt{1 - \cos 2\theta}} \right]$$

$$= \tan^{-1} \left[ \frac{\sqrt{1 + 2\cos^2 \theta - 1} - \sqrt{1 - 1 + 2\sin^2 \theta}}{\sqrt{1 + 2\cos^2 \theta - 1} + \sqrt{1 - 1 + 2\sin^2 \theta}} \right]$$

$$\begin{aligned}
 &= \tan^{-1} \left[ \frac{\cos \theta - \sin \theta}{\cos \theta + \sin \theta} \right] \\
 &= \tan^{-1} \left[ \frac{1 - \tan \theta}{1 + \tan \theta} \right] \\
 &= \tan^{-1} \left[ \frac{\tan(\pi/4) - \tan \theta}{1 + \tan(\pi/4) \cdot \tan \theta} \right] \\
 &= \tan^{-1} [\tan(\pi/4 - \theta)] \\
 &= \frac{\pi}{4} - \theta \quad \text{as } \begin{cases} x = \cos 2\theta \\ \text{so, } \theta = \frac{\cos^{-1} x}{2} \end{cases} \\
 &= \frac{\pi}{4} - \frac{1}{2} \cos^{-1} x = \text{RHS} \quad \text{proved}
 \end{aligned}$$

अथवा

$$\tan^{-1} \left( \frac{x-2}{x-4} \right) + \tan^{-1} \left( \frac{x+2}{x+4} \right) = \frac{\pi}{4} \quad \dots(i)$$

Use formula,  $\tan^{-1} \left[ \frac{\frac{x-2}{x-4} + \frac{x+2}{x+4}}{1 - \left( \frac{x-2}{x-4} \right) \left( \frac{x+2}{x+4} \right)} \right] = \frac{\pi}{4}$

$$\Rightarrow \tan^{-1} \left[ \frac{(x-2)(x+4) + (x+2)(x-4)}{(x-4)(x+4) - (x-2)(x+2)} \right] = \frac{\pi}{4}$$

$$\Rightarrow \frac{(x-2)(x+4) + (x+2)(x-4)}{(x-4)(x+4) - (x-2)(x+2)} = 1$$

$$\Rightarrow \frac{x^2 - 8 - 2x + x^2 - 8 + 2x}{x^2 - 16 - x^2 + 4} = 1$$

$$\Rightarrow \frac{2x^2 - 16}{-12} = 1$$

$$\Rightarrow 2x^2 = -12 + 16 = 4$$

$$\Rightarrow x^2 = 2 \quad \Rightarrow x = \pm\sqrt{2}$$

18. दिया है  $(x^2 + y^2)^2 = xy$ , find  $\frac{dy}{dx}$

$$x^4 + y^4 + 2x^2y^2 = xy$$

x के सापेक्ष दोनों ओर अवकलन करने पर

$$4x^3 + 4y^3 \frac{dy}{dx} + 2 \left[ 2xy^2 + x^2 \cdot 2y \frac{dy}{dx} \right] = x \frac{dy}{dx} + y$$

$$\frac{dy}{dx} (4y^3 + 4x^2y - x) = (y - 4x^3 - 4xy^2)$$

$$\Rightarrow \frac{dy}{dx} = \frac{y - 4x^3 - 4xy^2}{4y^3 + 4x^2y - x}$$

अथवा

$$\text{फलन प्राचलिक है } \frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta} \quad \dots(1)$$

$$\text{अब, } x = a(2\theta - \sin 2\theta)$$

$\theta$  के सापेक्ष अवकलन करने पर

$$\frac{dx}{d\theta} = a(2 - 2\cos 2\theta)$$

$$\Rightarrow \frac{dx}{d\theta} = 2a(1 - \cos 2\theta)$$

$$\Rightarrow \frac{dx}{d\theta} = 4a \sin^2 \theta \quad \dots(2)$$

$$y = a(1 - \cos 2\theta)$$

$\theta$  के सापेक्ष अवकलन करने पर

$$\frac{dy}{d\theta} = 2a \sin 2\theta$$

$$\Rightarrow \frac{dy}{d\theta} = 4a \sin \theta \cos \theta \quad \dots(3)$$

समीकरण (2) और (3) में (1) का मान रखने पर

$$\frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta}$$

$$\frac{dy}{dx} = \frac{4a \sin \theta \cos \theta}{4a \sin^2 \theta}$$

$$\frac{dy}{dx} = \frac{\cos \theta}{\sin \theta}$$

$$\left(\frac{dy}{dx}\right)_{\theta=\frac{\pi}{3}} = \frac{\cos\left(\frac{\pi}{3}\right)}{\sin\left(\frac{\pi}{3}\right)} = \frac{1}{\sqrt{3}}$$

19.  $I = \int e^{2x} \frac{(2x-5)}{(2x-3)^3} dx$

$$I = \int e^{2x} \left[ \frac{(2x-3)}{(2x-3)^3} - \frac{2}{(2x-3)^3} \right] dx$$

$$I = \int e^{2x} \left[ \frac{1}{(2x-3)^2} - \frac{2}{(2x-3)^3} \right] dx$$

Put  $2x = t$

$$dx = \frac{dt}{2}$$

$$I = \frac{1}{2} \int e^t \left[ \frac{1}{(t-3)^2} - \frac{2}{(t-3)^3} \right] dt$$

$$I = \frac{1}{2} \frac{e^t}{(t-3)^2} + C \quad \left[ \because \int e^x [f(x) + f'(x)] dx = e^x f(x) + C \right]$$

$$I = \frac{1}{2} \frac{e^{2x}}{(2x-3)^2} + C$$

अथवा

$$I = \int \frac{x^2 + x + 1}{(x^2 + 1)(x + 2)} dx$$

$$\text{Let } \frac{x^2 + x + 1}{(x + 2)(x^2 + 1)} = \frac{A}{x + 2} + \frac{Bx + C}{x^2 + 1} \quad \dots\dots\dots (1)$$

$$x^2 + x + 1 = A(x^2 + 1) + (Bx + C)(x + 2)$$

$$x = -2 \text{ पर } A = \frac{3}{5}$$

$$\text{और } x = 0 \quad A + 2C = 1 \Rightarrow C = \frac{1}{5}$$

$x^2$  के गुणांक की तुलना करने पर

$$1 = A + B \Rightarrow B = 1 - \frac{3}{5} = \frac{2}{5}$$

समीकरण (1) में मान रखने पर और समाकलन करने पर

$$\begin{aligned} \therefore I &= \int \frac{x^2 + x + 1}{(x + 2)(x^2 + 1)} dx = \frac{3}{5} \int \frac{1}{x + 2} dx + \int \frac{\frac{2}{5}x + \frac{1}{5}}{x^2 + 1} dx \\ &= \frac{3}{5} \int \frac{1}{x + 2} dx + \frac{1}{5} \int \frac{2x}{x^2 + 1} dx + \frac{1}{5} \int \frac{1}{x^2 + 1} dx \\ &= \frac{3}{5} \log(x + 2) + \frac{1}{5} \log(x^2 + 1) + \frac{1}{5} \tan^{-1} x + C \end{aligned}$$

20. माना सदिश समान अक्ष के साथ  $\alpha$  कोण बनाता है।

$$\text{तब } \vec{r} = 3\sqrt{3} (\ell \hat{i} + m \hat{j} + n \hat{k}) \quad (\because |\vec{r}| = 3\sqrt{3})$$

जहाँ  $\ell = \cos \alpha$ ,  $m = \cos \alpha$ ,  $n = \cos \alpha$

$$\therefore \ell^2 + m^2 + n^2 = 1$$

$$\Rightarrow 3 \cdot \cos^2 \alpha = 1 \quad \Rightarrow \cos \alpha = \pm \frac{1}{\sqrt{3}}$$

$$\therefore \vec{r} = 3\sqrt{3} \left( \pm \frac{\hat{i}}{\sqrt{3}} \pm \frac{\hat{j}}{\sqrt{3}} \pm \frac{\hat{k}}{\sqrt{3}} \right)$$

$$\text{या } \vec{r} = \pm 3(\hat{i} + \hat{j} + \hat{k})$$



अथवा

दिया है  $\vec{a}$  तथा  $\vec{b}$  मात्रक सदिश  $|\vec{a}| = 1 = |\vec{b}|$

तथा  $|\sqrt{3}\vec{a} - \vec{b}| = 1 \Rightarrow |\sqrt{3}\vec{a} - \vec{b}|^2 = 1$

$\Rightarrow (\sqrt{3}\vec{a} - \vec{b}) \cdot (\sqrt{3}\vec{a} - \vec{b}) = 1$

या  $3|\vec{a}|^2 - \sqrt{3}\vec{a} \cdot \vec{b} - \sqrt{3}\vec{b} \cdot \vec{a} + |\vec{b}|^2 = 1 \quad (\because \vec{a} \cdot \vec{a} = |\vec{a}|^2)$

$\Rightarrow 3 - 2\sqrt{3}\vec{a} \cdot \vec{b} + 1 = 1 \quad (\because \vec{a} \cdot \vec{b} = \vec{b} \cdot \vec{a})$

या  $\vec{a} \cdot \vec{b} = \frac{\sqrt{3}}{2} \Rightarrow |\vec{a}| |\vec{b}| \cos \theta = \frac{\sqrt{3}}{2} \quad (\because \vec{a} \cdot \vec{b} = |\vec{a}| |\vec{b}| \cos \theta)$

$\Rightarrow \cos \theta = \frac{\sqrt{3}}{2} \Rightarrow \theta = \frac{\pi}{6}$

(खण्ड-द)

21.  $I = \int_0^\pi \frac{x dx}{a^2 \cos^2 x + b^2 \sin^2 x}$

$I = \int_0^\pi \frac{(\pi - x) dx}{a^2 \cos^2 x + b^2 \sin^2 x} \quad \left[ \because \int_0^a f(x) dx = \int_0^a f(a - x) dx \right]$

$2I = \int_0^\pi \frac{\pi dx}{a^2 \cos^2 x + b^2 \sin^2 x}$

$I = \frac{\pi}{2} \int_0^\pi \frac{dx}{a^2 \cos^2 x + b^2 \sin^2 x}$

$I = \frac{\pi}{2} \times 2 \int_0^{\frac{\pi}{2}} \frac{dx}{a^2 \cos^2 x + b^2 \sin^2 x} \quad \left[ \because \int_0^a f(x) dx = 2 \int_0^{\frac{a}{2}} f(x) dx, \text{ if } f(2a - x) = f(x) \right]$

$I = \pi \int_0^{\frac{\pi}{2}} \frac{\sec^2 x dx}{a^2 + b^2 \tan^2 x}$

Put  $\tan x = t \Rightarrow \sec^2 x dx = dt$

when  $x = 0 \Rightarrow t = 0$  &  $x = \pi/2 \Rightarrow t = \infty$

$I = \pi \int_0^\infty \frac{dt}{a^2 + b^2 t^2} = \frac{\pi}{b^2} \int_0^\infty \frac{dt}{t^2 + \left(\frac{a}{b}\right)^2}$

$I = \left(\frac{\pi}{b^2}\right) \frac{1}{\left(\frac{a}{b}\right)} \left| \tan^{-1} \left(\frac{t}{a/b}\right) \right|_0^\infty = \frac{\pi}{ab} \cdot \frac{\pi}{2} = \frac{\pi^2}{2ab}$

अथवा

$$\begin{aligned} & \int \frac{dx}{\cos^4 x + \sin^4 x} \\ &= \int \frac{1}{\cos^4 x + \sin^4 x} dx \\ &= \int \frac{\cos^4 x}{1 + \tan^4 x} dx \\ &= \int \frac{\sec^2 x \cdot \sec^2 x}{1 + \tan^4 x} dx \\ &= \int \frac{(1 + \tan^2 x) \sec^2 x}{1 + \tan^4 x} dx \end{aligned}$$

put  $\tan x = t \Rightarrow \sec^2 x dx = dt$ 

$$\begin{aligned} &= \int \frac{(1+t^2)dt}{1+t^4} \\ &= \int \frac{\left(\frac{1}{t^2}+1\right)dt}{\frac{1}{t^2}+t^2} \\ &= \int \frac{\left(1+\frac{1}{t^2}\right)dt}{\left(t-\frac{1}{t}\right)^2+2} \end{aligned}$$

put  $t - \frac{1}{t} = z \Rightarrow \left(1 + \frac{1}{t^2}\right) dt = dz$ 

$$\begin{aligned} &= \int \frac{dz}{z^2 + (\sqrt{2})^2} = \frac{1}{\sqrt{2}} \tan^{-1} \frac{z}{\sqrt{2}} + C \\ &= \frac{1}{\sqrt{2}} \tan^{-1} \frac{1}{\sqrt{2}} \left(t - \frac{1}{t}\right) + C \\ &= \frac{1}{\sqrt{2}} \tan^{-1} \frac{1}{\sqrt{2}} \left(\tan x - \frac{1}{\tan x}\right) + C \\ &= \frac{1}{\sqrt{2}} \tan^{-1} \left(\frac{\tan x - \cot x}{\sqrt{2}}\right) + C \end{aligned}$$

22.  $x \frac{dy}{dx} = y - x \tan(y/x)$

$$\Rightarrow \frac{dy}{dx} = \left(\frac{y}{x}\right) - \tan\left(\frac{y}{x}\right)$$

उपरोक्त समीकरण समघात अवकल समीकरण है

$$\text{रखने पर } \frac{y}{x} = v \text{ i.e. } y = vx \Rightarrow dy/dx = v + x \frac{dv}{dx}$$

$$\therefore v + x \frac{dv}{dx} = v - \tan v$$

$$\Rightarrow x \frac{dv}{dx} = -\tan v$$

$$\Rightarrow \frac{dv}{\tan v} = -\frac{dx}{x}$$

दोनों तरफ समाकलन करने पर

$$\int \frac{dv}{\tan v} = \int \frac{-dx}{x}$$

$$\Rightarrow \int \cot v \, dv = -\int dx/x$$

$$\Rightarrow \log |\sin v| = -\log |x| + \log C$$

$$\Rightarrow \log |\sin v| = \log \left| \frac{C}{x} \right|$$

$$\Rightarrow \sin \left( \frac{y}{x} \right) = C/x$$

या  $x \sin \left( \frac{y}{x} \right) = C$

अथवा

$$\frac{dy}{dx} = -\left[ \frac{x + y \cos x}{1 + \sin x} \right]$$

$$\Rightarrow \frac{dy}{dx} + \left( \frac{\cos x}{1 + \sin x} \right) y = \frac{-x}{1 + \sin x} \quad \dots(i)$$

समीकरण (i) रैखिक अवकल समीकरण है  $\frac{dy}{dx} + Py = Q$ ;

जहाँ,  $P = \left( \frac{\cos x}{1 + \sin x} \right)$  and  $Q = \left( \frac{-x}{1 + \sin x} \right)$

$$\Rightarrow \text{I.F.} = e^{\int P dx} = e^{\int \frac{\cos x}{1 + \sin x} dx} = e^{\log |1 + \sin x|} = 1 + \sin x$$

अतः हल है

$$y \times \text{I.F.} = \int Q \times \text{I.F.} dx + C$$

$$\Rightarrow y \times (1 + \sin x) = \int \left( \frac{-x}{1 + \sin x} \right) \times (1 + \sin x) dx + C$$

$$\Rightarrow y(1 + \sin x) = \int -x \, dx + C$$

या  $y(1 + \sin x) = \frac{-x^2}{2} + C$

23. माना  $E_1 =$  थैले X को चुनना  
 $E_2 =$  थैले Y को चुनना  
 $E =$  एक सफेद और एक काली गेंद को निकालना

$$\Rightarrow P(E_1) = P(E_2) = \frac{1}{2}$$

$$\text{Also, } P\left(\frac{E}{E_1}\right) = \left(\frac{4}{6}\right)\left(\frac{2}{5}\right) + \left(\frac{2}{6}\right)\left(\frac{4}{5}\right) = \frac{16}{30}$$

$$P\left(\frac{E}{E_2}\right) = \left(\frac{3}{6}\right)\left(\frac{3}{5}\right) + \left(\frac{3}{6}\right)\left(\frac{3}{5}\right) = \frac{18}{30}$$

$$\begin{aligned} \text{आवश्यक प्रायिकता} = P(E_2/E) &= \frac{P(E_2) \times P(E/E_2)}{P(E_1) \times P(E/E_1) + P(E_2) \times P(E/E_2)} \quad (\text{बेज प्रमेय द्वारा}) \\ &= \frac{\frac{1}{2} \times \frac{18}{30}}{\left(\frac{1}{2} \times \frac{16}{30}\right) + \left(\frac{1}{2} \times \frac{18}{30}\right)} = \frac{9}{17} \end{aligned}$$

अथवा

माना A तथा B द्वारा पासों पर अंको का योग 10 आने की घटनायें क्रमशः  $A_i$  तथा  $B_i$  हैं।

$$P(A_i) = P(B_i) = \frac{3}{36} = \frac{1}{12}$$

$$P(\bar{A}_i) = P(\bar{B}_i) = \frac{11}{12}$$

$$\begin{aligned} P(\text{A के जीतने की प्रायिकता}) &= P(A) + P(\bar{A})P(\bar{B})P(A) + \dots \\ &= \frac{1}{12} + \frac{1}{12} \times \left(\frac{11}{12}\right)^2 + \frac{1}{12} \times \left(\frac{11}{12}\right)^4 + \dots \\ &= \frac{1}{12} \left[ 1 + \left(\frac{11}{12}\right)^2 + \left(\frac{11}{12}\right)^4 + \dots \right] = \frac{1}{12} \left[ \frac{1}{1 - \frac{11^2}{12^2}} \right] \end{aligned}$$

$$P(\text{A जीतने की प्रायिकता}) = \frac{1}{12} \times \left[ \frac{12^2}{144 - 121} \right] = \frac{12}{23}$$

$$P(\text{B के जीतने की प्रायिकता}) = 1 - P(\text{A के जीतने की प्रायिकता}) = 1 - \frac{12}{23} = \frac{11}{23}$$