Board of Secondary Education Rajasthan, Ajmer

Practice Question Paper Sr. Secondary Examination-2022

SUBJECT: MATHEMATICS
CLASS-XII

Time: 2 Hours 45 Minutes **Marks: 80**

(SECTION-A)

Q.1 Multiple Questions :

(i) If f = {(1,2), (3,5), (4,1)} and g={(1,3), (2,3), (5,1)} are two functions, then range of function gof is :

(A) {1,3,4} (B) {1,3} (C) {1,2,3} (D) {2,5} **[1 mark]**

(ii) Evaluate: **[1 mark]**

$\sin^{-1}\left(\sin\frac{2\pi}{3}\right)$

(A) $\frac{\pi}{3}$ (B) $\frac{2\pi}{3}$ (C) $\frac{\pi}{6}$ (D) None of these

(iii) If $A = \begin{bmatrix} \alpha & \beta \\ \gamma & -\alpha \end{bmatrix}$ is such that $A^2 = I$, then : **[1 mark]**

(A) $1 + \alpha^2 + \beta\gamma = 0$ (B) $1 - \alpha^2 + \beta\gamma = 0$ (C) $1 - \alpha^2 - \beta\gamma = 0$ (D) $1 + \alpha^2 - \beta\gamma = 0$

(iv) The value of the determinant $\begin{vmatrix} 1+a & 1 & 1 \\ 1 & 1+a & 1 \\ 1 & 1 & 1+a \end{vmatrix}$ is zero, then value of a is : **[1 mark]**

(A) –3 (B) 2 (C) 1 (D) 3

(v) If $y = \sqrt{\sin x + y}$, then $\frac{dy}{dx}$ is equal to **[1 mark]**

(A) $\frac{\cos x}{2y-1}$ (B) $\frac{\cos x}{1-2y}$ (C) $\frac{\sin x}{1-2y}$ (D) $\frac{\sin x}{2y-1}$

(vi) If $\int_0^a \frac{1}{9x^2+1}dx = \frac{\pi}{12}$, then a is equal to **[1 mark]**

(A) $\frac{\pi}{4}$ (B) $\frac{1}{3}$ (C) 3 (D) None of these

(vii) The integrating factor of differential equation $\cos x \frac{dy}{dx} + y\sin x = 1$ is : **[1 mark]**

(A) cos x (B) tan x (C) sec x (D) sin x

(viii) The unit vector in the direction of vector $\hat{i}+\hat{j}+\hat{k}$ is : **[1 mark]**

(A) $\frac{1}{\sqrt{3}}(\hat{i}+\hat{j}+\hat{k})$ (B) $\sqrt{3}(\hat{i}+\hat{j}+\hat{k})$ (C) $\frac{1}{\sqrt{2}}(\hat{i}+\hat{j}+\hat{k})$ (D) $\sqrt{2}(\hat{i}+\hat{j}+\hat{k})$

(ix) Three persons A, B and C, fire at a target in turn, starting with A. Their probabilities of hitting the target are 0.4, 0.3 and 0.2, respectively. The probability of two hits is **[1 mark]**

(A) 0.024 (B) 0.188 (C) 0.336 (D) 0.452

(x) If A = $\begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$, then A^n, is equal to : **[1 mark]**

(A) $\begin{bmatrix} 1 & 2n \\ 0 & 1 \end{bmatrix}$ (B) $\begin{bmatrix} 2 & n \\ 0 & 1 \end{bmatrix}$ (C) $\begin{bmatrix} 1 & 2^n \\ 0 & 1 \end{bmatrix}$ (D) $\begin{bmatrix} 1 & n \\ 0 & 2 \end{bmatrix}$

(xi) The derivative of $\cos^{-1}(2x^2 - 1)$ w.r.t. $\cos^{-1} x$ is **[1 mark]**

(A) 2 (B) $\frac{-1}{2\sqrt{1-x^2}}$ (C) $\frac{2}{x}$ (D) $1 - x^2$

(xii) The angle between the two vectors $\vec{a} = 2\hat{i}+\hat{j}-3\hat{k}$ and $\vec{b} = 3\hat{i}-2\hat{j}-2\hat{k}$ is : **[1 mark]**

(A) 30° (B) 60° (C) $\cos^{-1}\left(\frac{10}{\sqrt{238}}\right)$ (D) 90°

Q.2 Fill in the blanks :

(i) If f and g are two invertible functions such that their composite function gof be defined, then $(gof)^{-1}$ = ______________ **[1 mark]**

(ii) If $\tan^{-1}x + \tan^{-1}y = \frac{\pi}{4}$, then the value of x + y + xy is ________________ **[1 mark]**

(iii) Any square matrix A is symmetric if .... **[1 mark]**

(iv) For the curve $\sqrt{x} + \sqrt{y} = 1$, $\frac{dy}{dx}$ at $\left(\frac{1}{4}, \frac{1}{4}\right)$ is .... **[1 mark]**

(v) $\int_0^a \frac{1}{1+4x^2} dx = \frac{\pi}{8}$, then a = ... **[1 mark]**

(vi) The direction cosines of the vector $6\hat{i}+2\hat{j}-3\hat{k}$ are .... **[1 mark]**

Q.3 Short Answer Type Questions :

(i) If f : R→R, $f(x) = 2x - 3$ and g : R→R, $g(x) = x^3 + 5$, then find $(fog)^{-1}(x)$. **[1 mark]**

(ii) Find the value of $\cos^{-1}\left(\cos\frac{5\pi}{3}\right) + \sin^{-1}\left(\sin\frac{2\pi}{3}\right)$. **[1 mark]**

(iii) If matrix $A = \begin{bmatrix} 3 & -3 \\ -3 & 3 \end{bmatrix}$ and $A^2 = \lambda A$, then write the value of λ **[1 mark]**

(iv) If A and B are matrices of order 3 and $|A| = 5$, $|B| = 3$ then $|3AB|$ is equal to. **[1 mark]**

(v) If $y = \tan^{-1} x$, then find $\frac{d^2y}{dx^2}$ in terms of y alone. **[1 mark]**

(vi) Evaluate $\int \frac{\cos x - \sin x}{\sqrt{1+\sin 2x}} dx$ **[1 mark]**

(vii) Find the solution of the following differential equation : **[1 mark]**

$$x\sqrt{(1+y^2)}dx + y\sqrt{(1+x^2)}dy = 0$$

(viii) Find the area of parallelogram whose adjacent sides are $\vec{a} = \hat{i} - \hat{j} + 3\hat{k}$ and $\vec{b} = 2\hat{i} - 7\hat{j} + \hat{k}$.

[1 mark]

(ix) A box has 5 blue and 4 red balls. One ball is drawn at random and not replaced. Its colour is also noted. Then, another ball is drawn at random. What is the probability of second ball being blue?

[1 mark]

(x) Find the inverse of the matrix $\begin{bmatrix} 2 & -1 \\ 3 & 4 \end{bmatrix}$. **[1 mark]**

(xi) Verify that $x + y = \tan^{-1} y$ is a solution of differential equation $y^2y' + y^2 + 1 = 0$. **[1 mark]**

(xii) If the vectors $\vec{a} = 5\hat{i} + \lambda\hat{j}$ and $\vec{b} = 3\hat{i} + 9\hat{j}$ are parallel, then find the value of λ. **[1 mark]**

(SECTION-B)

Q.4 Consider $f : \mathbb{R} - \left\{-\frac{4}{3}\right\} \to R - \left\{\frac{4}{3}\right\}$ given by $f(x) = \frac{4x+3}{3x+4}$. Show that f is bijective. **[2 marks]**

Q.5 Given $A = \begin{bmatrix} 2 & -3 \\ -4 & 7 \end{bmatrix}$, compute A^{-1} and show that $2A^{-1} = 9I - A$. **[2 marks]**

Q.6 Using properties of determinants, prove that $\begin{vmatrix} a^2+2a & 2a+1 & 1 \\ 2a+1 & a+2 & 1 \\ 3 & 3 & 1 \end{vmatrix} = (a-1)^3$. **[2 marks]**

Q.7 If y = sin(sinx), prove that $\frac{d^2y}{dx^2}+\tan x\frac{dy}{dx}+y\cos^2 x=0$. **[2 marks]**

Q.8 Find : $\int \frac{x-5}{(x-3)^3}e^x\,dx$ **[2 marks]**

Q.9 A purse contains 3 silver and 6 copper coins and a second purse contains 4 silver and 3 copper coins. If a coin is drawn at random from one of the two purses, find the probability that it is a silver coin. **[2 marks]**

Q.10 If $A=\begin{bmatrix}1 & 3\\ 3 & 2\\ 2 & 5\end{bmatrix}$ and $B=\begin{bmatrix}-1 & -2\\ 0 & 5\\ 3 & 1\end{bmatrix}$ and A + B – D = 0 (zero matrix), then D matrix will be-

[2 marks]

Q.11 If x = a(cos 2t + 2t sin 2t) and y = a(sin 2t – 2t cos 2t), then find $\frac{d^2y}{dx^2}$. **[2 marks]**

Q.12 For what values of k, the system of linear equations

$x + y + z = 2$

$2x + y - z = 3$

$3x + 2y + kz = 4$

has a unique solution ? **[2 marks]**

Q.13 Evaluate : $\int_{\pi/6}^{\pi/3}\frac{dx}{1+\sqrt{\tan x}}$ **[2 marks]**

Q.14 Find the particular solution of the differential equation $\log\left(\frac{dy}{dx}\right)$ = 3x + 4y, given that y = 0 when x = 0. **[2 marks]**

Q.15 The scalar product of the vector $\hat{i}+\hat{j}+\hat{k}$ with the unit vector along the sum of vectors $2\hat{i}+4\hat{j}-5\hat{k}$ and $\lambda\hat{i}+2\hat{j}+3\hat{k}$ is equal to one. Find the value of λ. **[2 marks]**

Q.16 Of the students in a school, it is known that 30% have 100% attendance and 70% students are irregular. Previous year results report that 70% of all students who have 100% attendance attain A grade and 10% irregular students attain A grade in their annual examination. At the end of the year, one student is chosen at random from the school and he was found to have an A grade. What is the probability that the student has 100% attendance ? **[2 marks]**

(SECTION-C)

Q.17 Prove that $\tan^{-1}\left[\frac{\sqrt{1+x}-\sqrt{1-x}}{\sqrt{1+x}+\sqrt{1-x}}\right]=\frac{\pi}{4}-\frac{1}{2}\cos^{-1}x, \frac{-1}{\sqrt{2}}\le x\le 1$

OR

If $\tan^{-1}\left(\frac{x-2}{x-4}\right)+\tan^{-1}\left(\frac{x+2}{x+4}\right)=\frac{\pi}{4}$, find the value of x. **[3 marks]**

Q.18 If $(x^2 + y^2)^2 = xy$, find $\frac{dy}{dx}$.

OR

If $x = a(2\theta - \sin 2\theta)$ and $y = a(1 - \cos 2\theta)$, find $\frac{dy}{dx}$ when $\theta=\frac{\pi}{3}$. **[3 marks]**

Q.19 Evaluate : $\int \frac{(2x-5)e^{2x}}{(2x-3)^3}dx$

OR

Evaluate : $\int \frac{x^2+x+1}{(x^2+1)(x+2)}dx$ **[3 marks]**

Q.20 Find a vector $\vec{r}$ equally inclined to the three axes and whose magnitude is $3\sqrt{3}$ units.

OR

Find the angle between unit vectors $\vec{a}$ and $\vec{b}$ so that $\sqrt{3}\,\vec{a}-\vec{b}$ is also a unit vector. **[3 marks]**

(SECTION-D)

Q.21 Evaluate : $\int_0^{\pi} \frac{x\,dx}{a^2\cos^2 x+b^2\sin^2 x}$ **[4 marks]**

OR

Evaluate : $\int \frac{1}{\cos^4 x+\sin^4 x}dx$

Q.22 Solve the differential equation : $x\frac{dy}{dx}=y-x\tan\left(\frac{y}{x}\right)$

OR

Solve the differential equation : $\frac{dy}{dx}=-\left[\frac{x+y\cos x}{1+\sin x}\right]$ **[4 marks]**

Q.23 A bag X contains 4 white balls and 2 black balls, while another bag Y contains 3 white balls and 3 black balls. Two balls are drawn (without replacement) at random from one of the bags and were found to be one white and one black. Find the probability that the balls where drawn from bag Y.

OR

A and B throw a pair of dice alternately, till one of them gets a total of 10 and wins the game. Find their respective probabilities of winning, if A starts first. **[4 marks]**