MODEL QUESTION SET-2 : 2021-22
MATHEMATICS (THEORY)

MM : 80 **Time : 3 Hrs.**

SOLUTION

Entire Syllabus .

SECTION A

Q.1 Select and Write the correct Answer **16M**

i. **d)** $\frac{5\pi}{6}$ 2m

ii. **b)** 2, -1 2m

iii. **a)** $\frac{4}{5},\frac{3}{5},0$ 2m

iv. **a)** $\frac{-1}{2}$ 2m

v. **c)** 2, 2 2m

vi. **a)** $\frac{30}{91}$ 2m

vii. **d)** 9ab 2m

viii. **b)** $\frac{\pi^3}{192}$ 2m

Q.2 Answer the following (1 Mark Each) **4M**

i. Given $y=\tan^{-1}\left(\frac{4x}{1+5x^2}\right)$ 1m

$$=\tan^{-1}\left[\frac{5x-x}{1+5x\times x}\right]$$

$$=\tan^{-1}(5x)-\tan^{-1}(x)$$

Diff .w.r.t x

$$\frac{dy}{dx}=\frac{1\times 5}{1+(5x)^2}-\frac{1}{1+x^2}$$

ii. 1m

$(p \rightarrow q) \rightarrow p$

For writing the contrapositive of a logical statement we change the order and sign.

We first write the equivalent of

$p \rightarrow q \equiv \sim p \vee q$

$\therefore$ Given statement becomes

$(\sim p \vee q) \rightarrow p$

$\therefore$ Contrapositive is

$\sim p \rightarrow \sim (\sim p \vee q)$

i.e. $\sim p \rightarrow (p \wedge \sim q)$

iii. 1m

Since $\frac{9\pi}{4} \notin [0, \pi]$, we write

$$\cos^{-1}\left(\cos\frac{9\pi}{4}\right) = \cos^{-1}\cos\left(2\pi + \frac{\pi}{4}\right)$$

$$= \cos^{-1}\cos\left(\frac{\pi}{4}\right)$$

$$= \frac{\pi}{4}, \text{ since } \frac{\pi}{4} \in [0, \pi]$$

iv. 1m

Given $\frac{dy}{dx} = \frac{x}{y}$

$\Rightarrow y\,dy = x\,dx$

Integrating,

$\Rightarrow \int y\,dy = \int x\,dx + c$

$\Rightarrow \frac{y^2}{2} = \frac{x^2}{2} + c$

$y^2 = x^2 + 2c$ .....(i)

If x = 3 and y = 2

then 4 = 9 + 2c $\Rightarrow$ 2c = - 5

$\therefore$ Particular solution is $y^2 = x^2 - 5$.

SECTION B

Attempt Any Eight Questions **16M**

Q.3 2m

Given equation is $x^2 + kxy - 3y^2 = 0$

Comparing it with $ax^2 + 2hxy + by^2 = 0$,

We get a 1, 2h = k and b = - 3.

Let m_1 and m_2 be the slopes of the lines represented by $X^2 + kxy - 3y^2 = 0$

$\therefore m_1 + m_2 = \frac{-2h}{b} = \frac{k}{3}$ and $m_1 m_2 = \frac{a}{b} = \frac{-1}{3}$

According to the given condition,

$m_1 + m_2 = 2(m_1 m_2)$

$\Rightarrow \frac{k}{3} = 2\left(\frac{-1}{3}\right) = -2 \Rightarrow \frac{k}{3} = -\frac{2}{3}$

$\therefore k = -2$

Q.4 Let $\bar{a} = 2\hat{i} - 3\hat{j},\ \bar{b} = \hat{i} + \hat{j} - \hat{k},\ \bar{c} = 3\hat{i} - \hat{k}$ — 2m

Volume of parallelopiped

$= [\bar{a}\ \ \bar{b}\ \ \bar{c}]$

$= \begin{vmatrix} 2 & -3 & 0 \\ 1 & 1 & -1 \\ 3 & 0 & -1 \end{vmatrix}$

$= 2(-1 - 0) + 3(-1 + 3)$

$= -2 + 6$

$= 4$ cub. units

Q.5 Given $\frac{d\theta}{dt} = -k(\theta - \theta_0)$, — 2m

$\Rightarrow \frac{d\theta}{\theta - \theta_o} = -k\,dt$

Integrating both sides

$\int \frac{d\theta}{\theta - \theta_o} = \int -k\,dt$

$\therefore \log|\theta - \theta_0| = -kt + c_1$

Where c_1 is the constant of integration

$\therefore \theta - \theta_0 = e^{-kt+c_1} \quad \Rightarrow \theta = \theta_0 + e^{-kt}.e^{c_1}$

$\therefore \theta = \theta_0 + ce^{kt} \quad$ where $c = e^{c_1}$

Q.6 — 2m

$\text{Let } I = \int \frac{x\sin^{-1}(x^2)}{\sqrt{1-x^4}}\,dx$

$= \int \sin^{-1}(x^2)\cdot\left(\frac{x}{\sqrt{1-x^4}}\right)dx$

Put $\sin^{-1}(x^2) = t$

$$\frac{1\times 2x}{\sqrt{1-x^4}}dx = dt$$
$$\therefore \frac{x}{\sqrt{1-x^4}}dx = \frac{dt}{2}$$
$$\therefore I = \int t\cdot\frac{dt}{2} = \frac{1}{2}\int t\,dt$$
$$= \frac{1}{2}\times\frac{t^2}{2} = \frac{t^2}{4}+c$$
$$\therefore I = \frac{1}{4}[\sin^{-1}(x^2)]^2 + c$$

Q.7 The vectors along the given lines are : 2m

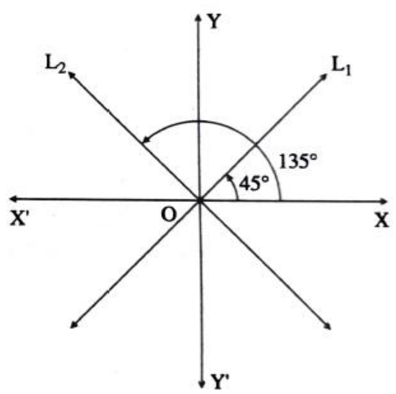
$\bar{a} = 3\hat{i} - 2\hat{j} + \hat{k}$ and $\bar{b} = 2\hat{i} + 4\hat{j} - 2\hat{k}$

The vector perpendicular to both the vectors is given by $\bar{a}\times\bar{b}$

$$\therefore \bar{a}\times\bar{b} = \begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 3 & -2 & 1\\ 2 & 4 & -2\end{vmatrix}$$
$$= \hat{i}(4-4) - \hat{j}(-6-2) + \hat{k}(12+4)$$
$$= 0\hat{i} + 8\hat{j} + 16\hat{k}$$

$\therefore$ The direction ratios of required line are 0, 8, 16 i.e. 0, 1, 2.

Q.8 Let L_1 and L_2 be the lines bisecting the angles between the coordinate axes. 2m



Then these lines make angles of 450 and 135° with positive direction of X-axis.

$\therefore$ slope of line L_1 = tan 45° = 1 and slope of line L_2 = tan 135°

= tan (180° - 45°)

= - tan45°= -1

Since these lines pass through the origin,

their equations are y = x and y = – x

i.e. x – y = 0 and x + y = 0

$\therefore$ the combined equation of the lines is (x - y)(x + y) = 0

$\therefore$ $x^2 - y^2 = 0$.

Q.9 2m

$$\int_2^a (x+1)\,dx = \frac{7}{2}$$

$$\therefore \left[\frac{x^2}{2}+x\right]_2^a = \frac{7}{2}$$

$$\therefore \left(\frac{a^2}{2}+a\right)-\left(\frac{4}{2}+2\right)=\frac{7}{2}$$

$$\therefore \frac{a^2}{2}+a-\frac{8}{2}=\frac{7}{2}$$

$$\therefore a^2+2a=15$$

$$\therefore a^2+2a-15=0, \quad \therefore (a+5)(a-3)=0$$

$$\therefore a+5=0 \text{ or } a-3=0$$

$$\therefore a=-5 \text{ or } a=3$$

Q.10 2m

$$A=\begin{bmatrix}-1 & 5\\ -3 & 2\end{bmatrix}$$

$$A_{11}=(-1)^2(2)=2, \quad A_{12}=(-1)^3(-3)=3$$

$$A_{21}=(-1)^3(5)=-5, \quad A_{22}=(-1)^4(-1)=-1$$

$$\therefore \text{Matrix of cofactors of } A=\begin{bmatrix}2 & 3\\ -5 & -1\end{bmatrix}$$

$$\therefore \text{adj } A=\begin{bmatrix}2 & -5\\ 3 & -1\end{bmatrix}$$

$$\text{Also } |A|=\begin{vmatrix}-1 & 5\\ -3 & 2\end{vmatrix}=-2+15=13\neq 0$$

$$\therefore A^{-1}=\frac{1}{|A|}(\text{adj} A)=\frac{1}{13}\begin{bmatrix}2 & -5\\ 3 & -1\end{bmatrix}$$

Q.11 Given $e^x + e^y = e^{x+y}$ .... (i) 2m

Diff. w.r.t. x

$$e^x + y^y \cdot \frac{dy}{dx} = e^{x+y} \cdot \left[1 + \frac{dy}{dx}\right]$$
$$= e^{x+y} + e^{x+y}\frac{dy}{dx}$$

$$\therefore e^y \cdot \frac{dy}{dx} - e^{x+y} \cdot \frac{dy}{dx} = e^{x+y} - e^x$$
$$\frac{dy}{dx}[e^y - e^{x+y}] = e^{x+y} - e^x$$

But $e^{x+y} = e^x + e^y$ by equation (i)

$$\therefore \frac{dy}{dx}[e^y - (e^x + e^y)] = e^x + e^y - e^x$$
$$\Rightarrow \frac{dy}{dx}[e^y - e^x - e^y] = e^y$$
$$\Rightarrow \quad \frac{dy}{dx}[-e^x] = e^y$$
$$\Rightarrow \frac{dy}{dx} = \frac{-e^y}{e^x} = -e^{y-x}$$

Q.12 The displacement s of a particle at time t is given by 2m

$s = t^3 - 4t^2 - 5t$

Velocity, $\upsilon = \frac{ds}{dt} = 3t^2 - 8t - 5$

Velocity at $t = 2$ is

$\upsilon = 3(2)^2 - 8(2) - 5 = 12 - 16 - 5 = -9$.

$$\frac{d^2s}{dt^2} = 6t - 8.$$

Acceleration = $\left(\frac{d^2s}{dt^2}\right)_{t-2} = 6(2) - 8 = 4$

Q.13 Let $I = \int x^3 \cdot \log x \cdot dx$ 2m

$$= \int \log x . x^3 dx$$

Integrating by parts

$$I=\log x.\int x^3dx-\int\left[\frac{d}{dx}(\log x)\cdot\int x^3dx\right]dx$$

$$I=\log x\cdot\left(\frac{x^4}{4}\right)-\int\frac{1}{x}\left(\frac{x^2}{4}\right)dx$$

$$=\frac{x^2.\log x}{4}-\frac{1}{4}\int x^2dx$$

$$=\frac{x^4\cdot\log x}{4}-\frac{1}{4}\times\frac{x^4}{4}+c$$

$$=\frac{x^4\cdot\log x}{4}-\frac{x^4}{16}+c$$

Q.14 Here $n = 6$ — 2m

Let p = Getting head in a single toss (success)

$\therefore\ p=\frac{1}{2},\quad \therefore\ q=1-p=\frac{1}{2}$

Let x = Number of heads out of 6.

Then $X \sim B\left(n=6, p=\frac{1}{2}\right)$

The p.m.f. of X is given by

$$P(X=x)=P(x)={}^6C_x\left(\frac{1}{2}\right)^x\left(\frac{1}{2}\right)^{6-x}$$

$x = 0, 1, 2, 3, 4, 5, 6$

$$\therefore\ P(X=4)=P(4)={}^6C_4\left(\frac{1}{2}\right)^4\left(\frac{1}{2}\right)^2$$

$$=\frac{6\times5}{2}\cdot\frac{1}{16}\cdot\frac{1}{4}=\frac{15}{64}=0.234$$

SECTION C

Attempt Any Eight Questions — **24M**

Q.15 Let $a=\frac{\pi}{6}$ *and* $\gamma=\frac{\pi}{3}$ are the angles made by X and Z axes respectively. — 3m

Let β be the angle made by a line with Y-axis

Now $\cos^2\alpha+\cos^2\beta+\cos^2\gamma=1.$

$$\cos^2\left(\frac{\pi}{6}\right)+\cos^2\beta+\cos^2\left(\frac{\pi}{3}\right)=1$$

$$\left(\frac{\sqrt{3}}{2}\right)^2+\cos^2\beta+\left(\frac{1}{2}\right)^2=1$$

$$\Rightarrow \frac{3}{4}+\cos^2\beta+\left(\frac{1}{2}\right)^2=1$$

$$\Rightarrow \quad 1+\cos^2\beta=1$$

$$\Rightarrow \quad \cos^2\beta=0$$

$$\Rightarrow \quad \cos\beta=0$$

$$\therefore \beta=\cos^{-1}(0)=\frac{\pi}{2}$$

Q.16 3m

The vectors along the lines are

$\overline{a}=\hat{i}+2\hat{j}+2\hat{k}$ and $\overline{b}=3\hat{i}+2\hat{j}+6\hat{k}$

The angle between the given lines is the same as the angle between $\overline{a}$ and $\overline{b}$.

Let θ be the acute angle between $\overline{a}$ and $\overline{b}$.

Then

$$\cos\theta=\frac{|\overline{a}\cdot\overline{b}|}{|\overline{a}|\,|\overline{b}|} \qquad \ldots(1)$$

Now $\overline{a}\cdot\overline{b}=\left(\hat{i}+2\hat{j}+2\hat{k}\right)\cdot\left(3\hat{i}+2\hat{j}+6\hat{k}\right)$

$= (1)(3) + (2)(2) + (2)(6)$

$= 3 + 4 + 12 = 19$

$|\overline{a}| = \sqrt{1+4+4}=\sqrt{9}=3$

$|\overline{b}| = \sqrt{9+4+36}=\sqrt{49}=7$

$\therefore$ From (1):

$$\cos\theta=\frac{|19|}{(3)(7)}=\frac{19}{21}$$

$$\therefore \quad \theta=\cos^{-1}\left(\frac{19}{21}\right)$$

Q.17 3m

$$\log\left(\frac{dy}{dx}\right)=3x+4y$$

$$\frac{dy}{dx}=e^{3x+4y}=e^{3x}\cdot e^{4y}$$

$$\frac{dy}{e^{4y}}=e^{3x}dx$$

Integrating both sides, we get

$$\int e^{-4y}dy=\int e^{3x}dx$$

$$\therefore \frac{e^{-4y}}{-4}=\frac{e^{3x}}{3}+c \qquad ....(i)$$

When $x = 0$, y = 0

$$\frac{e^0}{-4}=\frac{e^o}{3}+c\Rightarrow\frac{1}{-4}=\frac{1}{3}+c$$

$$\therefore c=\frac{1}{-4}-\frac{1}{3}=\frac{-7}{12}.$$

$\therefore$ Equation (i) becomes

$$\frac{e^{-4y}}{-4} = \frac{e^{3x}}{3} - \frac{7}{12}$$
$$4e^{3x} + 3e^{-4y} - 7 = 0$$

Q.18 3m

Number on the cards are 1, 1, 2, 3

X = sum of the numbers on two cards

Then X = 2, 3, 4, 5

Also 2 cards can be drawn from 4 cards in $^4C_2$ ways.

$\therefore$ $n(S) = {^4C_2} = 6$

Only one point is favourable for the sum 2 i.e. (1, 1)

$\therefore$ $P(X = 2) = \frac{1}{6}$

The points favourable for the sum 3 are (1, 2) and (1, 2)

$\therefore$ $P(X = 3) = \frac{2}{6}$

Similary $P(X = 4) = \frac{2}{6}$, $P(X = 5) = \frac{1}{6}$

$\therefore$ The probability distribution is

X	2	3	4	5
P (X)	$\frac{1}{6}$	$\frac{2}{6}$	$\frac{2}{6}$	$\frac{1}{6}$

$\therefore$ $E(X) = (2)\left(\frac{1}{6}\right) + (3)\left(\frac{2}{6}\right) + 4\left(\frac{2}{6}\right) + (5)\left(\frac{1}{6}\right)$

$$= \frac{2}{6} + \frac{6}{6} + \frac{8}{6} + \frac{5}{6}$$
$$= \frac{21}{6} = \frac{7}{2} = 3.5$$

Q.19 3m

Mean $= np = 4$ ...(1)

Variance $= npq = \frac{4}{3}$ ...(2)

From (1) and (2)

$$\frac{npq}{np} = \frac{4}{3} \times \frac{1}{4}$$

$\therefore$ $q = \frac{1}{3}$, $p = 1 - q = 1 - \frac{1}{3} = \frac{2}{3}$

From (1) : $n \cdot \frac{2}{3} = 4$, $\therefore n = 6$

$\therefore$ $P(x \geq 2) = 1 - P(x = 0) - P(x = 1)$

$$= 1 - {^6C_0}\left(\frac{2}{3}\right)^0\left(\frac{1}{3}\right)^6 - {^6C_1}\left(\frac{2}{3}\right)\left(\frac{1}{3}\right)^5$$
$$= 1 - \left(\frac{1}{3}\right)^6 - 6 \cdot \frac{2}{3}\left(\frac{1}{3}\right)^5$$
$$= 1 - \left(\frac{1}{3}\right)^5\left[\frac{1}{3} + 4\right]$$
$$= 1 - \frac{1}{243} \times \frac{13}{3}$$
$$= 1 - \frac{13}{729} = \frac{716}{729}$$

Q.20 3m

Let $I = \int \frac{1}{x(x-2)(x-4)} dx$

$$\int \frac{A}{x} + \frac{B}{x-2} + \frac{C}{x-4} dx$$

$$Let \frac{1}{x(x-2)(x-4)} = \frac{A}{x} + \frac{B}{x-2} + \frac{C}{x-4}$$
$$\Rightarrow 1 = A(x-2)(x-4) + Bx(x-4) + Cx(x-2)$$

By putting $x = 2$, we get B = $\frac{-1}{4}$

By putting $x = 4$, we get C = $\frac{1}{8}$

By putting $x = 0$, we get A = $\frac{1}{8}$

$$\therefore I = \int \frac{1}{8x} dx + \int \frac{-1}{4(x-2)} dx + \int \frac{1}{8(x-4)} dx$$
$$\therefore I = \frac{1}{8}\log|x| - \frac{1}{4}\log|x-2| + \frac{1}{8}\log|x-4| + c$$

Q.21 Given equation of plane is 3m

$$\bar{r} = (\bar{i} + \bar{j}) + s(\bar{i} - \bar{j} + 2\bar{k}) + t(\bar{i} + 2\bar{j} + \bar{k})$$

$\because$ Point P(1, 1, 0) lies on the plane. Plane is parallel to vectors $\bar{i} - \bar{j} + 2\bar{k}$ and $\bar{i} + 2\bar{j} + \bar{k}$

$\therefore$ Normal is perpendicular to the vectors $(\bar{i} - \bar{j} + 2\bar{k})$ and $(\bar{i} + 2\bar{j} + \bar{k})$

$$\therefore \bar{n} = \begin{vmatrix} \bar{i} & \bar{j} & \bar{k} \\ 1 & -1 & 2 \\ 1 & 2 & 1 \end{vmatrix}$$
$$= \bar{i}(-1-4) - \bar{j}(1-2) + \bar{k}(2+1)$$
$$= -5\bar{i} + \bar{j} + 3\bar{k}$$

$\therefore$ Equation of plane is $\bar{r}.\bar{n} = \bar{p}.\bar{n}$

$$\therefore \bar{r}.(-5\bar{i} + \bar{j} + 3\bar{k}) = (\bar{i} + \bar{j})\cdot(-5\bar{i} + \bar{j} + 3\bar{k})$$
$$\Rightarrow \bar{r}.(-5\bar{i} + \bar{j} + 3\bar{k}) = -5 + 1 + 0 = -4$$

For cartesian equation, put $\bar{r} = x\bar{i} + y\bar{j} + z\bar{k}$

$$\therefore (x\bar{i} + y\bar{j} + z\bar{k})\cdot(-5\bar{i} + \bar{j} + 3\bar{k}) = -4$$
$$\Rightarrow \quad -5x + y + 5z = -4$$
$$\Rightarrow \quad 5x - y - 3z = 4$$

Q.22 3m

$$f(x) = 5 + 36x + 3x^2 - 2x^3$$
$$\therefore f'(x) = 36 + 6x - 6x^2$$
$$= 6(6 + x - x^2)$$
$$= -6(x^2 - x - 6)$$
$$= -6(x-3)(x+2)$$

$f(x)$ increases for $f'(x) > 0$

i.e. $-6(x-3)(x+2) > 0$
i.e. $(x-3)(x+2) < 0$
i.e. $x-3 < 0$ and $x+2 > 0$
i.e. $x < 3$ and $x > -2$
OR
$x-3 > 0$ and $x+2 < 0$
i.e. $x > 3$ and $x < -2$
This is not possible.
$\therefore$ $x < 3$ and $x > -2$
i.e. $-2 < x < 3$
$\therefore$ $f(x)$ increases if $x \in (-2, 3)$.

Q.23 3m

$$\int_{-1}^{1} f(x)\,dx = \int_{-1}^{0} f(x)\,dx + \int_{0}^{1} f(x)\,dx$$

$$= \int_{-1}^{0} (4-3x)\,dx + \int_{0}^{1} (3x+4)\,dx$$

$$= \left[4x - 3\frac{x^2}{2}\right]_{-1}^{0} + \left[3\left(\frac{x^2}{2}\right) + 4x\right]_{0}^{1}$$

$$= \left[0 - \left(-4 - \frac{3}{2}\right)\right] + \left[\frac{3}{2} + 4 - 0\right]$$

$$= 4 + \frac{3}{2} + \frac{3}{2} + 4 = 11$$

Q.24 3m

Let $\sin^{-1}\left(\frac{3}{5}\right) = x \quad \ldots.(i)$

$\sin x = \frac{3}{5}, where\, 0 < x < \frac{\pi}{2}$
$\cos x > 0$

$$Now, \cos x = \sqrt{1-\sin^2 x} = \sqrt{1 - \frac{9}{25}}$$

$$= \sqrt{\frac{16}{25}} = \frac{4}{5}$$

$$\tan x = \frac{\sin x}{\cos x} = \frac{(3/5)}{(4/5)} = \frac{3}{4}$$

$$x = \tan^{-1}\left(\frac{3}{4}\right) \quad \ldots\ldots\ldots..(ii)$$

Form (i) and (ii)

$\therefore \sin^{-1}\left(\frac{3}{5}\right)=\tan^{-1}\left(\frac{3}{4}\right)$

Now L.H.S. $=2\sin^{-1}\left(\frac{3}{5}\right)$

$=2\tan^{-}\left(\frac{3}{4}\right)$

$=\tan^{-1}\left(\frac{3}{4}\right)+\tan^{-1}\left(\frac{3}{4}\right)$

$=\tan^{-1}\left[\frac{\frac{3}{4}+\frac{3}{4}}{1-\frac{3}{4}\times\frac{3}{4}}\right]$

$=\tan^{-1}\left(\frac{12+12}{16-9}\right)$

$=\tan^{-1}\left(\frac{24}{7}\right)=R.H.S.$

Q.25 $[(p \vee q) \wedge \sim p] \rightarrow q$ 3m

$= [(p \wedge \sim p) \vee (q \wedge \sim p)] \rightarrow q$

....Distributive Law)

$= [F \vee (q \wedge \sim p)] \rightarrow q$ ....(Complement Law)

$= (q \wedge \sim p) \rightarrow q$ .... (Identity Law)

$= \sim (q \wedge \sim p) \vee q$

.... (Implication equivalence)

$= (\sim q \vee p) \vee q$ .... (Demorgan's Law)

$= (\sim q \vee q) \vee p$ .... (Asspcoative Law)

$= T \vee p$ .... (Complement Law)

$= T$ .... (Identity Law)

Hence proved.

Q.26 Let $I=\int e^{\tan^{-1}x}\cdot\left[\frac{1+x+x^2}{1+x^2}\right]dx$ 3m

Put $\tan^{-1}x = t \Rightarrow x = \tan t$

$\Rightarrow \frac{1}{1+x^2}dx = dx$

$\therefore I = \int e^t[1+\tan t+\tan^2 t]dt$

$= \int e^t[(1+\tan^2 t)+\tan t].dt$

$= \int e^t[\sec^2 t+\tan t]dt$

$\therefore I = \int e^t[\tan t+\sec^2 t]dt$

$= e^t.\tan t + c$

.....by using the result

$\int e^x[f(x)+f'(x)]dx = e^x.f(x)+c$

$\therefore I = e^{\tan^{-1}x}.x + c$

SECTION D

Attempt Any Five Questions **20M**

Q.27 4m

$[p \wedge (\sim p \vee q)] \vee (\sim p \wedge q) \vee [(p \vee \sim q) \wedge r]$

$= [(p \wedge \sim p) \vee (p \wedge q)] \vee (\sim p \wedge q) \vee [(p \vee \sim q) \wedge r]$ ...(Distributive law)

$= [F \vee (p \wedge q)] \vee (\sim p \wedge q) \vee [(p \vee \sim q) \wedge r]$ ...(Complement law)

$= (p \wedge q) \vee (\sim p \wedge q) \vee [(p \vee \sim q) \wedge r]$ ...(Identity law)

$= [(p \vee \sim p) \wedge q] \vee [(p \vee \sim q) \wedge r]$ ... (Distributive law)

$= [T \wedge q] \vee [(p \vee \sim q) \wedge r]$ ...(Complement law)

$= q \vee [(p \vee \sim q) \wedge r]$ ....(Identity law)

$= [q \vee (p \vee \sim q)] \wedge (q \vee r)$ ...(Distributive law)

$= [(q \vee \sim q) \vee p] \wedge (q \vee r)$ ...(Associative law)

$= (T \vee p) \wedge (q \wedge r)$ ...(Complement law)

$= T \wedge (q \vee r)$ ...(Identity law)

$= q \vee r$ ...(Identity law)

Q.28 The reuired matrix equation is 4m

$$\begin{bmatrix} 1 & -1 & 1 \\ 2 & 1 & -3 \\ 1 & 1 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 4 \\ 0 \\ 2 \end{bmatrix}$$

This is of the form AX=B

$\therefore X = A^{-1}B$ …… (1)

Where, $A=\begin{bmatrix}1 & -1 & 1\\ 2 & 1 & -3\\ 1 & 1 & 1\end{bmatrix} X=\begin{bmatrix}x\\ y\\ z\end{bmatrix} \text{ and } B=\begin{bmatrix}4\\ 0\\ 2\end{bmatrix}$

Now $|A|=\begin{vmatrix}1 & -1 & 1\\ 2 & 1 & -3\\ 1 & 1 & 1\end{vmatrix} = 1(1+3)+1(2+3)+1(2-1)$
$= 4+5+1 = 10 \neq 0$

$\therefore A^{-1}$ exists

Co-factor matrix of $A=\begin{vmatrix}4 & -5 & 1\\ 2 & 0 & -2\\ 2 & 5 & 3\end{vmatrix}$

$\therefore$ Adj A = Transpose of the co-factor matrix

$=\begin{bmatrix}4 & 2 & 2\\ -5 & 0 & 5\\ 1 & -2 & 3\end{bmatrix}$

$\therefore A^{-1} = \frac{1}{|A|}(adj\ A) = \frac{1}{10}\begin{bmatrix}4 & 2 & 2\\ -5 & 0 & 5\\ 1 & -2 & 3\end{bmatrix}$

Hence, using (1)

$\therefore X = A^{-1}B = \frac{1}{10}\begin{bmatrix}4 & 2 & 2\\ -5 & 0 & 5\\ 1 & -2 & 3\end{bmatrix}\begin{bmatrix}4\\ 0\\ 2\end{bmatrix}$

$\therefore X = \frac{1}{10}\begin{bmatrix}20\\ -10\\ 10\end{bmatrix} = \begin{bmatrix}2\\ -1\\ 1\end{bmatrix}$

$\therefore x=2,\ y=-1, z=1$

Q.29 4m

By the sine rule,

$\frac{a}{\sin A} - \frac{b}{\sin B} = \frac{c}{\sin C} = k$

$\therefore a = k \sin A,\ b = k \sin B,\ c = k \sin C$

L.H.S. $= a^3\sin(B - C) + b^3\sin(C - A) + c^3\sin(A - B)$

$= a^3(\sin B \cos C - \cos B \sin C) + b^3(\sin C \cos A - \cos C \sin A) + c^3(\sin A \cos B - \cos A \sin B)$

$= a^3\left(\frac{b}{k}\cos C - \frac{c}{k}\cos B\right) + b^3\left(\frac{c}{k}\cos A - \frac{a}{k}\cos C\right) + c^3\left(\frac{a}{k}\cos B - \frac{b}{k}\cos A\right)$

$= \frac{1}{k}\left[a^3 b\cos C - a^3 c\cos B + b^3 c\cos A - b^3 a\cos C + c^3 a\cos B - c^3 b\cos A\right]$

$= \frac{1}{k}\left[a^3 b\left(\frac{a^2+b^2-c^2}{2ab}\right) - a^3 c\left(\frac{c^2+a^2-b^2}{2ca}\right) + b^3 c\left(\frac{b^2+c^2-a^2}{2bc}\right) - ab^3\left(\frac{a^2+b^2-c^2}{2ab}\right) + ac^3\left(\frac{c^2+a^2-b^2}{2ca}\right) - bc^3\left(\frac{b^2+c^2-a^2}{2bc}\right)\right]$

...[By cosine rule]

$= \frac{1}{2k}\left[a^2(a^2+b^2-c^2) - a^2(a^2+c^2-b^2) + b^2(b^2+c^2-a^2) - b^2(a^2+b^2-c^2) + c^2(c^2+a^2-b^2) - c^2(b^2+c^2-a^2)\right]$

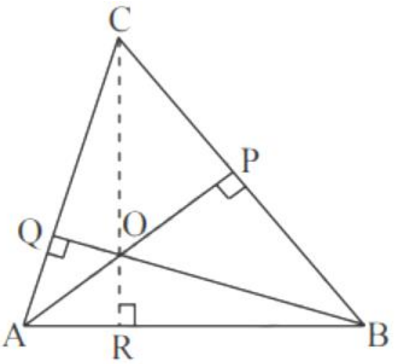
$= \frac{1}{2k}\left[a^4 + a^2b^2 - a^2c^2 - a^4 - a^2c^2 + a^2b^2 + b^4 + b^2c^2 - a^2b^2 - a^2b^2 - b^4 + b^2c^2 + c^4 + a^2c^2 - b^2c^2 - b^2c^2 - c^4 + a^2c^2\right]$

$= \frac{1}{2k}(0)$

$= 0$

= R.H.S.

Q.30 4m



Consider ΔABC.

Let AP ⊥ BC and BQ ⊥ AC.

Let AP and BQ intersect at O.

Join OC and extend OC to meet AB at R.

To prove that CR is also the altitude of ΔABC.

i.e., to prove that CR ⊥ AB

Let $\bar{a}, \bar{b}, \bar{c}$ be the position vectors of the points A, B, C respectively.

Consider $\overline{AP} \perp \overline{BC}$

$\therefore \overline{AO} \perp \overline{BC}$

$\therefore \overline{AO} \cdot \overline{BC} = 0$

$\therefore -\bar{a} \cdot \left(\bar{c} - \bar{b}\right) = 0$ .... $\left[\because \overline{AO} = -\overline{OA}\right]$

$\therefore \bar{a} \cdot \bar{c} - \bar{a}.\bar{b} = 0$ ....(i)

Now, $\overline{BQ} \perp \overline{AC}$

$\therefore \overline{BO} \perp \overline{AC}$

$\therefore \overline{BO} \cdot \overline{AC} = 0$

$\therefore -\bar{b} \cdot \left(\bar{c} - \bar{a}\right) = 0$ .... $\left[\because \overline{BO} = -\overline{OB}\right]$

$\therefore \bar{b} \cdot \bar{c} - \bar{b} \cdot \bar{a} = 0$ ....(ii)

Comparing equations (i) and (ii), we get

$\therefore \bar{a}\cdot\bar{c} - \bar{a}\cdot\bar{b} = \bar{b}\cdot\bar{c} - \bar{b}\cdot\bar{a}$

$\therefore \bar{a}\cdot\bar{c} = \bar{b}\cdot\bar{c}$

$\therefore \bar{a}\cdot\bar{c} - \bar{b}\cdot\bar{c} = 0$

$\therefore \bar{c}\cdot\left(\bar{a}-\bar{b}\right) = 0$

$\therefore -\bar{c}\cdot\left(\bar{a}-\bar{b}\right) = 0$

$\therefore \overline{CO} \perp \overline{BA}$

$\therefore \overline{CR} \perp \overline{BA}$

$\therefore$ CR ⊥ BA

$\therefore$ CR is also the altitude of ΔABC.

$\therefore$ AP, BQ, CR intersect at O.

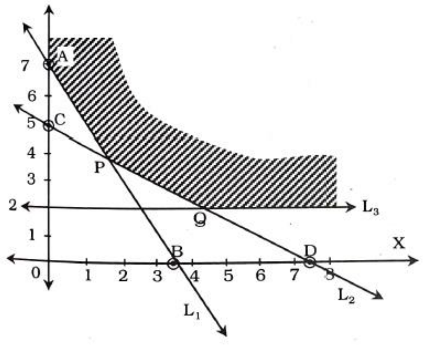
$\therefore$ All three altitudes of ΔABC intersect at a common point.

Thus, the altitudes of a triangle are concurrent.

Q.31 4m

To draw $2x + y \geq 7$, $2x + 3y \geq 15$, $y \geq 2$,
We draw lines $2x + y = 7$, $2x + 3y = 15$ and $y = 2$

To draw	x	y	Line passes through (x, y)	Sign	Region lies on
L_1 $2x + y = 7$	0	7	A (0, 7)	≥	Non-origin side of line L_1
	3.5	0	B (3.5, 0)		
L_2 $2x + 3y = 15$	0	5	C (0, 5)	≥	Non-origin side of line L_2
	7.5	0	D (7.5, 0)		
L_3 $y = 2$			Parallel to X-axis	≥	Non-origin side of line L_3



Solving equations of L_1 and L_2

$2x + y = 7$
$2x + 3y = 15$
$-2y = -8$

$\therefore y = 4, \; x = \frac{3}{2}$

$\therefore P = \left(\frac{3}{2}, 4\right)$

Solving equations of L_2 and L_3

$Q = \left(\frac{9}{2}, 2\right)$

$\therefore$ The common shaded region is the feasible region with vertices

A (0, 7), P $\left(\frac{3}{2}, 4\right)$, Q $\left(\frac{9}{2}, 2\right)$

Corner points	$z = 8x + 10y$
A (0, 7)	70
P $\left(\frac{3}{2}, 4\right)$	52
Q $\left(\frac{9}{2}, 2\right)$	56

From the table minimum value of z is 52 at $x = \frac{3}{2}, y = 4$

Q.32 4m

$x^y = e^{x-y}$

Taking logarithm of both the sides

$y \log x = (x - y) \log e$

$= x - y \quad ...(1)$

Diff. both the sides w.r.t. x ,

$y\frac{1}{x} + \log x.\frac{dy}{dx} = 1 - \frac{dy}{dx}$

$\therefore \log x.\dfrac{dy}{dx}+\dfrac{dy}{dx}=1-\dfrac{y}{x}$

$\therefore (1+\log x)\dfrac{dy}{dx}=\dfrac{x-y}{x}$

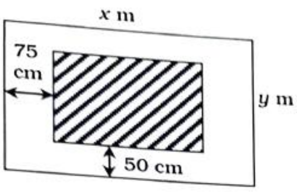
$\therefore (1+\log x)\dfrac{dy}{dx}=\dfrac{y\log x}{x}$ ...[by (1)]

$\dfrac{dy}{dx}=\dfrac{y\log x}{x.(1+\log x)}$

$=\dfrac{\log x}{(1+\log x)^2}$

$\left[\because \text{from (1) } \dfrac{y}{x}=\dfrac{1}{1+\log x}\right]$

Q.33 4m



Let length of the paper sheat be x meter and breadth be y meter.

$\therefore$ Area $= xy = 24$ (given)

$\therefore$ $y=\dfrac{24}{x}$ ... (1)

Now length of printed space (shaded) in the figure $= (x-1.5)$m and breadth $= (y-1)m$

$\therefore$ Area of printed space $= (x-1.5)(y-1)$

$= xy - x - 1.5y + 1.5$

Put $y=\dfrac{24}{x}$

$= x\cdot\dfrac{24}{x}-x-1.5\left(\dfrac{24}{x}\right)+1.5$

$= 25.5-x-\dfrac{36}{x}$

Let $f(x)=25.5-x-\dfrac{36}{x}$

$\therefore$ $f'(x)=-1+\dfrac{36}{x^2}$

and $f''(x)=-\dfrac{72}{x^3}$

For extreme values of $f(x)$, put $f'(x)=0$

$\therefore$ $-1+\dfrac{36}{x^2}=0$

$\therefore$ $\dfrac{36}{x^2}=1$, $x=6$

(Neglecting negative root as dimension can not be negative)

Now $f''(6)=-\dfrac{72}{216}<0$

$\therefore$ $f(x)$ is maximum at $x = 6$

From (1) : $y=\dfrac{24}{6}=4$

$\therefore$ Length = 6 m & breadth = 4 m

Q.34 Equation of the circle is $x^2 + y^2 = 16$ ..... (i) 4m

Equation of line is $y = x$ .... (ii)

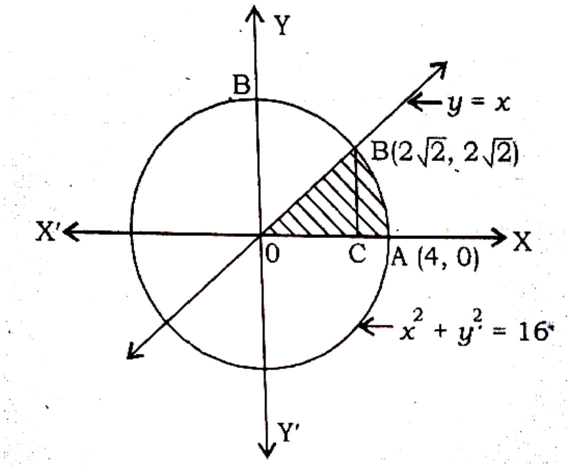
Solving (i) and (ii), we get

$x^2 + y^2 = 16$

$\Rightarrow 2x^2 = 16$ .....from (ii)

$\Rightarrow x = \pm 2\sqrt{2}$

$\therefore x = 2\sqrt{2} = y$ ($\because$ Area is in the first quasrant)



The Point of intersection of curve (i) and (ii) in the first quadrant is $B(2\sqrt{2}, 2\sqrt{2})$.

Draw BC perpendicular to X-axis

∴ A = Area of ΔOCB + Area of region CABC

$$= \int_0^{2\sqrt{2}} x\,dx + \int_{2\sqrt{2}}^{4} \sqrt{16 - x^2}\,dx$$

$$= \frac{1}{2}\left[x^2\right]_0^{2\sqrt{2}} + \left[\frac{x}{2}\sqrt{16 - x^2} + \frac{16}{2}\sin^{-1}\left(\frac{x}{4}\right)\right]_{2\sqrt{2}}^{4}$$

$$= \frac{1}{2}[8 - 0] + \left\{8\sin^{-1}(1) - \left[\frac{2\sqrt{2}}{2}\sqrt{8} + 8.\sin^{-1}\left(\frac{1}{\sqrt{2}}\right)\right]\right\}$$

$$= 4 + 8\cdot\frac{\pi}{2} - 4 - 8\cdot\frac{\pi}{4}$$

$$= 8\left(\frac{\pi}{2} - \frac{\pi}{4}\right)$$

$$\therefore A = 2\pi \text{ sq. units}$$

Together we will make a difference