MODEL QUESTION PAPER - SET-2 : 2021-22
MATHEMATICS (THEORY)

MM : 80 **Time : 3 Hrs.**

Entire Syllabus

The question paper is divided into Four Sections:

(1) Section A : Q. No. 1 contains Ten multiple choice type of questions carrying **Two marks** each.
Q. No. 2 contains Eight very short answer type of questions carrying **One mark** each.

(2) Section B: Q. No. 3 to Q. No. 14 contains Twelve short answer type of questions carrying **Two marks** each. Internal choice is provided (Any 8)

(3) Section C: Q. No. 15 to Q. No. 26 contains Twelve short answer type of questions carrying **Three marks** each. Internal choice is provided (Any 8)

(4) Section D: Q. No. 27 to Q. No. 34 contains Eight long answer type of questions carrying **Four marks** each. Internal choice is provided (Any 4)

(5) Use log – Table if necessary. Use of Calculator is not allowed.

SECTION A

Q.1 Select and Write the correct Answer **16M**

i. The principle solution of equation $\cot x = -\sqrt{3}$ is 2m

a) $\frac{\pi}{3}$ **b)** $\frac{2\pi}{3}$ **c)** $\frac{\pi}{6}$ **d)** $\frac{5\pi}{6}$

ii. If $\bar{a} = \hat{i} + 2\hat{j}, \bar{b} = -2\hat{i} + \hat{j}$ and $\bar{c} = 4\hat{i} + 3\hat{j}$, then the values of *x* and *y* such that $\bar{c} = x\bar{a} + y\bar{b}$ are 2m

a) 1,1 **b)** 2, -1 **c)** -1, 2 **d)** 1, 0

iii. The d.c.s. of line $\frac{x-2}{2} = \frac{2y-5}{3}; z = -1$ 2m

a) $\frac{4}{5}, \frac{3}{5}, 0$ **b)** $\frac{4}{5}, 0, \frac{3}{5}$ **c)** $\frac{-4}{5}, 0, \frac{3}{5}$ **d)** $0, \frac{4}{5}, \frac{3}{5}$

iv. If $y = \cot^{-1}\left(\frac{\sin x}{1+\cos x}\right)$ then $\frac{dy}{dx}$ is 2m

a) $\frac{-1}{2}$ **b)** $\frac{1}{2}$ **c)** 0 **d)** 1

v. The order and degree of the differential equation $\sqrt{1+\left(\frac{dy}{dx}\right)^2} = \left(\frac{d^2y}{dx^2}\right)^{\frac{1}{3}}$ is 2m

a) 2, 3 **b)** 3, 2 **c)** 1, 2 **d)** 2, 2

vi. The probability distribution of discrete random variable is defined as

$f(x) = kx^2$; $0 \le x \le 6$
0 ; elsewhere 2m

Then the value of F(4) is

a) $\frac{30}{91}$ **b)** $\frac{30}{97}$ **c)** $\frac{15}{47}$ **d)** $\frac{47}{99}$

vii. If slope of one of the lines represented by $ax^2 + 2hxy + by^2 = 0$ is 5 times the slope of the other, then $5h^2 =$ 2m

a) ab **b)** 2ab

c) 7ab **d)** 9ab

viii. $\int_0^1 \frac{(\tan^{-1} x)^2}{1+x^2} dx =$ 2m

a) $\frac{\pi^3}{187}$ **b)** $\frac{\pi^3}{192}$ **c)** $\frac{\pi^2}{192}$ **d)** $\frac{\pi^3}{172}$

Q.2 **Answer the following (1 Mark Each)** **4M**

i. Differentiate the following w.r.t. x $y = \tan^{-1}\left(\frac{4x}{1+5x^2}\right)$ 1m

ii. Write the contrapositive of the statement $(p \to q) \to p$. 1m

iii. Find the value of $\cos^{-1}\left(\cos\frac{9\pi}{4}\right)$. 1m

iv. Find the particular solution of the differential equation $\frac{dy}{dx} = \frac{x}{y}$, when x = 3 and y = 2. 1m

SECTION B

Attempt Any Eight Questions **16M**

Q.3 Find k if the sum of the slopes of the lines represented by $x^2 + kxy - 3y^2 = 0$ is twice their product. 2m

Q.4 Find the volume of the parallelepiped whose coterminous edges are $2\hat{i} - 3\hat{j}, \hat{i} + \hat{j} - \hat{k}$ and $3\hat{i} - \hat{k}$. 2m

Q.5 Solve the differential equation $\frac{d\theta}{dt} = -k(\theta - \theta_0)$, k is constant. 2m

Q.6 Evaluate : $\int \frac{x \sin^{-1} x^2 . dx}{\sqrt{1-x^4}}$ 2m

Q.7 Find the direction ratios of a line perpendicular to both the lines whose direction ratios are 3, -2, -1 and 2, 4, -2. 2m

Q.8 Find the combined equation of the lines bisecting the angles between coordinate axes. 2m

Q.9 Find the value of *'a'* if $\int_2^a (x+1)\, dx = \frac{7}{2}$. 2m

Q.10 Find the inverse of the matrix $A=\begin{bmatrix}-1 & 5\\ -3 & 2\end{bmatrix}$ by adjoint method. 2m

Q.11 If $e^x + e^y = e^{x+y}$ show that : $\frac{dy}{dx} = -e^{y-x}$ 2m

Q.12 The displacement of a particle at time t is given by $s = t^3 - 4t^2 - 5t$. Find its velocity and acceleration at t = 2. 2m

Q.13 Evaluate : $\int x^3 \cdot \log x \cdot dx$ 2m

Q.14 A fair coin in tossed 6 times. Find the probability of getting heads 4 times. 2m

SECTION C

Attempt Any Eight Questions **24M**

Q.15 A line makes angles angles of measures $\frac{\pi}{6}$ and $\frac{\pi}{3}$ with X and Z axes respectively. Find the angle it makes with Y –axis. 3m

Q.16 Find the angle between the lines 3m

$\bar{r} = 3\hat{i} + 2\hat{j} - 4\hat{k} + \lambda(\hat{i} + 2\hat{j} + 2\hat{k})$ and $\bar{r} = 5\hat{i} - 2\hat{j} + \mu(3\hat{i} + 2\hat{j} + 6\hat{k})$.

Q.17 Find the particular solution of the differential equation $\log\left(\frac{dy}{dx}\right) = 3x + 4y$ given that $y = 0$ when $x = 0$. 3m

Q.18 Two cards are drawn at random from a box which contains 4 cards numbered 1,1, 2, 3. Let X denote the sum of the numbers on two cards. Find the expected value of the sum. 3m

Q.19 If the mean and the variance of a binomial distribution are 4 and $\frac{4}{3}$, then find $P(x \geq 2)$. 3m

Q.20 Evaluate : $\int \frac{1}{x(x-2)(x-4)}\,dx.$ 3m

Q.21 Find the cartesian form of the equation of the plane 3m

$\bar{r} = \left(\bar{i} + \bar{j}\right) + s\left(\bar{i} - \bar{j} + 2\bar{k}\right) + t\left(\bar{i} + 2\bar{j} + \bar{k}\right)$.

Q.22 Find the interval in which the function $f(x) = 5 + 36x + 3x^2 - 2x^3$ is increasing. 3m

Q.23 Evaluate $\int_{-1}^{1} f(x)\,dx$, if 3m

$f(x) = 4 - 3x, \text{for } x \leq 0$
$= 3x + 4, \text{for } x > 0$

Q.24 Show that $2\sin^{-1}\left(\frac{3}{5}\right)=\tan^{-1}\left(\frac{24}{7}\right)$ 3m

Q.25 Without using truth table, prove that $[(p\vee q)\wedge\sim p]\rightarrow q$ is a tautology. 3m

Q.26 Evaluate : $\int e^{\tan^{-1}x}\cdot\left[\frac{1+x+x^2}{1+x^2}\right]dx$ 3m

SECTION D

Attempt Any Five Questions **20M**

Q.27 Simplify and show that the statement: 4m

$[p\wedge(\sim p\vee q)]\vee(\sim p\wedge q)\vee[(p\vee\sim q)\wedge r]$ is equivalent to $(q\vee r)$.

Q.28 Solve the following equations by method of inversion. 4m

$x - y + z = 4$

$2x + y - 3z = 0$

$x + y + z = 2$

Q.29 In ΔABC, prove that 4m

$a^3\sin(B-C)+b^3\sin(C-A)+c^3\sin(A-B)=0$

Q.30 Prove that the altitudes of triangle are concurrent. 4m

Q.31 Minimize $z = 8x + 10y$, subject to $2x+y\geq 7$, $2x+3y\geq 15$, $y\geq 2$, $x\geq 0$, $y\geq 0$. 4m

Q.32 If $x^y=e^{x-y}$ show that $\frac{dy}{dx}=\frac{\log x}{(1+\log x)^2}$ 4m

Q.33 A rectangular sheet of paper has the area 24 square meters. The margin at the top and bottom is 75 cm and sides 50 cm each. What are the dimensions of paper if the area of the printed space is maximum? 4m

Q.34 Find the area of sector of circle bounded by $x^2 + y^2 = 16$ and the line $y = x$ in the first quadrant. 4m

Together we will make a difference