MODEL QUESTION PAPER - SET-1 : 2021-22
MATHEMATICS (THEORY)

MM : 80 **SOLUTION** **Time : 3 Hrs.**

Entire Syllabus

SECTION A

Q.1 Select and Write the correct Answer **16M**

i. **d)** $\frac{5}{\sqrt{26}}$ 2m

ii. **a)** Circles 2m

iii. **a)** $\frac{2-4x^2}{\sqrt{1-x^2}}$ 2m

iv. **b)** - 0.85 2m

v. **d)** $\left(\sqrt{2},\sqrt{2}\right)$ 2m

vi. **b)** k = - 6 2m

vii. **a)** $\log x - f(x) + c$ 2m

viii. **a)** 4, 5, 7 2m

Q.2 Answer the following (1 Mark Each) **4M**

i. All triangles are not equilateral triangles. 1m

ii. Let $y = \tan^{-1}(\log x)$ 1m

Differentiating w.r.t. x, we get

$$\frac{dy}{dx} = \frac{d}{dx}[\tan^{-1}(\log x)]$$

$$= \frac{1}{1+(\log x)^2}\cdot\frac{d}{dx}(\log x)$$

$$= \frac{1}{1+(\log x)^2}\cdot\frac{1}{x}$$

$$\therefore \quad \frac{dy}{dx} = \frac{1}{x\left[1+(\log x)^2\right]}$$

iii. 1m

$$\sin\left[\frac{\pi}{2}+\sin^{-1}\left(\frac{-1}{2}\right)\right]$$

$$= \sin\left[\frac{\pi}{2}-\sin^{-1}\left(\frac{1}{2}\right)\right]$$

$$= \sin\left[\frac{\pi}{2}-\frac{\pi}{6}\right]$$

$$= \sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}$$

iv. $\frac{dy}{dx}=\frac{1+y^2}{1+x^2}$ 1m

$\therefore \quad \frac{dy}{1+y^2}=\frac{dx}{1+x^2}$

Integrating on both sides, we get

$\int\frac{dy}{1+y^2}=\int\frac{dx}{1+x^2}$

$\therefore \quad \tan^{-1}(y)=\tan^{-1}(x)+c$

SECTION B

Attempt Any Eight Questions **16M**

Q.3 Given equation of the lines is $kx^2 + 4xy - y^2 = 0$. 2m

Comparing with $ax^2 + 2hxy + by^2 = 0$, we get a = k, 2h = 4, b = –1.

Let m_1 and m_2 be the slopes of the lines represented by $kx^2 + 4xy - y^2 = 0$.

$\therefore \quad m_1 + m_2 = \frac{-2h}{b} = 4$ and

$m_1m_2 = \frac{a}{b} = -k$

According to the given condition,

$m_2 = m_1 + 8$

Now, $m_1 + m_2 = 4$

$\therefore \quad m_1 + (m_1 + 8) = 4$

$\therefore \quad 2m_1 = -4$

$\therefore \quad m_1 = -2$ ...(i)

Also, $m_1m_2 = -k$

$\therefore \quad m_1 + (m_1 + 8) = -k$

$\therefore \quad (-2)(-2 + 8) = -k$ ...[From (i)]

$\therefore \quad -2(6) = -k$

$\therefore \quad -12 = -k$

$\therefore \quad$ **k = 12**

Q.4 $\overline{AB} = \bar{b} - \bar{a}$ 2m

$= 4\hat{i} - 4\hat{j} - 2\hat{k}$

$\overline{AC} = \bar{c} - \bar{a}$

$= 3\hat{i} - \hat{j}$

$\overline{AD} = \bar{d} - \bar{a}$

$= -4\hat{j} + \hat{k}$

Volume of tetrahedron $= \frac{1}{6}\left[\overline{AB}\ \overline{AC}\ \overline{AD}\right]$

$$= \frac{1}{6}\begin{vmatrix} 4 & -4 & -2 \\ 3 & -1 & 0 \\ 0 & -4 & 1 \end{vmatrix}$$

$$= \frac{1}{6}\,[4\,(-1) + 4\,(3) - 2\,(-12)]$$

$$= \frac{1}{6}\,[-4 + 12 + 24]$$

$$= \frac{1}{6} \times 32 = \frac{16}{3} \text{ cubic units}$$

Q.5 Let $I = \int \left(\frac{x^2+2}{x^2+1}\right) a^{x+\tan^{-1}x} dx$ 2m

Put $x + \tan^{-1} x = t$

Differentiating w.r.t.x, we get

$$\left(1 + \frac{1}{1+x^2}\right) dx = dt$$

$$\therefore \quad \left(\frac{x^2+2}{x^2+1}\right) dx = dt$$

$$\therefore \quad I = \int a^t . dt = \frac{a^t}{\log a} + c$$

$$\therefore \quad I = \frac{a^{x+\tan^{-1}x}}{\log a} + c$$

Q.6 $\frac{x-2}{3} = \frac{y+1}{2} = \frac{z-k}{1}$ 2m

Equation of plane is $x - y - z + 8 = 0$

The given line passes through (2, -1, k).

Since the line lies on the plane, the point (2, -1, k). lies on the plane x–y–z+8=0

$\therefore 2 - (-1) - k + 8 = 0$

$\therefore 2 + 1 - k + 8 = 0$

$\therefore k = 11$

Q.7 Let $y = (\sin x)^x$ 2m

Taking log on both sides, we get

$\log y = x \log (\sin x)$

Differentiating w.r.t. x, we get

$$\frac{d}{dx}(\log y) = x.\frac{d}{dx}[\log(\sin x)]$$

$$+\log(\sin x).\frac{d}{dx}(x)$$

$$\therefore \quad \frac{1}{y}.\frac{dy}{dx} = x.\frac{1}{\sin x}.\frac{d}{dx}(\sin x) + \log(\sin x).1$$

$$\therefore \quad \frac{1}{y}.\frac{dy}{dx} = \frac{x}{\sin x}.\cos x + \log(\sin x)$$

$$\therefore \quad \frac{dy}{dx} = y[\cot x + \log(\sin x)]$$

$$\therefore \quad \frac{dy}{dx} = (\sin x)^x[x \cot x + \log(\sin x)]$$

Q.8 Polar coordinates are $\left(\frac{3}{4}, 135^\circ\right)$ 2m

Here $r = \frac{3}{4}$, $\theta = 135^\circ$

Now,

$$x = r\cos\theta = \frac{3}{4}\cos 135^\circ$$

$$= \frac{3}{4}\cos(90+45)^\circ$$

$$= \frac{-3}{4}\sin 45^\circ$$

$$= \frac{-3}{4\sqrt{2}}$$

Also,

$$y = r\sin\theta = \frac{3}{4}\sin 135^\circ$$

$$= \frac{3}{4}\sin(90+45)^\circ$$

$$= \frac{3}{4}\cos 45^\circ = \frac{3}{4}\times\frac{1}{\sqrt{2}} = \frac{3}{4\sqrt{2}}$$

$\therefore$ Cartesian coordinates $\equiv \left(\frac{-3}{4\sqrt{2}}, \frac{3}{4\sqrt{2}}\right)$

Q.9 Given equation of the lines is 2m

$3x^2 - 4\sqrt{3}xy + 3y^2 = 0$

Comparing with $ax^2 + 2hxy + by^2 = 0$, we get $a = 3$,

$h = -2\sqrt{3}$ and $b = 3$

Let θ be the acute angle between the lines.

$$\therefore \quad \tan\theta = \frac{2\sqrt{h^2 - ab}}{a + b} = \left|\frac{2\sqrt{\left(-2\sqrt{3}\right)^2 - 3(3)}}{3 + 3}\right|$$

$$= \left|\frac{2\sqrt{12 - 9}}{6}\right|$$

$$= \left|\frac{2\sqrt{3}}{6}\right|$$

$$\therefore \quad \tan\theta = \frac{1}{\sqrt{3}}$$

$$\therefore \quad \theta = \tan^{-1}\left(\frac{1}{\sqrt{3}}\right)$$

$$\therefore \quad \theta = 30^\circ$$

Q.10 Let $A = \begin{bmatrix} 2 & -3 \\ 3 & 5 \end{bmatrix}$ 2m

Here,

$a_{11} = 2$

$\therefore \quad M_{11} = 5 \quad$ and $A_{11} = (-1)^{1+1}(5) = 5$

$a_{12} = -3$

$\therefore \quad M_{12} = 3$ and $A_{12} = (-1)^{1+2}(3) = -3$

$a_{21} = 3$

$\therefore \quad M_{21} = -3$ and $A_{21} = (-1)^{2+1}(-3) = 3$

$a_{22} = 5$

$\therefore \quad M_{22} = 2$ and $A_{22} = (-1)^{2+2}(2) = 2$

$$\therefore \quad \left[A_{ij}\right]_{2\times2} = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} = \begin{bmatrix} 5 & -3 \\ 3 & 2 \end{bmatrix}$$

Now, adj $A = \left[A_{ij}\right]_{2\times2}^{T} = \begin{bmatrix} 5 & 3 \\ -3 & 2 \end{bmatrix}$

Q.11 Let, $I = \int \frac{x}{x+2}\,dx$ 2m

$$= \int \frac{(x+2) - 2}{x+2}\,dx$$

$$=\int\left(\frac{x+2}{x+2}-\frac{2}{x+2}\right)dx$$

$$=\int\left(1-\frac{2}{x+2}\right)dx$$

$$-\int 1\cdot dx-2\int\frac{1}{x+2}dx$$

$$\therefore\ I=x-2\log|x+2|+c$$

Q.12 Here $n = 400$, $p = 0.2$ 2m

$\therefore\ q = 1 - p = 1 - 0.2 = 0.8$

$\therefore$ Mean $= E(X) = np = 400 \times 0.2$

$= 80$

Var (X) $= npq$

$= 400 \times 0.2 \times 0.8$

$= 64$

$\therefore$ Standard deviation of $x=\sqrt{\text{Var X}}$

$=\sqrt{64}=8$

Q.13 2m

$$I=\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}}\frac{1}{1-\sin x}dx=\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}}\left(\frac{1}{1-\sin x}\times\frac{1+\sin x}{1+\sin x}\right)dx$$

$$=\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}}\frac{1+\sin x}{1-\sin^2 x}dx$$

$$=\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}}\frac{1+\sin x}{\cos^2 x}dx$$

$$=\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}}(\sec^2 x+\sec x\tan x)dx$$

$$=\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}}\sec^2 x+dx+\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}}\sec x\tan x\,dx$$

$$=[\tan x]_{-\pi/4}^{\pi/4}+[\sec x]_{-\pi/4}^{\pi/4}$$

$$=\left[\tan\frac{\pi}{4}-\tan\left(-\frac{\pi}{4}\right)\right]+\left[\sec\frac{\pi}{4}-\sec\left(-\frac{\pi}{4}\right)\right]$$

$$=[1-(-1)]+\left(\sqrt{2}-\sqrt{2}\right)$$

$$\therefore\ I=2$$

Q.14 $y = A\cos 2x + B\sin 2x$ ...(1) 2m

Since the solution contains two arbitrary constants A and B, we differentiate two times.

$\therefore$ Differentiating (1) *w.r.t.* x,

$$\frac{dy}{dx} = -2A\sin 2x + 2B\cos 2x$$

and $$\frac{d^2y}{dx^2} = -4A\cos 2x - 4B\sin 2x$$

$$= -4(A\cos 2x + B\sin 2x)$$

$$= -4y \quad \text{....[from (1)]}$$

$$\therefore \frac{d^2y}{dx^2} + 4y = 0$$

SECTION C

Attempt Any Eight Questions **24M**

Q.15 Let p : Surface area decreases. 3m

q : Pressure increases.

$\therefore$ The given statement is $p \to q$.

Its converse is $q \to p$.

If pressure increases, then surface area decreses.

Its inverse is $\sim p \to \sim q$.

If surface area does not decrease, then pressure does not increase.

Its contrapositive is $\sim q \to \sim p$.

If pressure does not increase, then surface area does not decrease.

Q.16 3m

$$\tan\theta + \tan 2\theta + \sqrt{3}\tan\theta\tan 2\theta = \sqrt{3}$$

$$\therefore \tan\theta + \tan 2\theta = \sqrt{3}(1 - \tan\theta\tan 2\theta)$$

$$\therefore \frac{\tan\theta + \tan 2\theta}{1 - \tan\theta\tan 2\theta} = \sqrt{3}$$

$$\therefore \tan(\theta + 2\theta) = \sqrt{3}$$

$$\therefore \tan 3\theta = \sqrt{3}$$

$$\therefore \tan 3\theta = \tan\frac{\pi}{3}$$

$$\therefore 3\theta = n\pi + \frac{\pi}{3}$$

$$\therefore \theta = \frac{n\pi}{3} + \frac{\pi}{9}$$

Q.17 The edge of a cube is decreasing at the rate of 0.6 cm/sec. Find the rate at which its volume is decreasing when the edge of the cube is 2cm. 3m

Let a be the length of each side of the cube and V be its volume.

Then, $\frac{da}{dt} = -\ 0.6$ cm/sec, a = 2cm ....[Given]

(where '-'ve sign represents rate of decrease.)

$V = a^3$

Differentiating w.r.t. t, we get

$$\frac{dV}{dt} = 3a^2 \frac{da}{dt}$$

$$\therefore \quad \frac{dV}{dt} = 3(2)^2(-0.6) = -7.2\text{cm}^3 / \sec$$

Thus, the volume is decreasing at the rate of 7.2 cm^3/sec.

Q.18 3m

$$x\sin\left(\frac{y}{x}\right)dy = \left[y\sin\left(\frac{y}{x}\right) - x\right]dx$$

$$\therefore \quad \frac{dy}{dx} = \frac{y\sin\left(\frac{y}{x}\right) - x}{x\sin\left(\frac{y}{x}\right)} \quad ...(i)$$

Put y = vx ...(ii)

differentiating w.r.t.x, we get

$$\frac{dy}{dx} = v + x\frac{dv}{dx} \quad ...(iii)$$

Substituting (ii) and (iii) in (i), we get

$$v + x\frac{dv}{dx} == \frac{vx\sin\left(\frac{vx}{x}\right) = x}{x\sin\left(\frac{vx}{x}\right)}$$

$$\therefore \quad v + x\frac{dv}{dx} = \frac{vx\sin v - x}{x\sin v}$$

$$\therefore \quad v + x\frac{dv}{dx} = v - \frac{1}{\sin v}$$

$$\therefore \quad x\frac{dv}{dx} = -\frac{1}{\sin v}$$

$\therefore \quad -\sin v\, dv = \frac{1}{x} dx$

Integrating on both sides, we get

$$-\int \sin v dv - \int \frac{1}{x} dx$$

$\therefore \quad -(-\cos v) = \log|x| + c$

$\therefore \quad \cos v = \log|x| + c$

$\therefore \quad \cos\left(\frac{y}{x}\right) = \log|x| + c$

Q.19 Let M be the foot of the perpendicular drawn from the point P(2, –3, 1) to the given line 3m

$$\frac{x+1}{2} = \frac{y-3}{3} = \frac{z+1}{-1}$$

Let $\frac{x+1}{2} = \frac{y-3}{3} = \frac{z+1}{-1} = \lambda$.

The co-ordinates of any point on the line are given by $x = 2\lambda - 1, y = 3\lambda + 3, z = -\lambda - 1$

$\therefore$ The co-ordinates of M are

$(2\lambda - 1, 3y + 3, -\lambda - 1)$ ...(i)

The direction ratios of PM are

$2\lambda - 2 - 2, 3\lambda + 3 + 3, -\lambda - 1 - 1$

i.e., $2\lambda - 3, 3\lambda + 6, -\lambda - 2$

The direction ratios of the given line are 2, 3, –1.

Since PM is perpendicular to the given line.

$\therefore \quad 2(2\lambda - 3) + 3(3\lambda + 6) - 1(-\lambda - 2) = 0$

$\therefore \quad 4\lambda - 6 + 9\lambda + 18 + \lambda + 2 = 0$

$\therefore \quad 14\lambda + 14 = 0$

$\therefore \quad \lambda = -1$

Substituting $\lambda = -1$ in (i), the co-ordinates of M are

$(-2 - 1, -3 + 3, 1 - 1)$

i.e., $(-3, 0, 0)$

$\therefore$ Length of the perpendicular from P to the given line

$$= PM = \sqrt{(-3-2)^2 + (0+3)^2 + (0-1)^2}$$

$$= \sqrt{(-5)^2 + 3^2 + (-1)^2}$$

$$= \sqrt{25 + 9 + 1}$$

$$= \sqrt{35} \text{ units}$$

Q.20 Let $I = \int \sqrt{x^2+a^2} \cdot 1\, dx$ ...(1) 3m

Integrating by parts,

$$I = \sqrt{x^2+a^2}\int 1 dx - \int\left(\int 1 dx \frac{d}{dx}\sqrt{x^2+a^2}\right)dx$$

$$= \sqrt{x^2+a^2}\,(x) - \int\left(x \cdot \frac{1}{2\sqrt{x^2+a^2}}(2x)\right)dx$$

$$= x\sqrt{x^2+a^2} - \int \frac{x^2}{\sqrt{x^2+a^2}}\,dx$$

$$= x\sqrt{x^2+a^2} - \int\left(\frac{a^2+x^2-a^2}{\sqrt{x^2+a^2}}\right)dx$$

$$= x\sqrt{x^2+a^2} - \int\left(\sqrt{x^2+a^2} - \frac{a^2}{\sqrt{x^2+a^2}}\right)dx$$

$$\therefore\ I = x\sqrt{x^2+a^2} - \int\sqrt{x^2+a^2}\,dx + a^2\int\frac{dx}{\sqrt{x^2+a^2}}$$

$$\therefore\ I = x\sqrt{x^2+a^2} - I + a^2\log\left(x+\sqrt{x^2+a^2}\right) + c_1$$

....[From (1)]

$$\therefore\ 2I = x\sqrt{x^2+a^2} + a^2\log\left(x+\sqrt{x^2+a^2}\right) + c_1$$

$$\therefore\ I = \frac{x}{2}\sqrt{x^2+a^2} + \frac{a^2}{2}\log\left(x+\sqrt{x^2+a^2}\right) + c$$

$$\left(\text{where } c = \frac{c_1}{2}\right)$$

Q.21 The cartesian equations of the line passing through A(x, y, z) and B(x_2, y_2, z_2) 3m

are $\frac{x-x_1}{x_2-x_1} = \frac{y-y_1}{y_2-y_1} = \frac{z-z_1}{z_2-z_1}$

Here, $(x_1, y_1, z_1) = (-2, 3, 4)$ and

$(x_2, y_2, z_2) = (1, 1, 2)$

$\therefore$ Required Cartesian equation are

$$\frac{x-(-2)}{1-(-2)} = \frac{y-3}{1-3} = \frac{z-4}{2-4}$$

$$\therefore\ \frac{x+2}{3} = \frac{y-3}{-2} = \frac{z-4}{-2}$$

Substituting C(4, -1, 0) in the above equation, we get

$$\frac{4+2}{3} = \frac{-1-3}{-2} = \frac{0-4}{-2}$$

$\therefore$ 2 = 2 = 2

Since C satisfies the equation of a line AB, points A, B, C are collinear.

Q.22 Evaluate: $\int \frac{3x-2}{x^2-3x+2}dx$ 3m

Let $I = \int \frac{3x-2}{x^2-3x+2}dx$

Consider, $x^2-3x+2 = x^2-2x-x+2$

$= x(x-2) - 1(x-2)$

$= (x-1)(x-2)$

$\therefore I = \int \frac{3x-2}{(x-1)(x-2)}dx$

Let $\frac{3x-2}{(x-1)(x-2)} = \frac{A}{x-1} + \frac{B}{x-2}$

$\therefore 3x - 2 = A(x-2) + B(x-1)$

Put $x = 2$, $\therefore 4 = B$

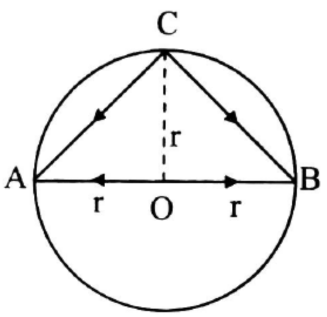
Put $x = 1$, $1 = -A$, $\therefore A = -1$

$\therefore I = \int \left(\frac{-1}{x-1} + \frac{4}{x-2}\right)dx$

$= -\log(x-1) + 4\log(x-2) + c$

$= 4\log(x-2) - \log(x-1) + c$

Q.23 3m



Let, r be the radius and O be the centre of the circle. A, B, and C are three points on the circle such that, AB is the diameter.

Let $\overline{a}, \overline{b}$ and $\overline{c}$ be the position vectors of points A, B, and C reaspectively.

$\therefore$ C is on the circle, $|\bar{c}| = r$

Also, $|\bar{a}| = |\bar{b}| = r$ and $\bar{b} = -\bar{a}$

Consider $\overline{CA}.\overline{CB} = (\bar{a} - \bar{c})\cdot(\bar{b} - \bar{c})$

$$= (\bar{a} - \bar{c})\cdot(-\bar{a} - \bar{c})$$

$$= (\bar{a} - \bar{c})\cdot(-1)(-\bar{a} - \bar{c})$$

$$= (-1)(\bar{a} - \bar{c})(\bar{a} - \bar{c})$$

$$= \left(|\bar{c}|^2 - |a|^2\right)$$

$$= r^2 - r^2$$

$$= 0$$

$\therefore$ $\overline{CA}\cdot\overline{CB} = 0$

$\therefore$ $\overline{CA}$ is perpendicular to $\overline{CB}$.

$\therefore$ The angle between $\overline{CA}$ and $\overline{CB}$ is a right angle.

$\therefore$ $m\angle ACB = 90°$

$\therefore$ The angle subtended on a semicircle is a right angle.

Q.24 Since $0 < a < 2a$ 3m

$\therefore \int_0^{2a} f(x)\,dx = \int_0^a f(x)\,dx + \int_a^{2a} f(x)\,dx$

In second integral put $x = 2a - t \therefore dx = -dt$

Also, when $x = a$, $t = a$ and

when $x = 2a$, $t = 0$

$\therefore \int_a^{2a} f(x)dx = \int_a^0 f(2a - t)\,(-dt)$

$$= -\int_a^0 f(2a - t)\,dt$$

$$= \int_0^a f(2a - t)\,dt \quad \text{...(by property)}$$

$$= \int_0^a f(2a - x)\,dx \quad \text{...(by property)}$$

$\therefore \int_0^{2a} f(x)\,dx = \int_0^a f(x)\,dx + \int_0^a f(2a - x)\,dx$

Q.25 A random variable X has the following probability distribution: 3m

X	0	1	2	3	4	5	6	7
P(X)	0	k	2k	2k	3k	k^2	$2k^2$	$7k^2 + k$

Determine: i. k ii. P(X < 3) iii. P(X > 4)

Sol. i. The table gives a probability distribution and

therefore $\sum_{i=1}^{8} p_i = 1$

$\therefore$ $0 + k + 2k + 2k + 3k + k^2 + 2k^2 + 7k^2 + k = 1$

$\therefore$ $10k^2 - 9k - 1 = 0$

$\therefore$ $10k^2 + 10k - k - 1 = 0$

$\therefore$ $10k(k + 1) - 1(k + 1) = 0$

$\therefore$ $(10k - 1)(k + 1) = 0$

$\therefore$ $k = \frac{1}{10}$ or $k = -1$

But k cannot be negative

$\therefore$ $k = \frac{1}{10}$

ii. $P(X < 3)$

$= P(X = 0 \text{ or } X = 1 \text{ or } X = 2)$

$= P(X = 0) + P(X = 1) + P(X = 2)$

$= 0 + k + 2k = 3k = \frac{3}{10}$

iii. $P(X > 4)$

$= P(X = 5 \text{ or } X = 6 \text{ or } X = 7)$

$= P(X = 5) + P(X = 6) + p(X = 7)$

$= k^2 + 2k^2 + 7k^2 + k$

$= 10k^2 + k = 10\left(\frac{1}{10}\right)^2 + \frac{1}{10} = \frac{1}{10} + \frac{1}{10} = \frac{1}{5}$

Q.26 Here $n = 5$ 3m

P (success) = P (target will be hit) = 0.2

$\therefore$ $p = 0.2$, $q = 1 - 0.2 = 0.8$

Let X = Number of times the target is hit.

Then $X \sim B(n = 5, p = 0.2)$

Then p.m.f. of X is given by

$P(X = x) = P(x)$

$= {}^5C_x (0.2)^x (0.8)^{5-x}$, $x = 0, 1, 2, \ldots, 5.$

$\therefore$ P (target is hit at least twice out of 5 shots)

$= 1 - [P(0) + P(1)]$

$= 1 - \left[{}^5C_0 (0.2)^0 (0.8)^5 + {}^5C_1 (0.2)^1 (0.8)^4\right]$

$= 1 - [(1)(1)(0.328) + (5)(0.2)(0.4096)]$

$= 1 - [0.328 + 0.4096]$

$= 1 - 0.7376$

$= 0.2624$

SECTION D

Attempt Any Five Questions **20M**

Q.27 $x = a\cos^3 t$ 4m

Differentiating w.r.t. t, we get

$$\frac{dx}{dt} = a\frac{d}{dt}(\cos t)^3 = a.3(\cos t)^2\frac{d}{dt}(\cos t)$$

$$= 3a\cos^2 t(-\sin t) = -3a\cos^2 t.\sin t$$

$y = a\sin^3 t$

Differentiating w.r.t. t, we get

$$\frac{dy}{dt} = a\frac{d}{dt}(\sin t)^3 = a.3(\sin t)^2\frac{d}{dt}(\sin t)$$

$$= 3a\sin^2 t.\cos t$$

$$\therefore \quad \frac{dy}{dx} = \frac{\left(\frac{dy}{dt}\right)}{\left(\frac{dx}{dt}\right)} = \frac{3a\sin^2 t\cos t}{-3a\cos^2 t\sin t} = -\frac{\sin t}{\cos t} \quad ....(i)$$

Now, $x = a\cos^3 t$

$$\therefore \quad \cos^3 t = \frac{x}{a}$$

$$\therefore \quad \cos t = \left(\frac{x}{a}\right)^{\frac{1}{3}}$$

$y = a\sin^3 t$

$$\therefore \quad \sin^3 t = \frac{y}{a}$$

$$\therefore \quad \sin t = \left(\frac{y}{a}\right)^{\frac{1}{3}}$$

From (i), we get

$$\frac{dy}{dx} = \frac{-\sin t}{\cos t} = -\frac{\frac{y^{\frac{1}{3}}}{a^{\frac{1}{3}}}}{\frac{x^{\frac{1}{3}}}{a^{\frac{1}{3}}}} = -\left(\frac{y}{x}\right)^{\frac{1}{3}}$$

Q.28 $A = \begin{bmatrix} 1 & 0 & 0 \\ 3 & 3 & 0 \\ 5 & 2 & -1 \end{bmatrix}$ 4m

Cofactors of matrix A are

$A_{11} = (-1)^2 \begin{vmatrix} 3 & 0 \\ 2 & -1 \end{vmatrix} = -3$

$A_{12} = (-1)^3 \begin{vmatrix} 3 & 0 \\ 5 & -1 \end{vmatrix} = -(-3) = 3$

$A_{13} = (-1)^4 \begin{vmatrix} 3 & 3 \\ 5 & 2 \end{vmatrix} = 6 - 15 = -9$

$A_{21} = (-1)^3 \begin{vmatrix} 0 & 0 \\ 2 & -1 \end{vmatrix} = 0$

$A_{22} = (-1)^4 \begin{vmatrix} 1 & 0 \\ 5 & -1 \end{vmatrix} = -1$

$A_{23} = (-1)^5 \begin{vmatrix} 1 & 0 \\ 5 & 2 \end{vmatrix} = -2$

$A_{31} = (-1)^4 \begin{vmatrix} 0 & 0 \\ 3 & 0 \end{vmatrix} = 0$

$A_{32} = (-1)^5 \begin{vmatrix} 1 & 0 \\ 3 & 0 \end{vmatrix} = 0$

$A_{33} = (-1)^6 \begin{vmatrix} 1 & 0 \\ 3 & 3 \end{vmatrix} = 3$

$\therefore$ Matrix of cofactors of A = $\begin{bmatrix} -3 & 3 & -9 \\ 0 & -1 & -2 \\ 0 & 0 & 3 \end{bmatrix}$

$\therefore$ adj A = $\begin{bmatrix} -3 & 0 & 0 \\ 3 & -1 & 0 \\ -9 & -2 & 3 \end{bmatrix}$

Also $|A| = \begin{vmatrix} 1 & 0 & 0 \\ 3 & 3 & 0 \\ 5 & 2 & -1 \end{vmatrix}$

$= 1\,(-3) = -3 \neq 0$

$\therefore$ A^{-1} exists.

$\therefore$ $A^{-1} = \frac{1}{|A|}$ (adj A)

$= \frac{-1}{3} \begin{bmatrix} -3 & 0 & 0 \\ 3 & -1 & 0 \\ -9 & -2 & 3 \end{bmatrix}$

Q.29 An open cylindrical tank whose base is a circle is to be constructed of metal sheet so as to contain a volume of πa^3 cu.cm of water. Find the dimensions so that sheet required is minimum. 4m

Let r be the radius, h be the height, V be the volume and A be the total surface area of open cylindrical tank.

Then, $V = \pi r^2 h = \pi a^3$ .....(i)

and $A = \pi r h = \pi r^2$ .....(ii)

From (i), we get

$r^2 h = a^3$

$\therefore \quad h = \dfrac{a^3}{r^2}$

Putting the value of h in (ii), we get

$$A = 2\pi r\left(\frac{a^3}{r^2}\right) + \pi r^2 = 2\pi a^3\left(\frac{1}{r}\right) + \pi r^2$$

$$\therefore \quad \frac{dA}{dr} = 2\pi a^3\left(-\frac{1}{r^2}\right) + 2\pi r = 2\pi\left(-\frac{a^3}{r^2} + r\right)$$

$$\therefore \quad \frac{d^2A}{dr^2} = 2\pi\left[(-a^3)(-2r^{-3}) + 1\right] = 2\pi\left(\frac{2a^3}{r^3} + 1\right)$$

Consider, $\dfrac{dA}{dr} = 0$

$$\therefore \quad 2\pi\left(-\frac{a^3}{r^2} + r\right) = 0$$

$$\therefore \quad -\frac{a^3}{r^2} + r = 0$$

$$\therefore \quad r = \frac{a^3}{r^2}$$

$\therefore \quad r^3 = a^3$

$\therefore \quad r = a$

For a = r,

$$\left(\frac{d^2A}{dr^2}\right) = 2\pi\left(\frac{2a^3}{a^3} + 1\right) = 6\pi > 0$$

Hence, A, i.e., total surface area is minimum when r = a.

From (i), we get

$\pi r^2 h = \pi a^3$

$\therefore \quad r^2 h = a^3$

$\therefore \quad a^2 . h = a^3$

$\therefore \quad h = a$

Thus, the quantity or metal sheet required is minimum when height = radius = a cm.

Q.30 L.H.S. 4m

$= (p \vee q) \wedge (p \vee \sim q)$

$\equiv p \vee (q \wedge \sim q)$ [Distributive law]

$\equiv p \vee F$ [Complement law]

$\equiv p$ **[Identify law]**

Q.31 In ΔABC by sine rule, we have 4m

$$\frac{a}{\sin A} = \frac{b}{\sin B} = k$$

$\therefore$ a = k sin A and b = k sin B ...(i)

Now, a cos A = b cos B ...[Given]

k sin A cos A = k sin B cos B ...[From (i)]

$\therefore$ sin A cos A = sin B cos B

$\therefore$ 2 sin A cos A = 2 sin B cos B

$\therefore$ sin 2A = sin 2B

$\therefore$ sin 2A – sin 2 B = 0

$\therefore$ $2\cos(A+B)\sin(A-B) = 0$

... $\left[\because \sin C - \sin D = 2\cos\left(\frac{C+D}{2}\right)\sin\left(\frac{C-D}{2}\right)\right]$

$2\cos(\pi - C)\sin A - B = 0$

... $[\because A + B + C = \pi]$

$\therefore$ –2 cos C. sin (A–B) = 0

$\therefore$ cos C = 0 or sin (A–B) = 0

$\therefore$ $C = \frac{\pi}{2}$ or $A - B = 0$

... $\left[\because \cos\frac{\pi}{2} = 0, \sin 0 = 0\right]$

$\therefore$ $C = \frac{\pi}{2}$ or A = B

$\therefore$ $C = \frac{\pi}{2}$ implies ΔABC is right angled triangle and A = B implies ΔABC is an isosceles triangle.

$\therefore$ The triangle is either right angled triangle or an isosceles triangle.

Q.32 Let, $\bar{a}, \bar{b}, \bar{c}$ be the position vectors of points A, B, C respectively. 4m

$\bar{a} = 3\hat{i} + 0\cdot\hat{j} + p\hat{k}, \bar{b} = -\hat{i} + q\hat{j} + 3\hat{k}$ and $\bar{c} = -3\hat{i} + 3\hat{j} + 0\cdot\hat{k}$

i. Let point C divides line segent AB in the ratio t : 1.

By using section formula,

$$\bar{c} = \frac{t\cdot\bar{b} + 1\cdot\bar{a}}{t+1}$$

$\therefore$ $-3\hat{i} + 3\hat{j} + 0\cdot\hat{k} = \frac{t\left(-\hat{i} + q\hat{j} + 3\hat{k}\right) + \left(3\hat{i} + 0\cdot\hat{j} + p\hat{k}\right)}{t+1}$

$\therefore$ $(t+1)\left(-3\hat{i} + 3\hat{j} + 0\hat{k}\right)$

$= -t\hat{i} + tq\hat{j} + 3t\hat{k} + 3\hat{i} + 0\cdot\hat{j} + p\hat{k}$

$\therefore$ $-3(t+1)\hat{i} + 3(t+1)\hat{j} + 0\cdot\hat{k}$

$$= (-t+3)\hat{i} + tq\hat{j} + (3t+p)\hat{k}$$

∴ By equality of vectors, we get

$-3(t-1) = -t+3$ ...(i)

$3(t+1) = tq$ ...(ii)

From (i), we get ...(iii)

$-3t - 3 = -t + 3$

∴ $-2t = 6$

∴ $t = -3$

∴ C divides segment AB externally, (since t is negative) in the ratio 3 : 1.

ii. Putting $t = -3$ in (ii), we get

$3(-3+1) = -3q$

∴ $-6 = -3q$

∴ $q = 2$

Putting $t = -3$ in (iii), we get

$0 = -9 + p$

∴ $p = 9$

∴ **p = 9 and q = 2**

Q.33 Given equation of the circle is 4m

$x^2 + y^2 = 4$ ...(i)

and equation of the

line is $x = y\sqrt{3}$

∴ $y = \dfrac{x}{\sqrt{3}}$ ....(ii)

From (i), we get

$y^2 = 4 - x^2$

∴ $y = \sqrt{4 - x^2}$ ...(iii)

[∵ In first quadrant, y > 0]

Find the point of intersection of

$x^2 + y^2 = 4$ and $x = y\sqrt{3}$.

Substituting (ii) in (i), we get

$$x^2 + \left(\frac{x}{\sqrt{3}}\right)^2 = 4$$

∴ $x^2 + \dfrac{x^2}{3} = 4$

$\therefore \quad \frac{4x^2}{3} = 4$

$\therefore \quad x^2 = 3$

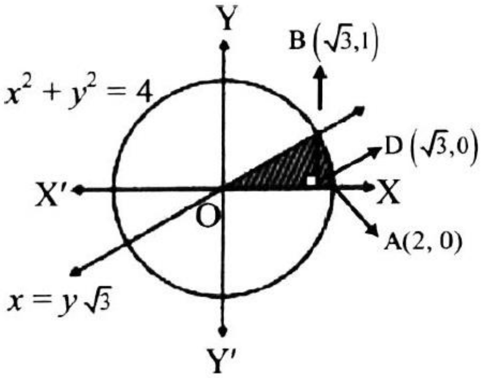
$\therefore \quad x = \pm\sqrt{3}$

$\therefore \quad x = \sqrt{3}$ ...[$\because$ in first quadrant, x > 0]

When $x = \sqrt{3}, y = 1$

$\therefore$ The point of intersection is $B\left(\sqrt{3},1\right)$.

Draw $\text{BD} \perp \text{OX}$



Required area

= area of the region OABO

= area of the region ODBO

+ area of the region BDAB

$$= \int_0^{\sqrt{3}} \frac{x}{\sqrt{3}} dx + \int_{\sqrt{3}}^{2} \sqrt{4 - x^2}\, dx \text{ ...[From (ii) and (iii)]}$$

$$= \frac{1}{\sqrt{3}}\left[\frac{x^2}{2}\right]_0^{\sqrt{3}} + \left[\frac{x}{2}\sqrt{4 - x^2} + \frac{4}{2}\sin^{-1}\left(\frac{x}{2}\right)\right]_{\sqrt{3}}^{2}$$

$$= \frac{1}{2\sqrt{3}}\left[\left(\sqrt{3}\right)^2 - 0\right] + \left[\begin{array}{l}\frac{2}{2}\sqrt{4-4} + 2\sin^{-1}(1) \\ \left\{-\frac{\sqrt{3}}{2}\sqrt{4-3} + 2\sin^{-1}\left(\frac{\sqrt{3}}{2}\right)\right\}\end{array}\right]$$

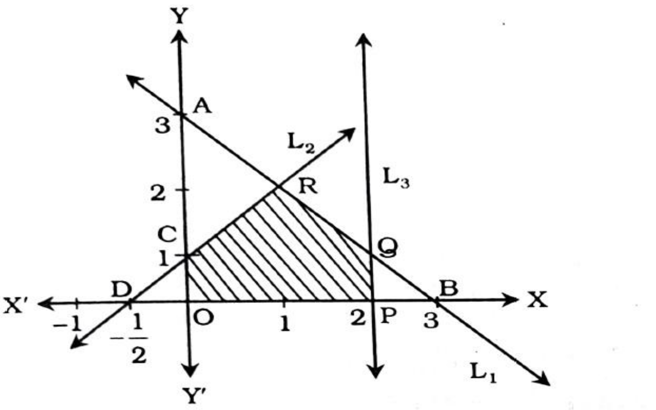
$$= \frac{\sqrt{3}}{2} + \left[0 + 2\cdot\frac{\pi}{2} - \left(\frac{\sqrt{3}}{2} + 2\cdot\frac{\pi}{3}\right)\right]$$

$$= \frac{\sqrt{3}}{2} + \pi - \frac{\sqrt{3}}{2} - \frac{2\pi}{3} = \frac{\pi}{3} \text{ sq. units.}$$

Q.34 To draw $x \le 2,\ x + y \le 3,\ -2x + y \le 1$ 4m

Draw lines $x = 2,\ x + y = 3,\ -2x + y = 1$

To draw	x	y	Lines passes through (x, y)	Sign	Region lies on
L_1 $x + y = 3$	0	3	A (0, 3)	$\le$	Origin side
	3	0	B (3, 0)		
L_2 $-2x + y = 1$	0	1	C (0, 1)	$\le$	Origin side
	$-\frac{1}{2}$	0	D $\left(-\frac{1}{2}, 0\right)$		
L_3 $x = 2$	The line is parallel to Y-axis			$\le$	Origin side



(1 mark)

From the figure the shaded region OPQRC is the feasible region.

Solving equations of L_1 and L_2

$$\begin{array}{r} x + y = 3 \\ -2x + y = 1 \\ + \quad - \quad - \\ \hline 3x = 2 \end{array}$$

$$\therefore x = \frac{2}{3},\ y = \frac{7}{3}$$

$$\therefore R = \left(\frac{2}{3}, \frac{7}{3}\right)$$

The corner points of feasible region are

$$O(0,0), P(2,0), Q(2,1), R\left(\frac{2}{3}, \frac{7}{3}\right), C(0,1)$$

Vertex of Feasible region (x, y)	Value of $Z = 6x + 4y$
O (0, 0)	0
P (2, 0)	12
Q (2, 1)	16
R $\left(\frac{2}{3}, \frac{7}{3}\right)$	$4 + \frac{28}{3} = 13\frac{1}{3}$
C (0, 1)	4

Z is maximum at $x = 2$, $y = 1$ and maximum value of Z is 16.

Together we will make a difference