

MODEL QUESTION PAPER - SET-1 : 2021-22
MATHEMATICS (THEORY)

MM : 80

Time : 3 Hrs.

Entire Syllabus

The question paper is divided into Four Sections:

- (1) **Section A :** Q. No. 1 contains Ten multiple choice type of questions carrying **Two marks** each.
Q. No. 2 contains Eight very short answer type of questions carrying **One mark** each.
- (2) **Section B:** Q. No. 3 to Q. No. 14 contains Twelve short answer type of questions carrying **Two marks** each.
Internal choice is provided (Any 8)
- (3) **Section C:** Q. No. 15 to Q. No. 26 contains Twelve short answer type of questions carrying **Three marks** each.
Internal choice is provided (Any 8)
- (4) **Section D:** Q. No. 27 to Q. No. 34 contains Eight long answer type of questions carrying **Four marks** each.
Internal choice is provided (Any 4)
- (5) Use log – Table if necessary. Use of Calculator is not allowed.

SECTION A

Q.1 Select and Write the correct Answer 16M

i. The shortest distance between the lines $\vec{r} = (\hat{i} + 2\hat{j} + \hat{k}) + \lambda(\hat{i} - \hat{j} - \hat{k})$ and $\vec{r} = (2\hat{i} - \hat{j} - \hat{k}) + \mu(2\hat{i} + \hat{j} + \hat{k})$ is 2m

- | | |
|--------------------------------|---------------------------------|
| a) $\frac{1}{\sqrt{3}}$ | b) $\frac{1}{\sqrt{2}}$ |
| c) $\frac{3}{\sqrt{2}}$ | d) $\frac{5}{\sqrt{26}}$ |

ii. The differential equation $y \frac{dy}{dx} + x = 0$ represents family of _____ 2m

- a)** Circles **b)** Parabolas **c)** Ellipses **d)** Hyperbolas

iii. If $y = \sin(2\sin^{-1} x)$, then $\frac{dy}{dx} =$ 2m

- | | | | |
|---|---|---|---|
| a) $\frac{2-4x^2}{\sqrt{1-x^2}}$ | b) $\frac{2+4x^2}{\sqrt{1-x^2}}$ | c) $\frac{4x^2-1}{\sqrt{1-x^2}}$ | d) $\frac{1-2x^2}{\sqrt{1-x^2}}$ |
|---|---|---|---|

iv. Find expected value of and variance of X for the following p.m.f. 2m

X	-2	-1	0	1	2
P(X)	0.3	0.4	0.2	0.15	0.25

- | | |
|----------------|------------------|
| a) 0.85 | b) - 0.85 |
| c) 0.15 | d) - 0.35 |

v. If polar co-ordinates of a point are $(2, \frac{\pi}{4})$, then its cartesian co-ordinates are 2m

- _____.
- | | | | |
|---------------------------|---------------------------|--------------------|----------------------------------|
| a) $(2, \sqrt{2})$ | b) $(\sqrt{2}, 2)$ | c) $(2, 2)$ | d) $(\sqrt{2}, \sqrt{2})$ |
|---------------------------|---------------------------|--------------------|----------------------------------|

- vi.** If the lines represented by $kx^2 - 3xy + 6y^2 = 0$ are perpendicular to each other, then _____ 2m
a) $k = 6$ **b)** $k = -6$
c) $k = 3$ **d)** $k = -3$

- vii.** $\int \frac{1}{x+x^5} dx = f(x) + c$, then $\int \frac{x^4}{x+x^5} dx =$ 2m
a) $\log x - f(x) + c$ **b)** $f(x) + \log x + c$
c) $f(x) - \log x + c$ **d)** $\frac{1}{5} x^5 f(x) + c$

- viii.** The direction ratios of the line which is perpendicular to the two lines 2m
 $\frac{x-7}{2} = \frac{y+17}{-3} = \frac{z-6}{1}$ and $\frac{x+5}{1} = \frac{y+3}{2} = \frac{z-6}{-2}$
a) 4, 5, 7 **b)** 4, -5, 7 **c)** 4, -5, -7 **d)** -4, 5, 8

Q.2 Answer the following (1 Mark Each) 4M

- i.** Write the negations of ‘Some triangles are equilateral triangle’. 1m
ii. Differentiate the following w.r.t. x, $\tan^{-1}(\log x)$ 1m
iii. Evaluate: $\sin\left[\frac{\pi}{2} + \sin^{-1}\left(-\frac{1}{2}\right)\right]$. 1m
iv. Solve the following differential equation. 1m
 $\frac{dy}{dx} = \frac{1+y^2}{1+x^2}$

SECTION B

Attempt Any Eight Questions 16M

- Q.3** Find k, if slope of one of the lines given by $kx^2 + 4xy - y^2 = 0$ exceeds the slope of the other by 8. 2m
Q.4 Find the volume of tetrahedron whose vertices are A(-1, 2,3), B(3, -2, 1), C(2,1,3) and D(-1, -2, 4). 2m
Q.5 Integrate the following functions w.r.t.x: $\frac{(x^2+2)}{(x^2+1)} \cdot a^{x+\tan^{-1}x}$ 2m
Q.6 Find the value of k such that the line $\frac{x-2}{3} = \frac{y+1}{2} = \frac{z-k}{1}$ lies in the plane $x - y - z + 8 = 0$. 2m
Q.7 Differentiate the following w.r.t. x. $(\sin x)^x$ 2m
Q.8 Find the Cartesian coordinates of the point whose polar coordinates are $\left(\frac{3}{4}, 135^\circ\right)$ 2m

- Q.9** Find the measure of the acute angle between the lines represented by: 2m
 $3x^2 - 4\sqrt{3}xy + 3y^2 = 0$
- Q.10** Find the adjoint of the following matrices: $\begin{bmatrix} 2 & -3 \\ 3 & 5 \end{bmatrix}$ 2m
- Q.11** Evaluate: $\int \frac{x}{x+2} dx$ 2m
- Q.12** If the probability of a defective bolt is 0.2, find the mean and standard deviation for the distribution of defective bolt in a total of 400 bolts. 2m
- Q.13** Evaluate : $\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{1}{1-\sin x} dx$ 2m
- Q.14** Form the differential equation whose general solution is 2m
 $y = A \cos 2x + B \sin 2x$

SECTION C

Attempt Any Eight Questions

24M

- Q.15** Write converse, inverse and contrapositive of the following statement. 3m
 If surface area decreases then pressure increases.
- Q.16** If $\tan \theta + \tan 2\theta + \sqrt{3} \tan \theta \tan 2\theta = \sqrt{3}$, then find the most general value of θ . 3m
- Q.17** The edge of a cube is decreasing at the rate of 0.6 cm/sec. Find the rate at which its volume is decreasing when the edge of the cube is 2cm. 3m
- Q.18** Solve the following differential equations : 3m
 $x \sin\left(\frac{y}{x}\right) dy = \left[y \sin\left(\frac{y}{x}\right) - x \right] dx$
- Q.19** Find the length of the perpendicular from (2, -3, 1) to the line 3m
 $\frac{x+1}{2} = \frac{y-3}{3} = \frac{z+1}{-1}$
- Q.20** Prove that : $\int \sqrt{x^2 + a^2} dx = \frac{x}{2} \sqrt{x^2 + a^2} + \frac{a^2}{2} \log\left(x + \sqrt{x^2 + a^2}\right) + c$ 3m
- Q.21** A(-2, 3, 4), B(1,1,2) and C(4, -1, 0) are the points. Find the Cartesian equations of the line AB and show that points A, B, C are collinear. 3m
- Q.22** Evaluate: $\int \frac{3x-2}{x^2-3x+2} dx$ 3m
- Q.23** Prove by vector method that the angle subtended on semicircle is a right angle. 3m
- Q.24** Prove that : $\int_0^{2a} f(x) dx = \int_0^a f(x) dx + \int_0^a f(2a-x) dx$. 3m

Q.25 A random variable X has the following probability distribution:

X	0	1	2	3	4	5	6	7
P(X)	0	k	2k	2k	3k	k ²	2k ²	7k ² + k

3m

Determine: i. k ii. P(X < 3) iii. P(X > 4)

Q.26 The probability of hitting a target in any shot is 0.2. If 5 shots are fired, find the probability that the target be hit at least twice.

3m

SECTION D

Attempt Any Five Questions

20M

Q.27 If $x = a \cos^3 t, y = a \sin^3 t$, then show that $\frac{dy}{dx} = -\left(\frac{y}{x}\right)^{\frac{1}{3}}$

4m

Q.28 If $A = \begin{bmatrix} 1 & 0 & 0 \\ 3 & 3 & 0 \\ 5 & 2 & -1 \end{bmatrix}$, find A^{-1} by adjoint method.

4m

Q.29 An open cylindrical tank whose base is a circle is to be constructed of metal sheet so as to contain a volume of πa^3 cu.cm of water. Find the dimensions so that sheet required is minimum.

4m

Q.30 Without using truth table prove that: $(p \vee q) \wedge (p \vee \sim q) = p$

4m

Q.31 In ΔABC , if $a \cos A = b \cos B$ then prove that the triangle is right angled or an isosceles triangle.

4m

Q.32 If the points A(3,0,p), B(-1,q,3) and C(-3,3,0) are collinear, then find

4m

i. the ratio in which the point C divides the line segment AB.

ii. the value of p and q.

Q.33 Find the area of the region in first quadrant bounded by the circle $x^2 + y^2 = 4$ and the X axis and the line $x = y\sqrt{3}$.

4m

Q.34 Maximize $z = 6x + 4y$ subject to $x \leq 2, x + y \leq 3, -2x + y \leq 1, x \geq 0, y \geq 0$. Also find the maximum value of z.

4m

Together we will make a difference