MATHEMATICS SAMPLE PAPER # 2

ANSWER AND SOLUTIONS

SECTION-A

1. Let $a_n < 0$

$a + (n-1)d < 0$

$120 + (n-1)(-4) < 0$

$120 < 4(n-1)$

$31 < n$

$\therefore$ a_{32} is first negative term

OR

$S_n = 5n^2 - 3n$

$n = 1 \quad S_1 = 2 = a_1$

$n = 2 \quad S_2 = 20 - 6 = 14$

$a_1 + a_2 = 14$

$2 + a_2 = 14$

$a_2 = 12$

$d = a_2 - a_1 = 12 - 2 = 10$

AP $\rightarrow$ 2, 12, 22..........

$a_{16} = a + 15d$

$= 2 + 15(10)$

$= 2 + 150$

$a_{16} = 152$

2. For equal roots

$D = 0$

$b^2 - 4ac = 0$

$[2(k-12)]^2 - 8(k-12) = 0$

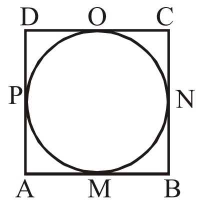
$4(k-12)^2 - 8(k-12) = 0$

$4[k-12][k-12-2] = 0$

$(k-12)(k-14) = 0$

k = 12 Not possible, $\boxed{k = 14}$

3.



Let ABCD be a parallelogram circumscribing a circle.

To prove : ABCD is rhombus.

i.e. to prove AB = BC = CD = DA

Proof : We know that, the tangents to a circle from an external point are equal in length.

$\therefore$ AM = AP, BM = BN, CO = CN and DO = DP

On adding all above equations, we get

(AM + BM) + (CO + DO) = AP + BN + CN + DP

$\Rightarrow$ AB + CD = (AP + PD) + (BN + NC)

= AD + BC .... (i)

Given, ABCD is a parallelogram.

$\therefore$ AB = CD and BC = AD

[$\because$ opposite sides of a parallelogram are equal]

Then, from Eq.(i), we get

2AB = 2BC $\Rightarrow$ AB = BC ..... (iii)

From Equation (ii) and (iii), we get

AB = BC = CD = DA $\Rightarrow$ ABCD is rhombus.

Hence, the parallelogram circumscribing a circle is a rhombus.

4. $\text{Ratio} = \dfrac{\text{Total surface area of cylinder}}{\text{Lateral surface area of cylinder}}$

$$= \frac{2\pi r(r+h)}{2\pi rh} = \frac{80(80+20)}{80 \times 20} = \frac{5}{1}$$

Ratio is 5 : 1

5.

Class	Frequency	x	xf
0 – 10	8	5	40
10 – 20	P	15	15P
20 – 30	12	25	300
30 – 40	13	35	455
40 – 50	10	45	450
	43 + P		1245+15P

$$\frac{1245+15P}{43+P} = 27$$

$1245 + 15P = 1161 + 27P$

$P = 7$

6. Sides be x & y

$x^2 + y^2 = 640$ .... (1)

$4x - 4y = 64$

$4(x - y) = 64$

$x - y = 16$

$x = 16 + y$ .... (2)

from equation (2) & (1)

$(16 + y)^2 + y^2 = 640$

$256 + y^2 + 32y + y^2 = 640$

$2y^2 + 32y - 384 = 0$

$y^2 + 16y - 192 = 0$

$y + 24y - 8y - 192 = 0$

$y(y + 24) - 8(y + 24) = 0$

$(y - 8)(y + 24) = 0$

$y = 8, y = -24$ (rejected)

$x = 24$

Thus, sides are 24 m and 8m

OR

$(x^2 - 2x)^2 - 4(x^2 - 2x) + 3 = 0$

Let $x^2 - 2x = y$

Then, $y^2 - 4y + 3 = 0$

By using the quadratic formula we get

$$y = \frac{4 \pm \sqrt{16-12}}{2}$$

$y = 3,1$

So, $x^2 - 2x = 3$ or $x^2 - 2x = 1$

Taking $x^2 - 2x - 3 = 0$

$$\Rightarrow x = \frac{2 \pm \sqrt{4+12}}{2}$$

$x = 3, -1$

Also when $x^2 - 2x - 1 = 0$

$$\Rightarrow x = \frac{2 \pm \sqrt{4+4}}{2}$$

Also when $x^2 - 2x - 1 = 0$

$\Rightarrow \quad x = 1 \pm \sqrt{2}$

Hence, $x = 1, 3, 1 - \sqrt{2}, 1 + \sqrt{2}$

SECTION-B

7.

	f	
0 – 50	2	
50 – 100	3	
100 – 150	5	f_0
150 – 200	6	f_1
200 – 250	5	f_2
250 – 300	3	
300 – 350	1	

Mode

$$= \ell + \frac{f_1 - f_0}{2f_1 - f_0 - f_2} \times h$$

Modal class $\rightarrow$ 150 – 200

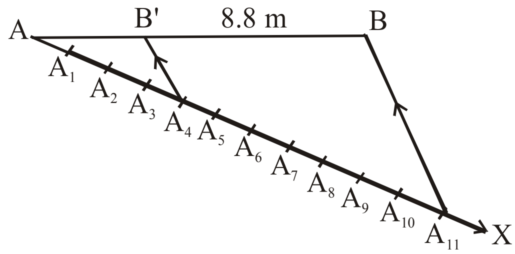
$f_1 = 6;\ f_0 = 5,\ f_2 = 5$

$$\text{Mode} = 150 + \frac{6-5}{12-5-5} \times 50$$

$$\text{Mode} = 150 + \frac{1}{2} \times 50$$

$\boxed{\text{Mode} = 175}$

8.



$$AB' = \frac{4}{11} \times 8.8 = 3.2 \text{ cm}$$

$$B'B = \frac{7}{11} \times 8.8 = 5.6 \text{ cm}$$

9.

Wages (in Rs.)	Number of workers	cf
800 – 820	7	7
820 – 840	14	21
840 – 860	19	40
860 – 880	25	65
880 – 900	20	85
900 – 920	10	95
920 – 940	5	100

$\frac{N}{2} = \frac{100}{2} = 50$

Median class = 860 – 880

Median $= l + \frac{\frac{N}{2} - cf}{f} \times h$

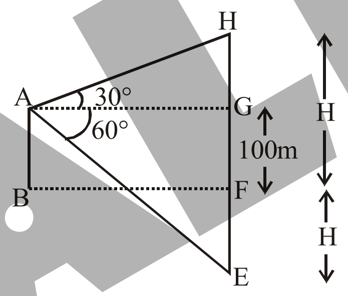
$= 860 + \frac{50-40}{25} \times 20$

$= 860 + \frac{200}{25}$

= 860 + 8 = 868

10. HG = H – 100

GE = H + 100



In ΔAHG

$\tan 30° = \frac{HG}{AG}$

$\frac{1}{\sqrt{3}} = \frac{H-100}{AG}$

$AG = \sqrt{3}\ (H - 100)$ ....(1)

In Δ AGE

$\tan 60° = \frac{GE}{AG}$

$\sqrt{3} = \frac{H+100}{AG} \Rightarrow AG = \frac{H+100}{\sqrt{3}}$ ....(2)

From (1) and (2) $\sqrt{3}\ (H - 100) = \frac{H+100}{\sqrt{3}}$

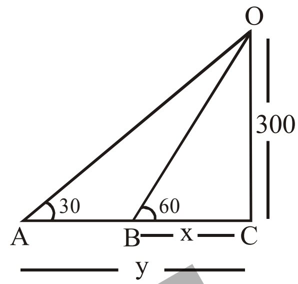
3 (H – 100) = H + 10

3H – 300 = H + 100

2H = 400

H = 200 m

OR



In ΔOBC, $x = \frac{300}{\sqrt{3}} = 100\sqrt{3}$

In ΔOAC, $y = 300\sqrt{3}$

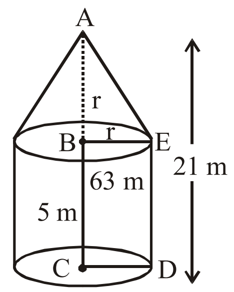
Distance travelled = y – x = $300\sqrt{3} - 100\sqrt{3}$

$= 200\sqrt{3}$

SECTION-C

11. AD = 21 m

AB = 21 – 5 = 16 m



$l = \sqrt{(16)^2 + (63)^2}$

$l = \sqrt{3969 + 256}$

$l = \sqrt{4225} = 65$

CSA of Tent = CSA of cylinder + CSA of cone (πrl)

$= 2 \times \frac{22}{7} \times 63 \times 5 + \frac{22}{7} \times 63 \times 65$

⇒ 22 × 9 (10 + 65)

⇒ 22 × 9 × 75

⇒ 14850 m^2

∴ Cost (a) Rs 12/m^2 = 14850 × 12

= ₹ 178,200

OR

Given, the height of deep well which is in the form of a cylinder, (b_1) = 14 m and radius of deep well, $(r_1) = \frac{3}{2}$ m

Let the width and height of the embankment which is in the form of a circular ring be d and b_2, respectively.

Here, d = 4 cm

Now, the volume of deep well (cylinder)

$$= \pi r_1^2 b_1 = \frac{22}{7} \times \frac{3}{2} \times \frac{3}{2} \times 14 = 99 \text{ m}^3$$

Here, we observe that the embankment form a hollow cylinder. So, for finding the volume of embankment, we apply the concept of hollow cylinder.

Let the height of the embankment be b_2

$\therefore$ Volume of hollow cylinder (embankment)

$= \pi(r_1 + d)^2 \cdot b_2 - \pi r_1^2 b_2$

[Here, radius of outer circle = r_1 + d]

$$= \pi\left(\frac{3}{2}+4\right)^2 \times b_2 - \pi\left(\frac{3}{2}\right)^2 b_2 = \pi\left[\left(\frac{11}{2}\right)^2 b_2 - \frac{9}{4} b_2\right] \text{m}^3$$

Now, according to the question,

Volume of deep well (cylinder)

= Volume of embankment (hollow cylinder)

$$\Rightarrow 99 = \pi\left[\left(\frac{11}{2}\right)^2 \times b_2 - \frac{9}{4} b_2\right]$$

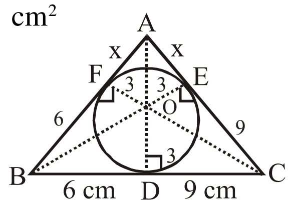
$$\Rightarrow \frac{7 \times 99}{22} = \frac{121}{4} \times b_2 - \frac{9}{4} b_2 \qquad \left[\because \pi = \frac{22}{7}\right]$$

$$\Rightarrow \frac{7 \times 99}{22} = \frac{112 b_2}{4}$$

$$\therefore b_1 = \frac{7 \times 99 \times 4}{22 \times 112} = \frac{9}{8} = 1.125 \text{ m}$$

Hence, the height of the embankment is 1.125 m

12. Given ar (ABC) = 54 cm^2



BD = 6 cm

DC = 9 cm

To find $\Rightarrow$ AB ; AC

Construction Join BO, CO, AO

Proof BD = BF = 6 cm (Ext point tangents)

DC = EC = 9 cm (Ext point tangents)

Let AF = AE = x (Ext point tangents)

ar(ABC) = ar(BOC) + ar(AOC) + ar(BOA)

$$54 = \frac{1}{2} \times 15 \times 3 + \frac{1}{2}(9 + x) \times 3 + \frac{1}{2}(6 + x) \times 3$$

$$54 = \frac{3}{2}[15 + 9 + x + 6 + x]$$

36 = 30 + 2x

6 = 2x

x = 3

$\boxed{AB = 9\text{cm}, AC = 12\text{cm}}$

13.

(i)

Time (in sec)	Number of students (f_1)	Mid-value (x_1)	f_1x_1
0-50	10	25	250
50-100	15	75	1125
100-150	7	125	875
150-200	8	175	1400
Total	40		Sf_1x_1=3650

The average time taken to complete the 200 m race =

$$\frac{\Sigma f_1 x_1}{\Sigma f_i} = \frac{3650}{40} = 91.25\text{s}$$

(ii) The highest frequency in the given data is 15, whose modal class is 50-100.

Here, ℓ = 50, f = 15, f_1 = 10, f_2 = 7 and h = 50.

$\because$ Mode = $l+\frac{f-f_1}{2f-f_1-f_2}\times h$

$= 50 + \frac{15-10}{2\times15-10-7}\times50$

$= 50 + \frac{5\times50}{30-17}$

$= 50 + \frac{250}{13}$

= 50 + 19.23

= 69.23

14. (i) For first metre = 150

for two metres = 150 + 50

= 200

for three metres = 200 + 50

Thus, it forms or A.P.

for 10m

a_{10} = a + 9d

= 150 + 9 × 50 = 600

(ii) If Ram agrees for Rs 550.

Savings = 600 – 550

= Rs. 50