# MATHEMATICS SAMPLE PAPER # 2

## ANSWER AND SOLUTIONS

### SECTION-A

**1.** Given, $21y^2 - 11y - 2 = 0$

$\Rightarrow 21y^2 - 14y + 3y - 2 = 0$

$\Rightarrow 7y(3y-2) + 1(3y-2) = 0$

$\Rightarrow (3y-2)(7y+1) = 0$

$\Rightarrow y = \frac{2}{3}$ or $y = -\frac{1}{7}$

$\therefore$ Zeroes of the given polynomials are $-\frac{1}{7}, \frac{2}{3}$.

**OR**

Given, quadratic equation is $5y^2 - 4y + 3 = 0$

On comparing with $ay^2 + by + c = 0$, we get

$a = 5$, $b = -4$ and $c = 3$

Now. $D = b^2 - 4ac = (-4)^2 - 4(5)(3)$

$= 16 - 60$

$= -44 < 0$

Since, $D < 0$, so the given quadratic equation has no real roots.

**2.** Given radius of base of cone, $r = 12$ cm and height $h = 24$ cm

$\therefore$ Volume of metal cone

$$= \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi \times (12)^2 \times 24 cm^3$$

Also, given diameter of sphere, $d = 6cm$

$\therefore$ Radius of sphere, $r = \frac{d}{2} = \frac{6}{2} = 3cm$

Volume of each spherical solid ball

$$= \frac{4}{3}\pi r^3 = \frac{4}{3}\pi \times (3)^3 = \frac{4}{3}\pi \times 27$$

Let the number of solid spherical balls formed be n.

$$\therefore \quad n = \frac{\text{Volume of solid metal cone}}{\text{Volume of 1 spherical solid ball}}$$

$$= \frac{\frac{1}{3}\pi \times (12)^2 \times 24}{\frac{4}{3}\pi \times 27} = \frac{(12)^2 \times 24}{4 \times 27}$$

$$= \frac{144 \times 24}{4 \times 27} = 32 \text{ balls}$$

**3.** Let assumed mean, $A = 145$

Table for deviation is given below

| Height (in cm) | Number of girls ($f_i$) | Class marks $x_i$ | $d_i = x_i - A$ | $f_i d_i$ |
|---|---|---|---|---|
| 120-130 | 2 | 125 | –20 | –40 |
| 130-140 | 8 | 135 | –10 | –80 |
| 140-150 | 12 | 145=A | 0 | 0 |
| 150-160 | 20 | 155 | 10 | 200 |
| 160-170 | 8 | 165 | 20 | 160 |
| Total | $\Sigma f_i = 50$ | | | $\Sigma f_i d_i = 240$ |

Here, $\Sigma f_i = 50$ and $\Sigma f_i d_i = 240$

$\therefore$ Mean $= A + \frac{\Sigma f_i d_i}{\Sigma f_i} = 145 + \frac{240}{50}$

$= 145 + 4.8 = 149.8$

**4.** $a = 4$, $d = 7 - 4 = 3$

$a_n = 112$

$a + (n-1)d = 112$

$4 + (n-1)3 = 112$

$(n-1)3 = 108$

$n - 1 = 36$

$\Rightarrow n = 37$

There are 37 terms in the sequence.

**5.**

| No. of cars | Frequency |
|---|---|
| 0-10 | 7 |
| 10-20 | 13 |
| 20-30 | 14 |
| 30-40 | 11 |
| 40-50 | 20 |
| 50-60 | 12 |
| 60-70 | 15 |
| 70-80 | 8 |

Modal class is 40–50

$\ell = 40$, $f_1 = 20$, $f_0 = 11$, $f_2 = 12$, $h = 10$

Mode $= \ell + \frac{f_1 - f_0}{2f_1 - f_0 - f_2} \times h$

$= 40 + \dfrac{20-11}{2\times 20 - 11 - 12} \times 10$

$= 40 + \dfrac{9}{40-23} \times 10$

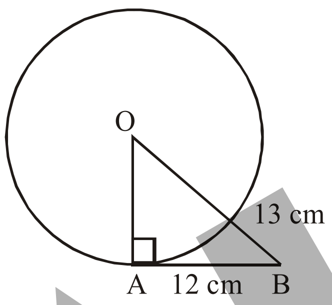
$= 40 + \dfrac{90}{17}$

$= 40 + 5.29$

$= 45.29$

**6.** Let AB be a tangent drawn from point B to a circle with centre O such that AB = 12 cm and OB = 13 cm. We know that, the tangent at any point of a circle is perpendicular to the radius through the point of contact.

$\therefore$ In $\Delta AOB$, $OA \perp AB$




Now, in right angled $\Delta OAB$,

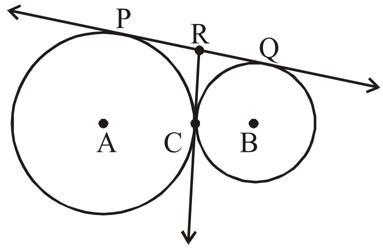
$OB^2 = OA^2 + AB^2$

$\Rightarrow (13)^2 = OA^2 + (12)^2 \Rightarrow 169 = OA^2 + 144$

$\Rightarrow OA^2 = 169 - 144 = 25 \Rightarrow OA = 5$ cm

**OR**

We know that the tangents drawn from an external point to the circle are equal.




$\therefore$ RP = RC and RC = RQ

$\Rightarrow$ RP = RQ

$\Rightarrow$ R is the mid-point of PQ.

## SECTION-B

**7.** The given sequence $7, 10\frac{1}{2}, 14, .... 84$.

$\because \; 10\frac{1}{2} - 7 = 14 - 10\frac{1}{2} = ... = \frac{7}{2}$

$\therefore$ The given sequence is an AP

Here, first term, a = 7,

common difference, $d = \frac{7}{2}$

and last term, $l = a_n = 84$

$\because \; a_n = a + (n-1)d$

$\therefore \; 84 = 7 + (n-1)\frac{7}{2} \quad \left[\because a = 7 \text{ and } d = \frac{7}{2}\right]$

$\Rightarrow \frac{7}{2}(n-1) = 84 - 7$

$\Rightarrow \frac{7}{2}(n-1) = 77$

$\Rightarrow n - 1 = 77 \times \frac{2}{7}$

$\Rightarrow n - 1 = 22 \Rightarrow n = 23$

$\because$ Sum of n terms of an AP,

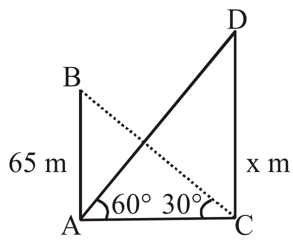
$$S_n = \frac{n}{2}(a + l)$$

$\therefore$ Sum of 23 terms,

$$S_{23} = \frac{23}{2}(7 + 84) = \frac{23}{2} \times 91$$

$$= \frac{2093}{2} = 1046\frac{1}{2}$$

**8.**



Let AB be the tower and CD be the hill.
Then, $\angle CAD = 60°$, $\angle ACB = 30°$ and AB = 65m.
Let CD = x m
From right angled $\Delta BAC$

$$\cot 30° = \frac{AC}{AB} \Rightarrow \sqrt{3} = \frac{AC}{65} \Rightarrow AC = 65\sqrt{3}\text{ m}$$

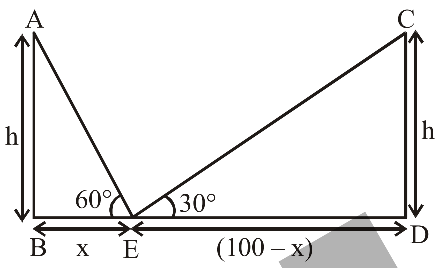
From right angled $\Delta ACD$

$$\tan 60° = \frac{CD}{AC}$$

$$\Rightarrow \sqrt{3} = \frac{CD}{65\sqrt{3}} \Rightarrow CD = 65\sqrt{3} \times \sqrt{3} = 195\text{m}$$

**OR**

Let AB and CD be two pillars of equal height h m and distance between them be BD = 100 m. Let E be a point on the road such that BE = x m, DE = (100 – x)m, $\angle AEB = 60°$ and $\angle CED = 30°$.




In right angled $\Delta ABE$,

$$\frac{AB}{BE} = \tan 60° \Rightarrow \frac{h}{x} = \sqrt{3} \quad [\because \tan 60° = \sqrt{3}]$$

$$\Rightarrow h = \sqrt{3}x \quad \text{...(i)}$$

In right angled $\Delta CDE$,

$$\frac{CD}{DE} = \tan 30°$$

$$\Rightarrow \frac{h}{100 - x} = \frac{1}{\sqrt{3}} \quad [\because \tan 30° = \frac{1}{\sqrt{3}}]$$

$$\Rightarrow h = \frac{100 - x}{\sqrt{3}} \quad \text{.....(ii)}$$

From equation (i) and (ii), we get

$$\sqrt{3}x = \frac{100 - x}{\sqrt{3}}$$

$\Rightarrow 3x = 100 - x$

$\Rightarrow 4x = 100$

$\therefore x = 25$

On putting x = 25 in equation (i), we get

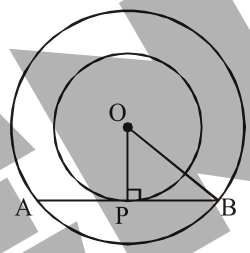
$h = \sqrt{3} \times 25 = 25 \times 1.732 = 43.3$ m

Hence, height of each pillar is 43.3 m and position of the point from a pillar making an angle 60° is 25 m.

**9.** Let OP is radius and AB is tangent of the inner circle.

$\therefore OP \perp AB$ [$\because$ tangent is perpendicular to the radius at the point of contact]

Again, let OB is radius of outer circle.
Here, OP = 3 cm, OB = 5 cm




In right angled $\Delta OPB$,

$(PB)^2 = (OB)^2 - (OP)^2$
[by using Pythagoras theorem]

$$\Rightarrow PB = \sqrt{(5)^2 - (3)^2}$$

[taking positive square root both sides]

$$= \sqrt{25 - 9} = \sqrt{16} = 4\text{cm}$$

$\therefore$ AB = 2PB = 2 × 4 = 8 cm
[$\because$ perpendicular from the centre of a circle to a chord bisects the chord]

Hence, the length of a chord of the circle which touches the inner circle is 8 cm.

**10.** $9x^2 - 9(a + b)x + (2a^2 + 5ab + 2b^2) = 0$

$D = [-9(a + b)]^2 - 4(2a^2 + 5ab + 2b^2) \times 9$

$= 81(a + b)^2 - 36(2a^2 + 5ab + 2b^2)$

$= 81a^2 + 81b^2 + 162ab - 72a^2 - 180ab - 72b^2$

$= 9a^2 + 9b^2 - 18ab$

$D = 9(a - b)^2$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \Rightarrow \frac{[9(a + b)] \pm \sqrt{9(a - b)^2}}{18}$$

$$x = \frac{9(a + b) \pm 3(a - b)}{18}$$

$$x = \frac{9a+9b+3a-3b}{18}$$

$$x = \frac{12a+6b}{18} = \frac{2a+b}{3}$$

$$x = \frac{9a+9b-3a+3b}{18}$$

$$x = \frac{6a+12b}{18} = \frac{a+2b}{3}$$

Thus $x = \frac{2a+b}{3}$ or $\frac{a+2b}{3}$

## SECTION-C

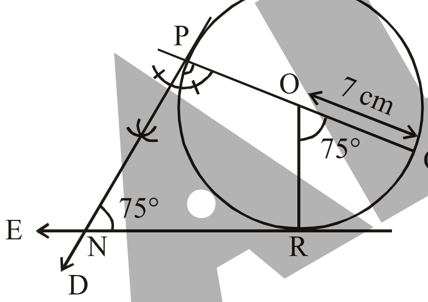
**11.** **Steps of construction**

(i) Draw a circle with O as centre and radius 7 cm.

(ii) Draw any diameter POQ of this circle.

(iii) Draw the radius OR meets the circle at R such that $\angle QOR = 75°$.

(iv) Draw PD $\perp$ PO and RE $\perp$ OR.




Let PD and RE intersect each other at point N. Then, NP and NR are the required tangents to the given circle inclined to each other at an angle of 75°

**OR**

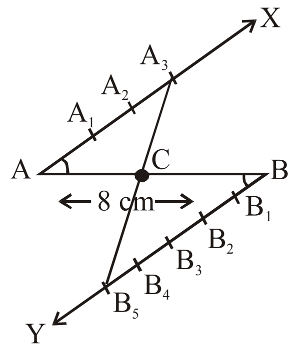
**Steps of construction**

(i) Draw a line segment AB = 8 cm and a ray AX making an acute angle with the line segment AB.

(ii) Draw another ray BY || AX such that $\angle ABY = \angle BAX$.

(iii) Mark 3 points, i.e. $A_1, A_2, A_3$ ($\because$ m = 3) on AX and 5 points, i.e. $B_1, B_2, B_3, B_4, B_5$ ($\because$ n = 5) on BY such that $AA_1 = A_1A_2 = A_2A_3 = BB_1 = B_1B_2 = B_2B_3 = B_3B_4 = B_4B_5$

(iv) Join $A_3B_5$ which intersects AB at point C Thus, C divides AB in the ratio 3 : 5, i.e. AC:CB = 3:5.




**12.** The given series is in inclusive form. Converting it to exclusive form and preparing the cumulative frequency table as given below

| Class interval | Frequency ($f_i$) | Comulative frequency |
|---|---|---|
| 159.5-162.5 | 15 | 15 |
| 162.5-165.5 | 117 | 132 |
| 165.5-168.5 | 136 | 268 |
| 168.5-171.5 | 118 | 386 |
| 171.5-174.5 | 14 | 400 |
| Total | $N=\Sigma f_i=400$ | |

Here, N = 400

Now, $\frac{N}{2} = \frac{400}{2} = 200$

The cumulative frequency just greater than 200 is 268 and the corresponding class is 165.5-168.5.

Thus, the median class is 165.5-168.5.

$\therefore$ $l$=165.5, h = 3 and f = 136 and cf = 132

$$\therefore \text{Median} = l + \left\{ h \times \frac{\frac{N}{2} - cf}{f} \right\}$$

$$= 165.5 + \left\{ 3 \times \frac{200-132}{136} \right\}$$

$$= 165.5 + \frac{3 \times 68}{136}$$

= 165.5 + 1.5 = 167

Hence, the median height is 167 cm.

**13.** (i) In $\Delta BEF$

$$\tan 60^\circ = \frac{BF}{EF}$$

$$\sqrt{3} = \frac{35.5 - 2.5}{EF}$$

$$EF = \frac{33}{\sqrt{3}} = \frac{33\sqrt{3}}{3}$$

$$= 11\sqrt{3}\text{ m}$$

(ii) In $\Delta BFD$

$$\tan 30^\circ = \frac{BF}{DF}$$

$$\frac{1}{\sqrt{3}} = \frac{33}{DE + 11\sqrt{3}}$$

$$DE + 11\sqrt{3} = 33\sqrt{3}$$

$$DE = 22\sqrt{3}\text{ m}$$

**14.** (i) Volume of spherical part = $\frac{4}{3}\pi r^3$

$$= \frac{4}{3} \times 3.14 \times \left(\frac{8.5}{2}\right)^3$$

$= 321.39 \text{ cm}^3$

(ii) Volume of solid figure = vol. of cylinder + vol of sphere

$= \pi\ (1)^2 \times 8 + 321.39$

$= 3.14 \times 8 + 321.39$

$= 25.12 + 321.39$

$= 346.51 \text{ cm}^3$