MATHEMATICS SAMPLE PAPER # 1

ANSWER AND SOLUTIONS

SECTION-A

1. 17th term from end means 17th term from begining of 216, 211, ..., 6, 1 i.e.

$d = 211 - 216 = -5$

$a_{17} = a + 16d = 216 + 16(-5) = 216 - 80 = 136$

2. $4x^2 + 4\sqrt{3}x + 3 = 0$

Here $a = 4$, $b = 4\sqrt{3}$, $c = 3$

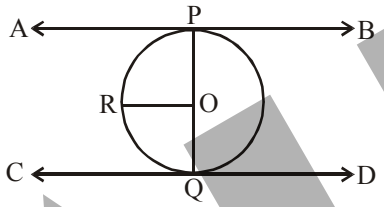
$D = b^2 - 4ac$

$D = (4\sqrt{3})^2 - 4(4)(3) = 48 - 48 = 0$

$\Rightarrow$ Roots are Real and equal

3. Let AB and CD be two parallel tangents to the circle at P and Q respectively.

To prove : POQ is a diameter.



Construction : Through O draw OR||BA or OR || CD as AB and CD are parallel tangents.

Proof : $\angle OPA = 90°$ (radius is always perpendicular to tangent)

Since OR || BA (by construction)

$\angle OPA + \angle POR = 180°$

$\angle POR = 180° - 90° = 90°$

Similarly $\angle QOR = 90°$

$\angle POR + \angle QOR = 180°$

$\Rightarrow$ POQ is straight line through O. So PQ is diameter.

OR

$PT = PS \Rightarrow \angle PTS = \angle PST$

But $\angle PTS + \angle PST + 60° = 180° \Rightarrow$

$2\angle PTS = 120° \Rightarrow \angle PTS = \angle PST = 60°$

$\therefore$ PTS is an equilateral triangle

$\therefore$ TS = PT = 5 cm

4. Volume of hemisphere $= \frac{2}{3}\pi\left(\frac{21}{2}\right)^3$

Volume of one cone $= \frac{1}{3}\pi\left(\frac{7}{2}\right)^2 \times 9$

Number of cones $= \dfrac{2\times\frac{21}{2}\times\frac{21}{2}\times\frac{21}{2}}{\frac{7}{2}\times\frac{7}{2}\times 9} = 21$

OR

Volume of cuboid = [66 × 20 × 27] cm^3

= Volume of the pipe

$= \pi[R^2 - r^2]l$

$\Rightarrow \pi[R^2 - r^2]l = 66 \times 20 \times 27$

Here R = 5 cm and r = 4 cm

$\frac{22}{7} \times [5^2 - 4^2]l = 66 \times 20 \times 27$

$\Rightarrow l = \dfrac{66\times 20\times 27\times 7}{9\times 22}$

$\Rightarrow l = 1260$ cm

5.

Class Interval	x	f	xf
0-20	10	7	70
20-40	30	P	30P
40-60	50	10	500
60-80	70	9	630
80-100	90	13	1170
		39+P	2370+30P

$\frac{2370+30P}{39+P} = 54$

$2370 + 30P = 2106 + 54P$

$264 = 24P$

$P = 11$

6. $a^2b^2x^2 + b^2x - a^2x - 1 = 0$

or $b^2x(a^2x + 1) - 1(a^2x + 1) = 0$

or $(b^2x - 1)(a^2x + 1) = 0$

i.e., $(b^2x - 1) = 0$ or $(a^2x + 1) = 0$

So, $x = \frac{1}{b^2}$ or $x = -\frac{1}{a^2}$

$\therefore$ The roots of the equation are $\frac{1}{b^2}$ and $-\frac{1}{a^2}$.

SECTION-B

7.

Class Interval	f	c.f
15 – 25	8	8
25 – 35	10	18
35 – 45	x	18 + x
45 – 55	25	43 + x
55 – 65	40	83 + x
65 – 75	y	83 + x + y
75 – 85	15	98 + x + y
85 – 95	7	105 + x + y
Total	**140**	

Median = 58

Median class : 55 – 65

$$\text{Median} = \ell + \frac{\frac{N}{2} - \text{c.f.}}{f} \times h$$

$$58 = 55 + \frac{70 - 43 - x}{40} \times 10$$

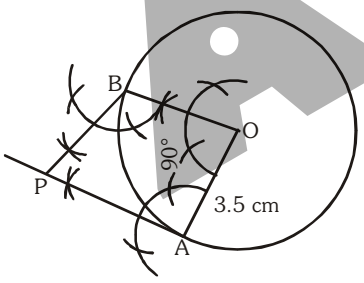
$$3 = \frac{70 - 43 - x}{4}$$

$12 = 27 - x$

$x = 27 - 12 = 15$

$y = 20$

8.



Steps of construction

1. Draw circle with centre at O and radius 3.5 cm.
2. Construct radii OA and OB such that $\angle AOB = 90°$.
3. Draw perpendiculars to OA and OB at A and B respectively and let they intersect at P.
4. Now, PA and PB is a pair of tangents inclined to each other at an angle of 90°.

9.

Lifetime	Frequency
0-20	10
20-40	35
40-60	52
60-80	61
80-100	38
100-120	29
Total	225

Modal class is 60-80

$$\text{Mode} = 60 + \frac{61 - 52}{2 \times 61 - 52 - 38} \times 20$$

$$= 60 + \frac{9}{122 - 90} \times 20$$

$$= 60 + \frac{9}{32} \times 20$$

$$= 60 + \frac{45}{8}$$

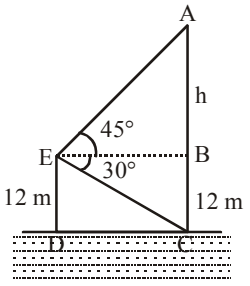
$= 60 + 5.625$

$= 65.625$

10. Let AC be the cliff and ED be the ship.

$AC = (h + 12)$ m

Now, in rt ΔAEB



$$\tan 45° = \frac{h}{EB}$$

$\tan 45° = 1$

$\Rightarrow h = EB$ ...(1)

In rt ΔEBC,

$$\tan 30° = \frac{BC}{EB}$$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{12}{h} \text{ [From (1) , h = EB]}$$

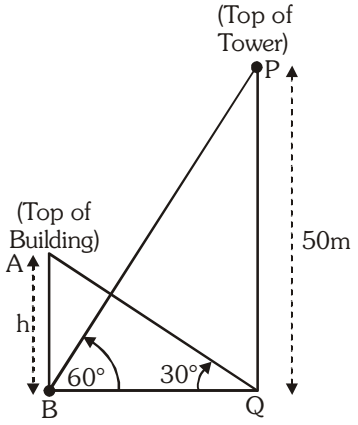
$\Rightarrow h = 12\sqrt{3}$

$\therefore$ Height of cliff $= (12 + 12\sqrt{3})$m.

$= 12\ (1 + \sqrt{3})$m.

OR

PQ = 50 metres is the height of the tower. Let AB = h metres be the height of the building. Angle of elevation of the top of the building from the foot of the tower = 30°, i.e., $\angle AQB = 30°$.



Angle of elevation of the top of the tower from the foot of the building = 60°, i.e., $\angle PBQ = 60°$.

In ΔAQB , $\frac{AB}{BQ} = \tan 30°$

$$\frac{h}{BQ} = \frac{1}{\sqrt{3}}$$

$\Rightarrow$ $BQ = h\sqrt{3}$ ..(i)

In ΔPBQ

$$\frac{50}{BQ} = \tan 60° = \sqrt{3}$$

$\Rightarrow$ $BQ = \frac{50}{\sqrt{3}}$ ..(ii)

From (i) and (ii), we have $h\sqrt{3} = \frac{50}{\sqrt{3}}$

$\Rightarrow$ $h = \frac{50}{3}m$, i.e., $h = 16\frac{2}{3}m$

$\therefore$ The heights of building is $16\frac{2}{3}m$.

SECTION-C

11. Volume of metal in spherical shell

$= \frac{4}{3}\pi[5^3 - 3^3]$ cm^3

$= \frac{4}{3}\pi \times (125 - 27)$ cm^3 $= \frac{4}{3}\pi \times 98$ cm^3

Let r be the radius of solid cylinder.

So, volume of the cylinder = $\pi r^2 h$

$= \pi r^2 \times 10\frac{2}{3}$ cm^3

$\therefore$ $\pi r^2 \times 10\frac{2}{3} = \frac{4}{3}\pi(98)$

$\Rightarrow \pi r^2 \times \frac{32}{3} = \frac{4}{3}\pi \times 98$

$\Rightarrow$ $r^2 = \frac{49}{4}$ $\Rightarrow r = \frac{7}{2}$cm

$\therefore$ Diameter of solid cylinder = 7 cm.

OR

Radius of hemispherical tank = $\frac{\text{Diameter}}{2} = \frac{1}{2}m$

Volume of tank = $\frac{2}{3}\pi r^3$

$= \frac{2}{3} \times \frac{22}{7} \times \frac{1}{8} = \frac{11}{42}$ m^3

Volume of water to be emptied

$= \frac{11}{42} \times 1000$ litres

Since water emptied per second is $\frac{11}{7}$ litres, So,

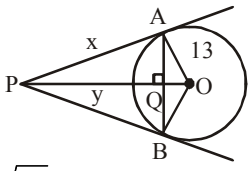
$\frac{11000}{42}$ litres will be emptied in

$\frac{11000}{42} \times \frac{7}{11}s = \frac{1000}{6}$ seconds

$= \frac{500}{3}$ seconds = 166.6 seconds.

12. BQ = 12 cm,

OB = 13 cm



$\therefore$ $OQ = \sqrt{13^2 - 12^2}$

$= \sqrt{160 - 144} = \sqrt{25}$

OQ = 5 cm

Let PQ = y and PA = x

In Δ POA : $x^2 + 13^2 = (y + 5)^2$

$x^2 + 169 = y^2 + 10y + 25$

$\therefore$ $x^2 - y^2 + 169 - 25 = 10y$ ..(1)

In ΔPQA : $x^2 = 12^2 + y^2$

$x^2 - y^2 = 144$ ..(2)

Put (2) in (1) $144 + 169 - 25 = 10y$

$10y = 288 \Rightarrow y = 28.8$

PA = $x = \sqrt{144+(28.8)^2} = \sqrt{973.44}$

= 31.2 cm

13. (i) Let assumed mean, A = 0.10 Table for assumed mean method and their product with corresponding frequency is given below

Concentration of SO_2 (in ppm)	**Frequency (f_i)**	**Class mark (x_i)**	**$d_i=x_i-A$**	**f_id_i**
0.00-0.04	4	0.02	– 0.08	– 0.32
0.04-0.08	9	0.06	– 0.04	– 0.36
0.08-0.12	9	0.10=A	0	0
0.12-0.16	2	0.14	0.04	0.08
0.16-0.20	4	0.18	0.08	0.32
0.20-0.24	2	0.22	0.12	0.24
Total	$\Sigma f_i=30$			$\Sigma f_id_i = -0.04$

Here, total observations (Σf_1) = 30 and Σf_1d_1 = – 0.4

According to formula,

$$\text{Mean} = A + \frac{\Sigma f_1 d_1}{\Sigma f_1} = 0.10 + \frac{(-0.04)}{30}$$

= 0.10 – 0.001 = 0.099 ppm

(ii) Here, N = 30

Concentration of SO_2 (in ppm)	Frequency (f_1)	Cummulative frequency (cf)
0.00-0.04	4	4
0.04-0.08	9	13
0.08-0.12	9	22
0.12-0.16	2	24
0.16-0.20	4	28
0.20-0.24	2	30

$$\therefore \frac{N}{2} = \frac{30}{2} = 15$$

Which lies in the cumulative frequency 22, whose median class interval is 0.08-0.12.

$$\text{Median} = \ell + \frac{\frac{N}{2} - cf}{f} \times h$$

$$= 0.08 + \frac{15-13}{9} \times 0.04 = 0.0888$$

14. (i) Total distance covered by competitor in placing the second potato in bucket.

= 2 × 6 + 2 × (6+4)

= 32 m

(ii) The distance run by the competitor to picks up the first potato, second potato, third potato, fourth potato.....are respectively 2x6 2x{6+4), 2x(6+4+ 4),

2x(6+4+ 4+4+4)... i.e. 12, 20, 28, 36.....

Clearly, it is an AP with first term, a = 12 and common difference, d = 20 –12 = 8

$$s_{10} = \frac{10}{2}[2\times12 + 9\times8]$$

= 5 [24 + 72] = 5 × 96

= 480 m