MODEL QUESTION PAPER SET- 1 : 2021 - 22

STD 10TH – MATHS - I (THEORY)

MM: 40 SOLUTIONS Time : 2 Hrs

ENTIRE SYLLABUS:

Q.1 A) Solve Multiple choice questions. **(4)**

1) Which one is a quadratic equation?

a. $\frac{5}{x} - 3 = x^2$ b. x (x + 5) = 2

c. n – 1 = 2n d. $\frac{1}{x^2}\left(x+2\right) = x$

Ans. Option b.

2) The sum of two natural number is 25 and their difference is 7. The numbers are

a. 17 and 8 b. 16 and 9 c. 18 and 7 d. 15 and 10

Ans. Option b.

3) A card is selected at random from a well-shuffled deck of 52 cards. The probability of its being a face card is

a. $\frac{3}{13}$ b. $\frac{4}{13}$ c. $\frac{6}{13}$ d. $\frac{9}{13}$

Ans. Option a.

Hint : Total face cards = 12

4) In an A.P. 1st term is 1 and the last term is 20. The sum of all terms is = 399 then n = ...

a. 42 b. 38 c. 21 d. 19

Ans. Option b.

B) Solve the following questions. **(4)**

1) Find the value of the following determinants.

$$\begin{vmatrix} -3 & 8 \\ 6 & 0 \end{vmatrix}$$

Ans. $\begin{vmatrix} -3 & 8 \\ 6 & 0 \end{vmatrix}$

=(- 3)$\times$ 0 - 8$\times$6

= 0 - 48

= - 48

2) Joseph kept 26 cards in a cap, bearing one English alphabet on each card. One card is drawn at random. What is the probability that the card drawn is a vowel card ?

Ans. S is the sample space

S = { A, B, C, D, ..., X, Y, Z }

$\therefore$ n(S) = 26

Event A : Card drawn is a Vowel Card.

A = { A, E, I, O, U }

$\therefore$ n(A) = 5

P (A) = $\frac{n(A)}{n(S)} = \frac{5}{26}$

$\therefore$ Probability of getting a vowel card is $\frac{5}{26}$

3) Write an A.P. whose first term is a and common difference is d in each of the following.
$a = -3, d = 0$

Ans. $a = t_1 = -3$
$t_2 = t_1 + d = -3 + 0 = -3, ...$
$t_3 = t_2 + d = -3 + 0 = -3,$
$t_4 = t_3 + d = -3 + 0 = -3.$
$\therefore$ Arithmetic progression is - 3, - 3, - 3, - 3, ...

4) Determine whether the values given against each of the quadratic equation are the roots of the equation.
$x^2 + 4x - 5 = 0, x = 1$

Ans. $x^2 + 4x 5 = 0$
By putting x = 1 in L.H.S. we get,
L.H.S. $= (1)^2 + 4(1) - 5$
$= 1 + 4 - 5$
$= 0$
Thus given equation is satisfied.

Q.2 A) Complete the following Activities. (Any Two) (4)

1) Form a 'Road safety committee' of two, from 2 boys (B1, B2) and 2 girls(G1, G2).
Complete the following activity to write the sample space.
(a) Committee of 2 boys = _____
(b) Committee of 2 girls = _____
(c) Committee of one boy and one girl = _____
$\therefore$ Sample space = {_____}

Ans. Form a 'Road safety committee' of two, from 2 boys (B1, B2) and 2 girls(G1, G2).
Complete the following activity to write the sample space.
(a) Committee of 2 boys = [B_1 B_2]
(b) Committee of 2 girls = [G_1 G_2]
(c) Committee of one boy and one girl = [B_1 G_1], [B_1 G_2], [B_2 G_1], [B_2 G_2]
$\therefore$ Sample space = { $B_1 B_2$, $G_1 G_2$, $B_1 G_1$, $B_1 G_2$, $B_2 G_1$, $B_2 G_2$}

2) The first term of an A. P. is 5 and the common difference is 4. Complete the following activity and find the sum of the first 12 terms of the A. P.
$a = 5, d = 4, s_{12} = ?$
$S_n = \frac{n}{2}$ _____
$S_{12} = \frac{12}{2}$ [10 + _____]
$= \frac{12}{2} [10 + 44]$
$= 6 \times$ _____
_____

Ans. The first term of an A. P. is 5 and the common difference is 4. Complete the following activity and find the sum of the first 12 terms of the A. P.
$a = 5, d = 4, s_{12} = ?$
$S_n = \frac{n}{2} [2a + (n - 1) d]$
$S_{12} = \frac{12}{2} [10 + 11 \times 4]$
$= \frac{12}{2} [10 + 44]$
$= 6 \times 54$
$S_n = 324$

3) Fill in the blanks with correct number

$$\begin{vmatrix} 3 & 2 \\ 4 & 5 \end{vmatrix}$$

= 3 × _____ – _____ × 4
= _____ – 8
= _____

Ans. Fill in the blanks with correct number

$$\begin{vmatrix} 3 & 2 \\ 4 & 5 \end{vmatrix}$$

= 3 × **5** – **2** × 4
= **15** – 8
= **7**

B) Solve the following questions. (Any four) **(8)**

1) Find the fourth term from the end in an A.P. – 11, – 8, – 5, . . . , 49.

Ans. Here a = – 11, d = $t_2 - t_1$ = – 8 – (– 11) = – 8 + 11 = 3, and t_n = 49

∴ $t_n = a + (n - 1) d$ ... (Formula)
∴ 49 = - 11 + (n – 1) × 3 ... (Substituting the values)
∴ 49 + 11 = (n – 1) × 3
∴ (n – 1) × 3 = 49 + 11
∴ (n – 1) × 3 = 60
∴ $n - 1 = \frac{60}{3}$
∴ 3n - 3 = 60
∴ n = 21
∴ There are 21 terms in this A.P.
The fourth term from the end means (21 – 3) = 18^{th} term.
$t_n = a + (n - 1) d$ ... (Formula)
∴ t_{18} = –11 + (18 – 1) × 3 ... (Substituting the values)
= –11 + 17 × 3
= – 11 + 51
t_{18} = 40
The fourth term from the end of the given A.P. is 40.

2) Two coins are tossed simultaneously, Find the probability of getting at least one head.

Ans. S = {HH, HT, TH, TT}
n (S) = 4
A = {HH, HT, TH}
n(A) = 3
$p(A) = \frac{n(A)}{n(S)} = \frac{3}{4}$

3) Solve : x + y = 4; x - y = 2.

Ans. x + y = 4 ... (1)
x - y = 2 ... (2)
Add equation 1 & 2
2x = 6
x = 3
3 + y = 4
y = 4 - 3
y = 1
x = 3, y = 1

4) Solve the following quadratic equations by factorization.
$3x^2 - x - 10 = 0$

Ans. $3x^2 - x - 10 = 0$

$\therefore$ $\underline{3x^2 - 6x} + \underline{5x - 10} = 0$

$\therefore$ $3x(x - 2) + 5(x - 2) = 0$

$\therefore$ $(3x + 5) = 0$ or $(x - 2) = 0$

$\therefore$ $x = -\frac{5}{3}$ or $x = 2$

$\therefore$ $-\frac{5}{3}$, and 2 are the roots of the given quadratic equation.

5) Find the sum of first n odd natural numbers.

Ans. First n natural numbers 1, 3, 5, 7, . . . , (2n - 1).

$a = t_1 = 1$ and $tn = (2n - 1)$, $d = 2$

$Sn = \frac{n}{2}[t_1 + tn]$

$= \frac{n}{2}[1 + (2n - 1)]$

$= \frac{n}{2}[1 + 2n - 1]$

$= \frac{n}{2} \times 2n$

$= n^2$

Q.3 A) Complete the following Activity (Any one) **(3)**

1) The roots of each of the following quadratic equations are real and equal, find k.
$3y^2 + ky + 12 = 0$

Here, a = 3, b = k, c = 12

Δ = _____

$= k^2 - 4(3)(12)$

= _____

The roots are real and equal ... (Given)

$\therefore$ Δ = _____

$\therefore$ $k^2 - 144 = 0$

$\therefore$ _____ = 0

$\therefore$ $k + 12 = 0$ or $k - 12 = 0$

$\therefore$ k = _____ or k = _____

Ans. The roots of each of the following quadratic equations are real and equal, find k.
$3y^2 + ky + 12 = 0$

Here, a = 3, b = k, c = 12

$\Delta = b^2 - 4ac$

$= k^2 - 4(3)(12)$

$= k^2 - 144$

The roots are real and equal ... (Given)

$\therefore$ $\Delta = 0$

$\therefore$ $k^2 - 144 = 0$

$\therefore$ $(k + 12)(k - 12) = 0$

$\therefore$ $k + 12 = 0$ or $k - 12 = 0$

$\therefore$ $k = -12$ or $k = 12$

2) Solve the following simultaneous equations using Cramer's method.

$4m + 6n = 54$
$3m + 2n = 28$

$D = \begin{vmatrix} 4 & 6 \\ 3 & 2 \end{vmatrix}$
$= (4 \times 2) - (6 \times 3)$
$= 8 - 18$

$\therefore$ D =_____

$Dm = \begin{vmatrix} 54 & 6 \\ 28 & 2 \end{vmatrix}$
$= (54 \times 2) - (6 \times 28)$
$= 108 - 168$

$\therefore$ Dm = _____

$Dn = \begin{vmatrix} 4 & 54 \\ 3 & 28 \end{vmatrix}$
$= (4 \times 28) - (54 \times 3)$
$= 112 - 162$

$\therefore$ Dn = _____

By Cramer's rule

$m = \frac{Dm}{D}$ = _____ = 6 and

n = _____ $= \frac{-50}{-10} = 5$

$\therefore$ m = _____, n = _____is the solution of given simultaneous equations.

Ans. Solve the following simultaneous equations using Cramer's method.

$4m + 6n = 54$
$3m + 2n = 28$

$D = \begin{vmatrix} 4 & 6 \\ 3 & 2 \end{vmatrix}$
$= (4 \times 2) - (6 \times 3)$
$= 8 - 18$

$\therefore$ D = - 10

$Dm = \begin{vmatrix} 54 & 6 \\ 28 & 2 \end{vmatrix}$
$= (54 \times 2) - (6 \times 28)$
$= 108 - 168$

$\therefore$ Dm = - 60

$Dn = \begin{vmatrix} 4 & 54 \\ 3 & 28 \end{vmatrix}$
$= (4 \times 28) - (54 \times 3)$
$= 112 - 162$

$\therefore$ Dn = - 50

By Cramer's rule

$m = \frac{Dm}{D} = \frac{-60}{-10} = 6$ and

$n = \frac{Dn}{D} = \frac{-50}{-10} = 5$

$\therefore$ m = 6, n = 5 is the solution of given simultaneous equations.

B) Solve the following questions. (Any two) **(6)**

1) In an A.P. 17^{th} term is 7 more than its 10^{th} term. Find the common difference.

Ans. Here $t_n = a + (n - 1)d$

$\therefore$ $t_{17} = a + (17 - 1)d$

$\therefore$ $t_{17} = a + 16d$ ... I

Also $t_{10} = a + (10 - 1)d$

$\therefore$ $t_{10} = a + 9d$ ... II

Now $t_{17} = t_{10} + 7$ ... [Given]

$\therefore$ $a + 16d = a + 9d + 7$ ... [From I, II]

$\therefore$ $a + 16d - a - 9d = 7$

$\therefore$ $7d = 7$

$\therefore$ $d = \frac{7}{7}$

$\therefore$ **$d = 1$**

Thus, the common difference is 1.

2) Solve: $5x^2 - 4x - 3 = 0$ by completing square method.

Ans. $5x^2 - 4x - 3 = 0$

Dividing the equation by 5,

$x^2 - \frac{4}{5}x - \frac{3}{5} = 0$

$x^2 - \frac{4}{5}x - \frac{3}{5} = 0$

$\therefore$ $x^2 - \frac{4}{5}x + \frac{4}{25} - \frac{4}{25} - \frac{3}{5} = 0$

$\therefore$ $\left(x - \frac{2}{5}\right)^2 - \left(\frac{4}{25} + \frac{3}{5}\right) = 0$

$\therefore$ $\left(x - \frac{2}{5}\right)^2 - \left(\frac{19}{25}\right) = 0$

$\therefore$ $\left(x - \frac{2}{5}\right)^2 = \left(\frac{19}{25}\right)$

$\therefore$ $x - \frac{2}{5} = \frac{\sqrt{19}}{5}$ or $x - \frac{2}{5} = -\frac{\sqrt{19}}{5}$

$\therefore$ $x = \frac{2}{5} + \frac{\sqrt{19}}{5}$ or $x = \frac{2}{5} - \frac{\sqrt{19}}{5}$

$\therefore$ $x = \frac{2 + \sqrt{19}}{5}$ or $x = \frac{2 - \sqrt{19}}{5}$

$\therefore$ $\frac{2 + \sqrt{19}}{5}$ and $\frac{2 - \sqrt{19}}{5}$ are roots of the equation.

3) There are three boys and three girls. An environment committee of two is to be formed. Write the sample space S, the number of sample points n(S). Express the following events and find the total number of elements in the following events: A is the event that the committee should contain at least two girls. B is the event that the committee should contain both the boys. C is the event that there is only one girl in the committee. D is the event that there is at the most one boy in the committee.

Ans. There are three boys says B_1, B_2, B_3 and three girls G_1, G_2,

An environment committee of two members is to be formed.

$\therefore$ The sample space :

$S = \{B_1B_2, B_1B_3, B_2B_3, B_1G_1, B_1G_2, B_1G_3, B_2G_1, B_2G_2, B_2G_3, B_3G_1, B_3G_2, B_3G_3, G_1G_2, G_1G_3, G_2G_3\}$

$\therefore$ $n(S) = 15$

A is the event tht the committee should contain at least two girl.

$\therefore$ $A = \{G_1G_2, G_1G_3, G_2G_3\}$ $\therefore$ $n(A) = 3$

B is the event that the commitee should contain both boys.

$\therefore$ $B = \{B_1B_2, B_1B_3, B_2B_3\}$ $\therefore$ $n(B) = 3$

C is the event that the committee should contain only one girl.

$\therefore$ C = {B_1G_1, B_1G_2, B_1G_3, B_2G_1, B_2G_2, B_2G_3, B_3G_1, B_3G_1, B_3G_1, B_3G_2, B_2G_3}

$\therefore$ n(C) = 9

And D is the event that there is at the most one boys in the committee.

$\therefore$ D = {B_1G_1, B_1G_2, B_1G_3, B_2G_1, B_2G_2, B_2G_3, B_3G_1, B_3G_2, B_3G_3, G_1G_2, G_1G_3, G_2G_3}

$\therefore$ n(D) = 12

4) Solve the following simultaneous equations.
99x + 101y = 499 ; 101x + 99y = 501

Ans. 99x + 101y = 499 ...I

101x + 99y = 501 ...II

Adding equation I and equation II

$$\begin{array}{r} 99x + 101y = 499 \\ + \; 101x + 99y = 501 \\ \hline 200x + 200y = 1000 \end{array}$$

$\therefore$ x + y = 5 ...III (divided by 200)

Subtracting equation I from equation II

$$\begin{array}{r} 101x + 99y = 501 \\ 99x + 101y = 499 \\ - \quad - \quad\quad - \\ \hline 2x - 2y = 2 \end{array}$$

$\therefore$ x - y = 1 ... iv (divided by 2)

Adding equation III and equation IV

$$\begin{array}{r} x + y = 5 \\ + \; x - y = 1 \\ \hline 2x \quad = 6 \end{array}$$

$\therefore$ x = $\frac{6}{2}$

$\therefore$ **x = 3**

Substituting x = 3 in equation III

x + y = 5

$\therefore$ 3 + y = 5

$\therefore$ y = 5 - 3

$\therefore$ **y = 2**

$\therefore$ x = 3, y = 2 is the solution of given simultaneous equations.

Q.4 Solve the following questions. (Any two) (8)

1) Two years ago, my age was $4\frac{1}{2}$ times the age of my son. Six years ago, my age was twice the square of the age of my son. What is the present age of my son?

Ans. Let the present age of my son be x years.

Then 2 years ago, his age was (x - 2) years.

Form the first condition,

2 years ago my age was $\frac{9}{2}$(x - 2) years

$\therefore$ my present age is $\left[\frac{9}{2}\left(x-2\right)+2\right]$ years.

6 years ago, my son's age was (x - 6) years and my age was

$\left[\frac{9}{2}\left(x-2\right)+2-6\right]$ = $\left[\frac{9(x-2)}{2}-4\right]$ years.

From the second condition,

$\frac{9(x-2)}{2}$ - 4 = $2(x - 6)^2$

$\therefore$ $\frac{9x-18-8}{2}$ = $2(x^2 - 12x + 36)$

∴ $9x - 26 = 4(x^2 - 12x + 36)$

∴ $9x - 26 = 4x^2 - 48x + 144$

∴ $4x^2 - 48x - 9x + 144 + 26 = 0$

∴ $4x^2 - 57x + 170 = 0$

∴ $4x^2 - 40x - 17x + 170 = 0$ ... [4 × 170 = 4 × 17 × 10 = 40 × 17]

∴ $4x(x - 10) - 17(x - 10) = 0$

∴ $(x - 10)(4x - 17) = 0$

∴ $x - 10 = 0$ or $4x - 17 = 0$

∴ $x = 10$ or $x = \frac{17}{4} = 4\frac{1}{4}$

If $x = 4\frac{1}{4}$ years, then 6 years ago the son was not born.

∴ $x \neq 4\frac{1}{4}$ $x = 10$

The present age of my son **10 years**.

2) Find three consecutive terms in an A.P. whose sum is - 3 and the product of their cubes is 512.

Ans. Let the three consecutive terms in an A.P. be a - d, a and a + d.

From the first condition,

$(a - d) + a + (a + d) = - 3$

∴ $3a = - 3$ ∴ $a = - 1$.

From the second condition,

$(a - d)^3 \times a^3 \times (a + d)^3 = 512$

∴ $(- 1 - d)^3 \times (- 1)^3 \times (- 1 + d)^3 = 512$ ... [Substituting a = - 1]

∴ $[(- 1) (- 1 - d)]^3 (- 1 + d)^3 = 512$

∴ $(1 + d)^3 (- 1 + d)^3 = (8)^3$

∴ $(1 + d) (- a + d) = 8$ ... (Taking cube root of both the sides)

∴ $d^2 - 1 = 8$ ∴ $d^2 = 9$ ∴ $d = \pm 3$.

Taking a = - 1 and d = 3,

$(a - d) = - 1 - 3 = - 4$;

$(a + d) = - 1 + 3 = 2$

∴ The terms are - 4, - 1 and 2

Taking a = - 1 and d = - 3,

$(a - d) = - 1 - (- 3) = - 1 + 3 = 2$;

$a = - 1$

$a + d = -1 - 3 = - 4$

∴ the terms are 2, -1, -4.

The three consecutive terms are **- 4, - 1** and **2** OR **2, - 1** and **-4.**

3) There are six cards in a box, each bearing a number from 0 to 5. Find the probability of each of the following events, that a card drawn shows,

(1) a natural number. (2) a number less than 1. (3) a whole number. (4) a number is greater than 5.

Ans. Sample space, S = { 0, 1, 2, 3, 4, 5 }

∴ n(S) = 6

Event A: The card drawn shows a natural number

∴ A = { 1, 2, 3, 4, 5 }

∴ n(A) = 5

$P(A) = \frac{n(A)}{n(S)} = \frac{5}{6}$

Event B : The card drawn shows a number less than 1

∴ B = { 0 }

P(B) = $\frac{n(B)}{n(S)} = \frac{1}{6}$

Event C : The card drawn shows a whole number

∴ C = { 0, 1, 2, 3, 4, 5 }

∴ n(C) = 6

P(C) = $\frac{n(C)}{n(S)}$

= $\frac{6}{6}$

= 1

∴ C is certain event.

Event D: The card drawn shows a number is greater than 5.

∴ D = { }

P(D) = $\frac{n(D)}{n(S)}$

∴ P(D) = $\frac{0}{6} = 0$

∴ D is an impossible event.

Q.5 Solve the following questions. (Any one) (3)

1) All the three face cards of spades are removed from a well-shuffled pack of 52 cards. A card is then drawn at random from the remaining pack. Find the probability of getting

i. a black face card

ii. a queen

iii. a black card

iv. a heart

v. a spade

vi. '9' of black colour

Ans. In a pack of 52 cards

All the three face cards of spade are = 3

Number of remaining cards = 52 - 3 = 49

One card is drawn at random

i. Probability of a black face card which are

= 6 - 3 = 3 = $\frac{3}{49}$

ii. Probability of being a queen which are

4 - 1 = 3

∴ Probability = $\frac{3}{49}$

iii. Probability of being a black card

(26 - 3 = 23) = $\frac{23}{49}$

iv. Probability of being a heart = $\frac{13}{49}$

v. Probability of being a spade = (13 - 3 = 10)

= $\frac{10}{49}$

vi. Probability of being 9 of black colour (which are 2)

= $\frac{2}{49}$

2) Solve: 15x + 17y = 21; 17x + 15y = 11

Ans. 15x + 17y = 21 ... I

17x + 15y = 11 ... II

Add the two given equations.

$$\begin{array}{r} 15x+17y=21 \\ +17x+15y=11 \\ \hline 32x+32y=32 \end{array}$$

Dividing both sides of the equation by 32.

x + y = 1 ... III

Subtract equation (II) from (I)

$$\begin{array}{r} 15x+17y=21 \\ 17x+15y=11 \\ - \quad - \quad - \\ \hline -2x+2y=10 \end{array}$$

Dividing the equation by 2.

-x + y = 5 ... IV

Now let's add equations (III) and (V).

$$\begin{array}{r} x+y=1 \\ + \quad -x+y=5 \\ \hline \therefore \quad 2y=6 \end{array}$$

∴ y = 3

Place this value in equation (III)

x + y = 1

∴ x + 3 = 1

∴ x = 1 - 3

∴ x = - 2

(x, y) = (- 2, 3) is the solution.

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