MODEL QUESTION PAPER-1 : 2020-21

MATHEMATICS

Time Allowed: 3 Hours **Maximum Marks: 80**

General Instructions:

1. This question paper contains two **parts A and B**. Each part is compulsory. Part A carries **24** marks and Part B carries **56** marks
2. **Part-A** has Objective Type Questions and **Part -B** has Descriptive Type Questions
3. Both Part A and Part B have choices.

Part – A:

1. It consists of two sections**- I and II**.
2. Section **I** comprises of 16 very short answer type questions.
3. Section **II** contains **2** case studies. Each case study comprises of 5 case-based MCQs. An examinee is to attempt **any 4 out of 5 MCQs**.

Part – B:

1. It consists of three sections**- III, IV and V.**
2. Section **III** comprises of 10 questions of **2 marks** each.
3. Section **IV** comprises of 7 questions of **3 marks** each.
4. Section **V** comprises of 3 questions of **5 marks** each.
5. Internal choice is provided in **3** questions of Section–III, **2** questions of Section-IV and **3** questions of Section-V. You have to attempt only one of the alternatives in all such questions.

Part – A

Section I

All questions are compulsory. In case of internal choices attempt any one.

1. State the reason for the relation R in the set {1, 2, 3} given by R {(1, 2), (2, 1)} not to be transitive. **[1]**

OR

If A = {1, 2, 3}, B = {4, 5, 6, 7} and f = {(1, 4), (2, 5), (3, 6)} is a function from A to B; state whether f is one-one or not. **[1]**

2. How many equivalence relations on the set {1, 2, 3} containing (1, 2) and (2, 1) are there in all ? **[1]**

3. If the set A contains 5 elements and the set B contains 6 elements; then find the number of one-one and onto mappings from A to B ? **[1]**

OR

Let A = {a, b, c} and the relation R be defined on A as follows :

R = {(a, a), (b, c), (a, b)}

Then; write minimum number of ordered pairs to be added in R to make R reflexive and transitive. **[1]**

4. Assume X, Y, Z, W and P are matrices of order $2 \times n$, $3 \times k$, $2 \times p$, $n \times 3$ and $p \times k$, respectively. If $n = p$, write the order of the matrix $(7X - 5Z)$? **[1]**

5. Let A be a non-singular square matrix of order 3×3. Then; find the value of $|\text{adj } A|$. **[1]**

OR

Simplify :

$$\cos\theta\begin{bmatrix}\cos\theta & \sin\theta \\ -\sin\theta & \cos\theta\end{bmatrix}+\sin\theta\begin{bmatrix}\sin\theta & -\cos\theta \\ \cos\theta & \sin\theta\end{bmatrix}$$ **[1]**

6. If A_{ij} is the co-factor of the element a_{ij} of the determinant $\begin{vmatrix}2 & -3 & 5 \\ 6 & 0 & 4 \\ 1 & 5 & -7\end{vmatrix}$;

then write the value of $a_{32}.A_{32}$? **[1]**

7. Evaluate : $\int_{-\pi/4}^{\pi/4}\frac{dx}{1+\cos 2x}$ **[1]**

OR

Evaluate : $\int\frac{x+3}{(x+4)^2}e^x\,dx$ **[1]**

8. Calculate the area of the region bounded by the ellipse $\frac{x^2}{25}+\frac{y^2}{16}=1$. **[1]**

9. Find the degree of the differential equation : $\left[1+\left(\frac{dy}{dx}\right)^2\right]^{3/2}=\frac{d^2y}{dx^2}$ **[1]**

OR

Calculate the number of arbitrary constants in the particular solution of a differential equation of third order. **[1]**

10. Write a unit vector in the direction of the sum of vectors : $\vec{a}=2\hat{i}-\hat{j}+2\hat{k}$ and $\vec{b}=-\hat{i}+\hat{j}+3\hat{k}$. **[1]**

11. If the vectors from origin to the points A and B are $\vec{a}=2\hat{i}-3\hat{j}+2\hat{k}$ and $\vec{b}=2\hat{i}+3\hat{j}+\hat{k}$ respectively, then find the area of triangle OAB. **[1]**

12. If $\vec{a},\vec{b}$ and $\vec{c}$ are unit vectors such that $\vec{a}+\vec{b}+\vec{c}=\vec{0}$, then find the value of $\vec{a}.\vec{b}+\vec{b}.\vec{c}+\vec{c}.\vec{a}$. **[1]**

13. Find the direction cosines of the vector $(2\hat{i}+2\hat{j}-\hat{k})$. **[1]**

14. Write the vector equation of the line passing through (1, 2, 3) and perpendicular to the plane $\vec{r}.(\hat{i}+2\hat{j}-5\hat{k})+9=0$. **[1]**

15. One bag contains 3 red and 5 black balls. Another bag contains 6 red and 4 black balls. A ball is transferred from first bag to the second bag and then a ball is drawn from the second bag. Find the probability that the ball drawn is red. **[1]**

16. The probability that atleast one of the two events A and B occurs is 0.6. If A and B occur simultaneously with probability 0.3; evaluate $P(\overline{A})+P(\overline{B})$. **[1]**

Section II

Both the Case study based questions are compulsory. Attempt any 4 sub parts from each question 17 and 18. Each question carries 1 mark

17. A doctor is to visit a patient. From the past experience, it is known that the probabilities that he will come by train, bus, scooter or by other means of transport are respectively $\frac{3}{10}, \frac{1}{5}, \frac{1}{10}$ and $\frac{2}{5}$. The probabilities that he will be late are $\frac{1}{4}, \frac{1}{3}$ and $\frac{1}{12}$, if he comes by train, bus and scooter respectively, but if he comes by other means of transport, then he will not be late.

Based on the above information, answer the following :

(i) The conditional probability that the doctor is late when he arrives; given that he comes by scooter is : **[1]**

a) 0.195 b) 0.083

c) 0.33 d) 0.25

(ii) The patient calls the doctor for confirmation of his arrival. If he finds out that the doctor is late, find the probability that he hasn't come by train ? **[1]**

a) 0.44 b) 0.05

c) 0.5 d) 0.37

(iii) The probability that the doctor came by bus and is late is : **[1]**

a) 0.0083 b) 0.075

c) 0 d) 0.066

(iv) Let E be the event of doctor arriving late while visiting the patient and T_1, T_2, T_3, T_4 be the events that he came by train, bus, scooter and by other means of transport; respectively. Then, the value of $\sum_{i=1}^{4} P(T_i / E)$ is : **[1]**

a) 0.20 b) 0.45

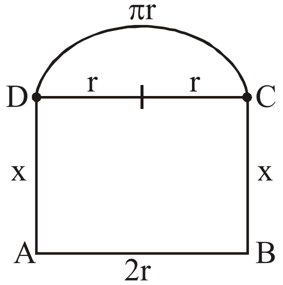
c) 1 d) 0

(v) The total probability of doctor coming late while travelling is : **[1]**

a) 0.15 b) 0.20

c) 0.25 d) 0.60

18. A window is in the form of a rectangle surmounted by a semi-circular opening; as shown below. The total perimeter of the window is 10 m.



Based on the above information, answer the following :

(i) If '2r' and 'x' represent the length and breadth of the rectangular region; then the relation between the variables is : **[1]**

a) $2x + r(\pi + 2) = 10$ b) $x + r(\pi + 2) = 10$

c) $2x + 2r(\pi + 2) = 10$ d) $x + 2r(\pi + 2) = 10$

(ii) The area of the window 'A' expressed as a function of 'r' is : **[1]**

a) $A = 10r - \pi r^2/2$ b) $A = 10r - 2r^2$

c) $A = 10r - \pi r^2/2 - 2r^2$ d) $A = 10r + \pi r^2/2 + 2r^2$

(iii) The maximum or minimum area will be obtained when the value of 'r' is given as : **[1]**

a) $r = \dfrac{10}{5+\pi}$ b) $r = \dfrac{10}{3+\pi}$

c) $r = \dfrac{10}{2+\pi}$ d) $r = \dfrac{10}{4+\pi}$

(iv) The dimensions of the window for maximum area will be : **[1]**

a) $2r = \dfrac{10}{\pi+4}, x = \dfrac{20}{\pi+4}$ b) $2r = \dfrac{20}{\pi+4}, x = \dfrac{10}{\pi+4}$

c) $2r = \dfrac{30}{\pi+4}, x = \dfrac{20}{\pi+4}$ d) $2r = \dfrac{20}{\pi+4}, x = \dfrac{30}{\pi+4}$

(v) The maximum area of the window is given as : **[1]**

a) $30/4+\pi$ b) $60/4+\pi$

c) $50/4+\pi$ d) $40/4+\pi$

Part – B

Section III

19. Express $\cot^{-1}\left[\dfrac{\sqrt{1+\sin x}+\sqrt{1-\sin x}}{\sqrt{1+\sin x}-\sqrt{1-\sin x}}\right]$ in the simplest form. **[2]**

20. If there are two values of 'a' which makes determinant;

$$\Delta = \begin{vmatrix} 1 & -2 & 5 \\ 2 & a & -1 \\ 0 & 4 & 2a \end{vmatrix} = 86;$$

then what will be the sum of these numbers ? **[2]**

OR

Given A = $\begin{bmatrix} 2 & -3 \\ -4 & 7 \end{bmatrix}$, compute A^{-1} and show that $2A^{-1} = 9I - A$ **[2]**

21. Find the value of k, so that f(x) defined below is continuous at x = 0;

where $f(x) = \begin{cases} \left(\dfrac{1-\cos 4x}{8x^2}\right), & \text{if } x \neq 0 \\ k, & \text{if } x = 0 \end{cases}$ **[2]**

22. Find the point on the curve $y = x^3 - 11x + 5$ at which the equation of tangent is $y = x - 11$. **[2]**

23. Evaluate : $\int \frac{\sin x}{3+4\cos^2 x}dx$ **[2]**

OR

Evaluate $\int_2^5 \left[|x-2|+|x-3|+|x-5|\right]dx$ **[2]**

24. The area between $x = y^2$ and $x = 4$ is divided into two equal parts by the line $x = a$; find the value of 'a'. **[2]**

25. Find the particular solution of the differential equation :

$(1-y^2)(1+\log x)dx + 2xy\ dy = 0$, given that $y = 0$ when $x = 1$. **[2]**

26. If $\vec{a} = 2\hat{i} - 3\hat{j} + \hat{k}$, $\vec{b} = -\hat{i} + \hat{k}$, $\vec{c} = 2\hat{j} - \hat{k}$ are three vectors, find the area of the parallelogram having diagonals $(\vec{a}+\vec{b})$ and $(\vec{b}+\vec{c})$. **[2]**

27. Find the equation of a plane which is at a distance $3\sqrt{3}$ units from origin and the normal to which is equally inclined to the co-ordinate axes. **[2]**

28. If A and B are two events such that $P(A) = \frac{1}{2}$, $P(B) = \frac{1}{3}$ and $P(A \cap B) = \frac{1}{4}$; then find :

(i) $P(A'/B)$ (ii) $P(A'/B')$ **[2]**

OR

The probability distribution of a random variable X is given below : **[2]**

X	0	1	2	3
P(X)	k	k / 2	k / 4	k / 8

(i) Determine the value of k.

(ii) Find $P(X \le 2) + P(X > 2)$.

Section IV

All questions are compulsory. In case of internal choices attempt any one.

29. Show that the relation S in the set R of real numbers defined as $S = \{(a, b) : a, b \in R \text{ and } a \le b^3\}$ is neither reflexive, nor symmetric nor transitive. **[3]**

30. Find $\frac{dy}{dx}$, if : $y = (\cos x)^x + (\sin x)^{1/x}$. **[3]**

31. Show that the function $f(x) = |x - 3|$; $x \in R$, is not differentiable at $x = 3$. **[3]**

OR

If $x = \cos t\,(3 - 2\cos^2 t)$ and $y = \sin t\,(3 - 2\sin^2 t)$, find the value of $\frac{dy}{dx}$ at $t = \frac{\pi}{4}$. **[3]**

32. Find the intervals in which the function $f(x) = 3x^4 - 4x^3 - 12x^2 + 5$ is : **[3]**

(a) strictly increasing (b) strictly decreasing

33. Evaluate : $\int \frac{2x^2+1}{x^2(x^2+4)}dx$ **[3]**

34. Find the area of the region in the first quadrant enclosed by the x-axis, the line $y = x$, and the circle $x^2 + y^2 = 32$. **[3]**

OR

Find the area of the region bounded by the parabola $y = x^2$ and $y = |x|$. **[3]**

35. Solve the following differential equation : **[3]**

$$(x^2 - 1)\frac{dy}{dx} + 2xy = \frac{2}{x^2 - 1}, |x| \neq 1$$

Section V

All questions are compulsory. In case of internal choices attempt any one.

36. Find A^{-1}, where $A = \begin{bmatrix} 1 & 2 & -3 \\ 2 & 3 & 2 \\ 3 & -3 & -4 \end{bmatrix}$. Hence; solve the system of equations : **[5]**

$x + 2y - 3z = -4$
$2x + 3y + 2z = 2$
$3x - 3y - 4z = 11$

OR

Determine the product :

$$\begin{bmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{bmatrix}\begin{bmatrix} 1 & -1 & 1 \\ 1 & -2 & -2 \\ 2 & 1 & 3 \end{bmatrix}$$

and use it to solve the system of equations. **[5]**
$x - y + z = 4$,
$x - 2y - 2z = 9$,
$2x + y + 3z = 1$

37. Find the equation of the plane which contains the line the intersection of the planes $\vec{r}.(\hat{i} - 2\hat{j} + 3\hat{k}) - 4 = 0$ and $\vec{r}.(-2\hat{i} + \hat{j} + \hat{k}) + 5 = 0$ and whose intercept on x-axis is equal to that on y-axis. **[5]**

OR

Find the coordinates of the point where the line through the points (3, –4, –5) and (2, –3, 1), crosses the plane determined by the points (1, 2, 3), (4, 2, –3) and (0, 4, 3). **[5]**

38. Solve the following problem graphically : **[5]**
Minimise and Maximise $Z = 3x + 9y$
subject to constraints :

$x + 3y \le 60$; $x + y \ge 10$; $x \le y$; $x \ge 0$; $y \ge 0$.

OR

Find graphically, the maximum value of $Z = 2x + 5y$, subject to constraints given below : **[5]**
$2x + 4y \le 8$; $3x + y \le 6$; $x + y \le 4$; $x \ge 0$; $y \ge 0$.

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