MATHEMATICS SAMPLE PAPER

ANSWER AND SOLUTIONS

PART-A

SECTION-I

1. $11 \times 13 \times 15 \times 17 + 17$

$= 17[11 \times 13 \times 15 + 1]$

Since, the given number has more than two factors.

1, 17 and number itself. Hence, it is a composite number.

OR

$A = b^2$ and $B = a^3b$

$\text{LCM}(A, B) = a^3b^2$

2. Given polynomial $4x^2 + 3x + 7$

$\alpha + \beta = -\frac{3}{4}$ .....(1)

$\alpha\beta = \frac{7}{4}$ ....(2)

Now, $\frac{1}{\alpha} + \frac{1}{\beta} = \frac{\alpha+\beta}{\alpha\beta}$

$= \frac{-\frac{3}{4}}{\frac{7}{4}}$ [From (1) and (2)]

$= \frac{-3}{7}$

3. Given equations $x + 2y = 5$ and $3x + ky = 15$

For unique solution $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

$\frac{1}{3} \neq \frac{2}{k}$

$k \neq 6$

Thus, for $k \neq 6$, the system of equation will have unique solution.

4. Let one's digit of the number be x and ten's digit by y

According to question

$x + y = 12$

$10x + y - (10y + x) = 18$

$9(x - y) = 18$

$x - y = 2$

Thus, required equation are

$x + y = 12$

$x - y = 2$

5. Let n^{th} term of the A.P. be 0

$a_n = 0$

$a + (n - 1)d = 0$

Here, $a = 92$, $d = 88 - 92 = -4$

$\Rightarrow 92 + (n - 1)(-4) = 0$

$\Rightarrow 92 = (n - 1)4$

$\Rightarrow 23 = n - 1$

$\Rightarrow n = 24$

Thus, 24^{th} term of the A.P. is 0.

OR

Here, $a = \sqrt{27} = 3\sqrt{3}, d = \sqrt{48} - \sqrt{27}$

$= 4\sqrt{3} - 3\sqrt{3} = \sqrt{3}$

$a_4 = a + 3d$

$= 3\sqrt{3} + 3\sqrt{3} = 6\sqrt{3} = \sqrt{108}$

So, next term is $\sqrt{108}$

6. Given, equation $2x^2 + kx + 3 = 0$

Here, $a = 2$, $b = k$, $c = 3$

For equal roots

$D = b^2 - 4ac = 0$

$k^2 - 4 \times 2 \times 3 = 0$

$k^2 = 4 \times 2 \times 3$

$k = \pm 2\sqrt{6}$

7. Given equation $6x^2 - x - 2 = 0$

$6x^2 - 4x + 3x - 2 = 0$

$2x(3x - 2) + 1(3x - 2) = 0$

$(3x - 2)(2x + 1) = 0$

$3x - 2 = 0$ or $2x + 1 = 0$

$x = \frac{2}{3}$ or $x = -\frac{1}{2}$

Thus, roots are $\frac{2}{3}$ and $-\frac{1}{2}$

OR

$r = 3$ is root of equation $kr^2 - kr - 3 = 0$

$\Rightarrow k.3^2 - k.3 - 3 = 0$

$\Rightarrow 9k - 3k = 3$

$\Rightarrow 6k = 3$

$\Rightarrow k = \frac{1}{2}$

8. CP = CQ [Length of tangents from an external point to a circle are equal]

CQ = 11 cm [$\because$ CP = 11 cm]

BQ = CQ – CB

= 11 – 7

= 4 cm

BR = BQ [Length of tangents from an external point to a circle are equal]

BR = 4 cm

9. AP = AQ = 4 cm

$\angle PAQ = 90°$ [Given]

Now, $OP \perp AP$, $OQ \perp AQ$

[Line joining the centre to point of contact of tangent is perpendicular to tangent.]

In quadrilateral OPQA

$\angle P + \angle Q + \angle A + \angle O = 360°$

$90° + 90° + 90° + \angle O = 360°$

$\angle O = 90°$

Also, OP = OQ (Radius of same circle)

Thus, OPAQ in a square

$\Rightarrow$ Radius of circle = 4 cm.

OR

$AB \parallel PR$

$\angle ABQ = \angle BQR$ (Alternate $\angle$s)

$\angle ABQ = 70°$

$\angle OQB = 180° - 90° - \angle ABQ = 20°$

10. In Δs ADE and ABC

$\angle ADE = \angle ABC$ [Corresponding angles]

$\angle AED = \angle ACB$ [Corresponding angles]

$\Delta ADE \sim \Delta ABC$ [by AA]

$\frac{AD}{AB} = \frac{DE}{BC}$

$\frac{3}{5} = \frac{4}{x}$

$x = \frac{20}{3}$ cm

11. Minimum number of points of be marked = 5

12. $\cos x = \cos 60° \cos 30° + \sin 60° \sin 30°$

$\cos x = \frac{1}{2} \times \frac{\sqrt{3}}{2} + \frac{\sqrt{3}}{2} \times \frac{1}{2}$

$= \frac{\sqrt{3}}{4} + \frac{\sqrt{3}}{4} = \frac{2\sqrt{3}}{4} = \frac{\sqrt{3}}{2}$

$= \cos 30°$

$x = 30°$

13. $\cos\theta = \frac{1}{2}$

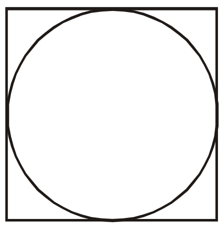
$\Rightarrow \sec\theta = 2$

Now, $\cos\theta(\cos\theta - \sec\theta) = \frac{1}{2}\left(\frac{1}{2} - 2\right)$

$= \frac{1}{2}\left(\frac{1-4}{2}\right) = \frac{-3}{4}$

14. Diameter of circle = side of square = 6 cm

Radius of circle = 3 cm



Area of circle $= \frac{22}{7} r^2 = \frac{22}{7} \times 3 \times 3 = \frac{198}{7}$

$= 28.28 \text{ cm}^2$

15. Volume of sphere = Volume of cuboid

$\frac{4}{3}\pi r^3 = 49 \times 33 \times 24$

r = 21 cm

16. Number of bed eggs = 0.035 × 400 = 14

OR

Number of favourable outcomes = 3

Total outcomes = 36

Probability $= \frac{3}{36} = \frac{1}{12}$

SECTION-II

17. (a) (i) The coordinates of points Q and S are (2, 3) and (6, 6)

(b) (ii) Distance between the vertices Q and S is

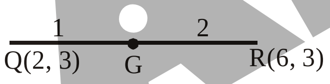
$8 = \sqrt{(6-2)^2+(6-3)^2} = \sqrt{4^2+3^2}$

$= \sqrt{16+9} = \sqrt{25} = 5$ units

(c) (i) As the coordinates of points Q and P are (2, 3) and (2, 6).

∴ The width of the rectangle = 6 – 3 = 3

(d) (ii) By using internal division formula.



Coordinate of

$G = \left(\frac{1\times6+2\times2}{1+2}, \frac{1\times3+2\times3}{1+2}\right)$

$= \left(\frac{6+4}{3}, \frac{3+6}{3}\right)$

$= \left(\frac{10}{3}, \frac{9}{3}\right) = \left(\frac{10}{3}, 3\right)$

(e) (iii) Area of rectangle PQRS

= Length × Breadth

= 4 × 3 = 12 sq. units

18. (a) (i) Total number of equilateral triangle in the given figure is 6.

(b) (iv) If area of two triangle are equal, then it is not necessary to be similar or congruent.

(c) (iii) As we know that there are six equilateral triangle all have equal side.

Hence, we get six similar triangles.

(d) (iv) 'SSA' property is not used for making triangles similar.

(e) (iv) Area of hexagon = 6 × Area of one equilateral triangle having side a.

$= 6 \times \frac{\sqrt{3}}{4} \times (a)^2$

$= \frac{3\sqrt{3}}{2}(a)^2$

19. (a) (i) Here, we see that shape of the parabola is upward, So, in quadratic polynomial $ax^2 + bx + c$, a is always > 0.

(b) (ii) Given curve intersect the x-axis at two point i.e. –2 and 1.

Hence, zeroes of given curve are –2 and 1.

(c) (ii) Since, zeroes of given polynomial are –2 and 1.

∴ Polynomial expression,

$p(x) = x^2 -$ (sum of zeroes)x + product of zeroes

$= x^2 - (-2 + 1)x + (-2)(1)$

$= x^2 + x - 2$

(d) (i) We have, $p(x) = x^2 + x - 2$

When x = 2, then $p(2) = 2^2 + 2 - 2 = 4$

(e) (iv) If we move that parabola right side of one unit, then zeroes polynomial becomes –1 and 2.

Now, polynomial expression

$= x^2 -$ (sum of zeroes)x + product of zeroes

$= x^2 - (-1 + 2)x + (-1) \times 2 = x^2 - x - 2$

20.

Weight (in kg)	Number of students	cf	x_i	$f_i x_i$
38 – 40	3	3	39	117
40 – 42	2	5	41	82
42 – 44	4	9	43	172
44 – 46	5	14	45	225
46 – 48	14	28	47	658
48 – 50	4	32	49	196
50 – 52	3	35	51	153
	$\sum f = 35$			$\sum f_i x_i = 1603$

(a) (ii) N = 35, $\frac{N}{2} = \frac{35}{2}$ = 17.5 median class 46 – 48

(b) (i) Here ℓ = 46, $\frac{N}{2}$ = 17.5, cf = 14, f = 14

Median = $46 + \frac{17.5 - 14}{14} \times 2$

$= 46 + \frac{3.5}{14} \times 2$

= 46.5

(c) (ii) Mean = $\frac{\sum f_i x_i}{\sum f_i} = \frac{1603}{35}$ = 45.8

(d) (i) Modal class is 46 – 48

(e) (i) Median > Mean

PART-B

SECTION-III

21. 72 = 2 × 2 × 2 × 3 × 3

80 = 2 × 2 × 2 × 2 × 5

120 = 2 × 2 × 2 × 3 × 5

LCM of 72, 80, 120 = 2 × 2 × 2 × 2 × 3 × 3 × 5

= 720

22. Since A and B both are on the same circle with centre O(2, 3))

⇒ OA = OB (Radius of circle)

$\Rightarrow \sqrt{(2-4)^2 + (3-3)^2} = \sqrt{(2-x)^2 + (3-5)^2}$

On squaring

4 = (2 – x) + 4

2 – x = 0

x = 2

OR

Let fourth vertex D be (x, y)

Since diagonals of parallelogram bisect each other

⇒ 1 + 4 = 1 + x

x = 4

2 + 0 = 0 + y

⇒ y = 2

Thus, D(4, 2)

23. $x^2 + x - 6$

$= x^2 + 3x - 2x - 6$

= x(x + 3) –2(x + 3)

= (x + 3)(x – 2)

For finding zeros

(x + 3)(x – 2) = 0

⇒ x = –3, or x = 2

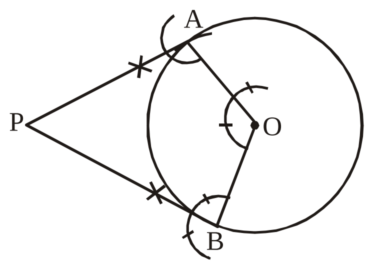
Thus, zeros are –3 and 2

Sum of zeros = –3 + 2 = –1 = $\frac{-\text{coefficient of } x}{\text{coefficient of } x^2}$

Product of zeros = –3 × 2 = –6 = $\frac{\text{constant}}{\text{coefficient of } x^2}$

Hence verified.

24. **Steps of construction**



1. A circle with centre O and radius 3.5 cm is drawn.
2. A radius OB is drawn.
3. OA ⊥ OB is drawn.
4. AP ⊥ OA and BP ⊥ OB is drawn intersecting each other at P.
5. PA and PB are required tangents.

25. LHS

$$\frac{\cos A}{1+\sin A}+\frac{1+\sin A}{\cos A}$$

$$=\frac{\cos^2 A+(1+\sin A)^2}{\cos A(1+\sin A)}$$

$$=\frac{\cos^2 A+1+\sin^2 A+2\sin A}{\cos A(1+\sin A)}$$

$$=\frac{1+1+2\sin A}{\cos A(1+\sin A)} \qquad [\because \sin^2 A+\cos^2 A=1]$$

$$=\frac{2(1+\sin A)}{\cos A(1+\sin A)}$$

$$=\frac{2}{\cos A}=2\sec A$$

= RHS

Hence proved

OR

$$\sqrt{3}\tan\theta=3\sin\theta$$

$$\sqrt{3}\frac{\sin\theta}{\cos\theta}=3\sin\theta$$

$$\cos\theta=\frac{1}{\sqrt{3}}$$

Now, $\sin^2\theta-\cos^2\theta = 1-\cos^2\theta-\cos^2\theta$

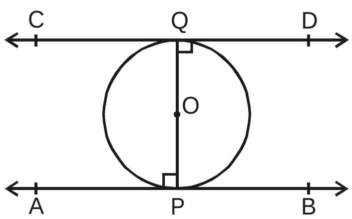
$= 1-2\cos^2\theta$

$=1-\frac{2}{3}=\frac{1}{3}$

26. In the figure, PQ is diameter of the given circle and O is its centre.

Let tangents AB and CD be drawn at the end points of the diameter PQ.

Since, the tangents at a point to a circle is perpendicular to the radius through the point.



$\therefore$ $PQ \perp AB$

$\Rightarrow$ $\angle APQ = 90°$ and $PQ \perp CD$

$\Rightarrow$ $\angle PQD = 90°$

$\Rightarrow$ $\angle APQ = \angle PQD$

But they form a pair of alternate angles.

$\therefore$ AB $\parallel$ CD.

Hence, the two tangents are parallel.

SECTION-IV

27. Let us assume, to the contrary, that $3-\sqrt{5}$ is rational. That is, we can find coprime integers a and b ($b \neq 0$) such that $3-\sqrt{5}=\frac{a}{b}$, $b \neq 0$,

a, b $\in$ I

Therefore, $\frac{a}{b}-3=-\sqrt{5}$

$\Rightarrow$ $\frac{a-3b}{b}=-\sqrt{5}$

$\Rightarrow$ $\frac{a-3b}{2b}=-\sqrt{5}$ $\Rightarrow$ $\frac{a}{2b}-\frac{3}{2}=-\sqrt{5}$

Since a and b are integers, we get $\frac{a}{2b}-\frac{3}{2}$ is rational, and so $\frac{a-3b}{2b}=-\sqrt{5}$ is rational.

But this contradicts the fact that $\sqrt{5}$ is irrational.

This contradiction has arisen because of our incorrect assumption that $3-\sqrt{5}$ is rational.

So, we conclude that $3-\sqrt{5}$ is irrational.

28. $$\frac{1}{a+b+x}-\frac{1}{x}=\frac{1}{a}+\frac{1}{b}$$

$$\frac{x-(a+b+x)}{x(a+b+x)}=\frac{a+b}{ab}$$

$-ab = x(a+b+x)$

$x^2+ax+bx+ab=0$

$x(x+a)+b(x+a)=0$

$(x+a)(x+b)=0$

$x=-a$ or $-b$

OR

$$\frac{x+3}{x-2}-\frac{1-x}{x}=\frac{17}{x}$$

$$\frac{x+3}{x-2}=\frac{17}{x}+\frac{1-x}{x}$$

$$\frac{x+3}{x-2}=\frac{17+1-x}{x}$$

$x(x+3)=(18-x)(x-2)$

$x^2+3x=18x-x^2-36+2x$

$2x^2 - 17x + 36 = 0$

$2x^2 - 9x - 8x + 36 = 0$

$x(2x - 9) - 4(2x - 9) = 0$

$(2x - 9)(x - 4) = 0$

$x = \frac{9}{2}, 4$

29. Let breadth of rectangle be x

Length of rectangle be 2x

Area of rectangle = $x \times 2x = 2x^2$

Area of shaded region = $2x^2 - \frac{1}{2}\pi x^2$

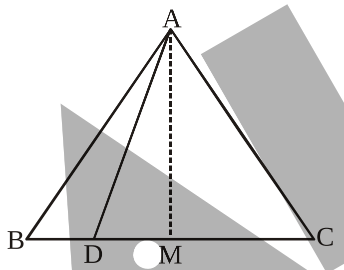
Ratio = $\frac{2x^2}{2x^2 - \frac{1}{2}\pi x^2} = \frac{2\times 2}{4-\pi} = \frac{4}{4-\pi} = \frac{4}{4-\frac{22}{7}}$

$\frac{28}{6} = \frac{14}{3}$

Ratio = 14 : 3

30. AM $\perp$ BC is drawn

In ΔADM, by pythagores theroem



$AD^2 = AM^2 + DM^2$

$AD^2 = AB^2 - BM^2 + DM^2$

$AD^2 = AB^2 - (BM^2 - DM^2)$

$= AB^2 - (BM + DM)(BM - DM)$

$= AB^2 - (CM + DM)(BD)$ [$\because$ BM = CM]

$= AB^2 - CD.BD$

$= AB^2 - \frac{2}{3}BC.\frac{1}{3}BC$

[$BD = \frac{1}{3}$ BC $\Rightarrow CD = \frac{2}{3}BC$]

$= AB^2 - \frac{2}{9}BC^2$

$AD^2 = \frac{9AB^2 - 2AB^2}{9}$ [$\because$ AB = BC]

$9AD^2 = 7AB^2$ Hence proved.

In ΔABC, DX $\|$ AC

$\frac{BX}{BC} = \frac{BD}{AB}$ ... (1) [By corollary of thales theroem]

In ΔABC, YE $\|$ AB

$\frac{CE}{AC} = \frac{CY}{CB}$ ... (2) [By corollary of thales theroem]

From (1) and (2)

$\frac{BD}{AB} = \frac{CE}{AC}$ [$\because$ BX = CY]

$\frac{AB}{BD} = \frac{AC}{CE}$

Subracting 1 from both sides

$\frac{AB - BD}{BD} = \frac{AC - CE}{CE}$

$\frac{AD}{BD} = \frac{AE}{CE}$

By inverse of BPT

DE $\|$ BC.

31.

Class	Frequency	cf
0–10	7	7
10–20	8	15
20–30	20	35
30–40	8	43
40–50	7	50

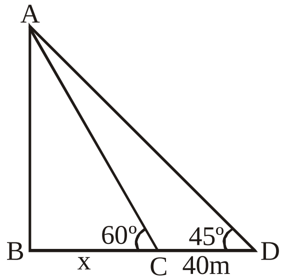
$\frac{N}{2} = \frac{50}{2} = 25$

Median class 20 – 30, cf = 15, f = 20, b = 10

Median = $20 + \frac{25-15}{20} \times 10$

= 20 + 5 = 25

32. Let AB be the tower of height h. D and B are points at a distance of 40m from each other and makes angle of elevation 45º and 60º. Let BC = x



In ΔABC

$\tan 60° = \frac{h}{x}$

$\sqrt{3} = \frac{h}{x}$ $[\because \tan 60° = \sqrt{3}]$

$x = \frac{h}{\sqrt{3}}$ ... (1)

In ΔABD

$\tan 45° = \frac{h}{x+40}$

$1 = \frac{h}{x+40}$

$h = x + 40$

$h = \frac{h}{\sqrt{3}} + 40$ (from 1)

$h - \frac{h}{\sqrt{3}} = 40$

$h\left(\frac{\sqrt{3}-1}{\sqrt{3}}\right) = 40$

$h = \frac{40\sqrt{3}\left(\sqrt{3}+1\right)}{2}$

$= 20\sqrt{3}(\sqrt{3} + 1)$

$= 20 \times 1.73 \times 2.73 = 94.46$ m

33.

Class	Frequency
0-10	5
10-20	10
20-30	18
30-40	30
40-50	20
50-60	12
60-70	5

Modal class is 30–40

$f_1 = 30$, $f_2 = 20$, $f_0 = 18$, $h = 10$

Mode $= \ell + \frac{f_1 - f_0}{2f_1 - f_0 - f_2} \times h$

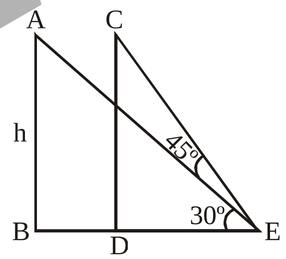
$= 30 + \frac{30-18}{60-20-18} \times 10$

$= 30 + \frac{12}{22} \times 10$

$= 30 + 5.46 = 35.46$

SECTION-V

34. Distance travelled in 20 seconds



$= 360 \times \frac{2}{3600}$

$= 2$ km

Let C and A be the positions of areoplanes and BD = 2 km

Let DE = x

In ΔCDE

$\tan 45° = \frac{CD}{x}$

$CD = x$ $[\because \tan 45° = 1]$

In ΔABE

$\tan 30^\circ = \dfrac{AB}{BE}$

$\dfrac{1}{\sqrt{3}} = \dfrac{x}{2+x}$ [$\because$ AB = CD]

$2 + x = \sqrt{3}x$

$2 = (\sqrt{3} - 1)x$

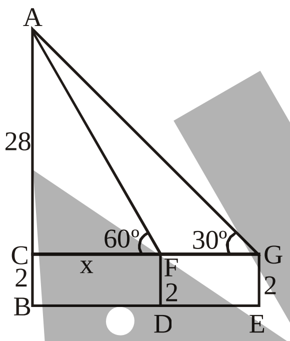
$x = \dfrac{2(\sqrt{3}+1)}{2}$

$= \sqrt{3} + 1 = 2.732$ km

OR

Let AB be the building & GE and DF be the position of boy. Let CF = x

In ΔACF



$\dfrac{28}{x} = \tan 60^\circ$

$x = \dfrac{28}{\sqrt{3}}$ ...(1) [$\because \tan 60^\circ = \sqrt{3}$]

In ΔACG

$\tan 30^\circ = \dfrac{28}{CG}$

$CG = 28\sqrt{3}$ [$\because \tan 30^\circ = \dfrac{1}{\sqrt{3}}$]

$CF + FG = 28\sqrt{3}$

$FG = 28\sqrt{3} - \dfrac{28}{\sqrt{3}}$ [From (1)]

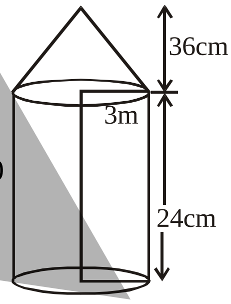
$= \dfrac{28(3-1)}{\sqrt{3}}$

$= \dfrac{28\times 2\sqrt{3}}{3} = 32.30$ m

35. Volume of pillar

= Volume of cone volume of cylinder

$= \dfrac{1}{3}\pi r^2 h + \pi r^2 H$



$= \dfrac{1}{3}\pi\, 8^2 \times 36 + \pi\, 8^2 \times 240$

$= \pi\, 8^2\, [12 + 240]$

$= \dfrac{22}{7} \times 64 \times 252$

$= 22 \times 64 \times 36 = 50688\ cm^3$

Weight of 1 cu cm of iron = 7.5 g

Weight of pillar = 50688 × 7.5

= 380160 gm

= 380.16 kg

36. Let speed of train be x km / hr and speed of car be y km / hr

ATQ

$\dfrac{320}{x} + \dfrac{280}{y} = 8 + \dfrac{40}{60} = \dfrac{26}{3}$

$\dfrac{200}{x} + \dfrac{400}{y} = 9 + \dfrac{10}{60} = \dfrac{55}{6}$

Let $\dfrac{1}{x} = u,\ \dfrac{1}{y} = v$

$320u + 280v = \dfrac{26}{3}$ ...(1)

$200u + 400v = \dfrac{55}{6}$ ...(2)

Multiplying equation (1) by 20 & (2) by 32 and subtracting

$6400\ u + 5600\ v = \frac{520}{3}$

$6400u + 12800\ v = \frac{880}{3}$

$\underline{-\qquad -\qquad\quad -\qquad\qquad}$

$7200\ v = 120$

$v = \frac{120}{7200} = \frac{1}{60}$

From (1)

$320u + 280 \times \frac{1}{60} = \frac{26}{3}$

$320u = \frac{26}{3} - \frac{14}{3}$

$320u = \frac{12}{3}$

$u = \frac{4}{320} = \frac{1}{80}$

Thus, speed of train $= x = \frac{1}{u} = 80$ km/hr

Thus, speed of train $= y = \frac{1}{v} = 60$ km/hr