

MATHEMATICS

SAMPLE PAPER

ANSWER AND SOLUTIONS

PART-A

SECTION-I

1. $\frac{51}{1500} = \frac{17}{500} \Rightarrow \frac{17}{5^3 \times 2^2} = \frac{17}{2^n \times 5^m}$

$\therefore m = 3, n = 2$

Hence $m + n = 5$

2. $f(x) = ax + b$

$\Rightarrow ax + b = 0 \Rightarrow x = -\frac{b}{a}$

OR

$\frac{c}{a} = 4$

$\therefore \frac{-6}{a} = 4$

$\Rightarrow a = -\frac{3}{2}$

3. $b^2 - 4ac = 0$

$k^2 - 4(k)(2) = 0$

$k^2 - 8k = 0$

$k(k - 8) = 0$

$k = 0$ Not possible

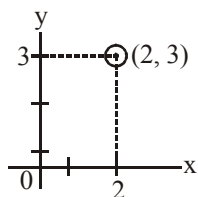
$k = 8$

4. $a = -3$

$d = 4 - (-3) = 7$

$a_{21} = a + 20d = -3 + 20(7) = -3 + 140 = 137$

5. Distance from x-axis is 3 units



6. $PT = PR = PQ = 3.8$ cm

(Tangents from external point)

$\therefore QR = 3.8 + 3.8 = 7.6$ cm

7. $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

$\frac{3}{2} = \frac{2k}{5} \Rightarrow k = \frac{15}{4}$

8. $a = -5$

$d = -\frac{5}{2} + 5 = \frac{5}{2}$

$a_{25} = a + 24d \Rightarrow -5 + 24 \times \frac{5}{2}$

$\Rightarrow -5 + 60 = 55$

9. Volume of big cube = $n \times$ volume of small cube

$(4)^3 = n \times (2)^3$

$\frac{64}{8} = n$

$\Rightarrow n = 8$

OR

$4\pi r^2 = 616$

$4 \times \frac{22}{7} \times r^2 = 616$

$\Rightarrow r^2 = \frac{616 \times 7}{4 \times 22}$

$\Rightarrow r^2 = 49$ or $r = 7$ cm

10. $S_n = \frac{n}{2}(a + a_n)$

$730 = \frac{n}{2}(8 + 65)$

$\Rightarrow 730 = \frac{n}{2} \times 73$

$\therefore n = 20$

OR

$\sqrt{2}; 2\sqrt{2}; 3\sqrt{2} \dots$

$a_{10} = a + 9d$

$= \sqrt{2} + 9\sqrt{2}$

$= 10\sqrt{2} = \sqrt{200}$

11. $OP = \sqrt{(x-0)^2 + (y-0)^2} = \sqrt{x^2 + y^2}$ units
12. Let P(x, -1) and Q(3, 2) be the given points.
Then,
 $PQ = 5$
 $\Rightarrow \sqrt{(x-3)^2 + (-1-2)^2} = 5$
 $\Rightarrow (x-3)^2 + 9 = 5^2$
 $\Rightarrow x^2 - 6x + 18 = 25$
 $\Rightarrow x^2 - 6x - 7 = 0$
 $\Rightarrow (x-7)(x+1) = 0$
 $\Rightarrow x = 7$ or, $x = -1$

13. Putting $x = \frac{7}{3}$ in $3x^2 - 13x - k = 0$

$$3\left(\frac{7}{3}\right)^2 - 13\left(\frac{7}{3}\right) - k = 0$$

$$\frac{49}{3} - \frac{91}{3} - k = 0$$

$$k = -\frac{42}{3} = -14$$

14. Let α, β be the zero of the polynomial $f(x) = x^2 - 8x + k$. Then,

$$\alpha + \beta = -\left(\frac{-8}{1}\right) = 8 \text{ and } \alpha\beta = \frac{k}{1} = k$$

It is given that

$$\alpha^2 + \beta^2 = 40$$

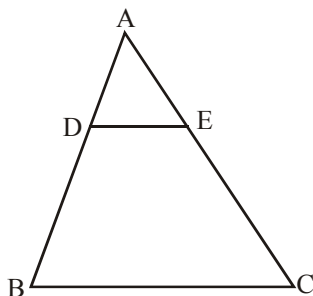
$$\Rightarrow (\alpha + \beta)^2 - 2\alpha\beta = 40$$

$$\Rightarrow 8^2 - 2k = 40$$

$$\Rightarrow 2k = 64 - 40 \quad [\because \alpha + \beta \text{ and } \alpha\beta = k]$$

$$\Rightarrow 2k = 24 \Rightarrow k = 12$$

15. We have,



$AB = 5.6$ cm, $AD = 1.4$ cm, $AC = 7.2$ cm and $AE = 1.8$ cm.

$\therefore BD = AB - AD = (5.6 - 1.4)$ cm = 4.2 cm
and, $EC = AC - AE = (7.2 - 1.8)$ cm = 5.4 cm

$$\text{Now, } \frac{AD}{DB} = \frac{1.4}{4.2} = \frac{1}{3} \text{ and } \frac{AE}{EC} = \frac{1.8}{5.4} = \frac{1}{3}$$

$$\Rightarrow \frac{AD}{DB} = \frac{AE}{EC}$$

Thus, DE divides sides AB and AC of $\triangle ABC$ in the same ratio, Therefore, by the converse of Basic Proportionality Theorem, we have

$DE \parallel BC$

OR

Since, $\triangle ADB \sim \triangle ADC$

$$\frac{BD}{DC} = \frac{AB}{AC}$$

$$\frac{P}{2} = \frac{18}{P}$$

$$P^2 = 36$$

$$P = 6$$

16. Clearly, the given sequence is an A.P. with first term $a (= 4)$ and common difference $d (= 5)$

Let 124 be the n th term of the given sequence.
Then,

$$a_n = 124$$

$$\Rightarrow a + (n-1)d = 124 \Rightarrow 4 + (n-1) \times 5 = 124$$

$$\Rightarrow 5n = 125 \Rightarrow n = 25$$

Hence, 25th term of the given sequence is 124.

OR

$$a_1 = 3, a_3 = 7$$

$$s_3 = \frac{3}{2}(3 + 7) = 15$$

SECTION-II

17. $P(E) = \frac{\text{Number of favourable outcomes}}{\text{Total outcomes}}$

(a) Favourable \rightarrow 2 cards
Total \rightarrow 52 cards
$$P(E) = \frac{2}{52} = \frac{1}{26}$$
 option (iii)

(b) Favourable \rightarrow 12 cards
Total \rightarrow 52 cards
$$P(E) = \frac{12}{52} = \frac{3}{13}$$
 option (iii)

(c) Favourable \rightarrow 26 cards
Total \rightarrow 52 cards
$$P(E) = \frac{26}{52} = \frac{1}{2}$$
 option (i)

(d) Favourable \rightarrow 13 cards
Total \rightarrow 52 cards
$$\text{Probability} = \frac{13}{52} = \frac{1}{4}$$
 option (iii)

(e) Favourable \rightarrow 8 cards
Total \rightarrow 52 cards
$$\text{Probability} = \frac{8}{52} = \frac{2}{13}$$
 option (iv)

18. (a) Curved surface area
 $= 4 \times 2\pi rh$
 $= 4 \times 2 \times \frac{22}{7} \times 2 \times 14$
 $= 704 \text{ m}^2$ option (iii)

(b) Volume $= \pi r^2 h$
 $= \frac{22}{7} \times 2 \times 2 \times 14$
 $= 176 \text{ m}^3$ option (i)

(c)
$$\frac{\text{Surface Area of Big Dome}}{4 \times \text{Surface Area of Smaller Dome}}$$

$$= \frac{2\pi R^2}{4 \times 2\pi r^2}$$

$$= \frac{21 \times 21}{4 \times 7 \times 7}$$

$$= \frac{9}{4}$$
 option (iv)

(d) $2\pi r^2 + 2\pi rh$ option (iii)

(e) Volume $= \frac{2}{3}\pi r^3 + \pi r^2 h$
 $= \frac{2}{3} \times \frac{22}{7} \times 21 \times 21 \times 21 + \frac{22}{7} \times 21 \times 21 \times 3$
 $= 19404 + 4158 = 23562 \text{ m}^3$ option (ii)

19. (a) $a_n = a + (n - 1)d$
 $= 41 + (16 - 1)(-2)$
 $= 41 - 30 = 11$ option (ii)

(b) $a_8 - a_{13} = a + 7d - (a + 12d)$
 $= -5d$
 $= -5(-2) = 10$ option (i)

(c) $a_{11} = 41 + (11 - 1) \times (-2)$
 $= 41 - 20 = 21$ option (iii)

(d) Number of decks $= \frac{\text{Total cards}}{\text{Cards in a deck}}$
 $= \frac{832}{52} = 16$ option (iv)

(e) $a_n = a + (n - 1)d$
 $a = 11$
 $n = 4$
 $d = 2$
 $a_n = 11 + 3(2) = 17$ option (iii)

20. (a) In $\triangle ABD$ $\tan 30^\circ = \frac{300}{BD}$
 $\Rightarrow \frac{1}{\sqrt{3}} = \frac{300}{BD}$
 $\Rightarrow BD = 300\sqrt{3} \text{ m}$ option (iii)

(b) In $\triangle ABC$ $\tan 60^\circ = \frac{300}{BC}$
 $\Rightarrow \sqrt{3} = \frac{300}{BC}$
 $\Rightarrow BC = \frac{300}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = 100\sqrt{3} \text{ m}$ option (i)

(c) Distance between two ships
 $300\sqrt{3} - 100\sqrt{3} = 200\sqrt{3} \text{ m}$ option (iii)

(d) In $\triangle ABD$ $\sin 30^\circ = \frac{AB}{AD}$

$$\frac{1}{2} = \frac{300}{AD}$$

$$\Rightarrow AD = 600 \text{ m} \quad \text{option (iv)}$$

(e) For $100\sqrt{3}$ m time taken 10 min

For $300\sqrt{3}$ m time taken

$$\frac{10}{100\sqrt{3}} \times 300\sqrt{3} = 30 \text{ min} \quad \text{option (ii)}$$

PART-B

SECTION-III

21. In 60 minutes $\rightarrow 360^\circ$

$$\therefore \text{In 5 minutes} \rightarrow \frac{360}{60} \times 5 = 30^\circ$$

$$\text{Area} = \frac{\pi r^2 \theta}{360} = \frac{22}{7} \times \frac{14 \times 14 \times 30}{360}$$

$$= \frac{154}{3} = 51\frac{1}{3} \text{ cm}^2$$

22. Out of 600 electric bulbs one bulb can be chosen in 600 ways.

\therefore Total number of elementary events = 600
There are 588 (= 600 - 12) non-defective bulbs out of which one bulb can be chosen in 588 ways.

\therefore Favourable number of elementary events = 588

Hence, P(Getting a non-defective bulb)

$$= \frac{588}{600} = \frac{49}{50} = 0.98$$

23. Let us assume, to the contrary, that $3 + 2\sqrt{5}$ is rational. That is, we can find coprime integers a and b ($b \neq 0$) such that $3 + 2\sqrt{5} =$

$$\frac{a}{b}, b \neq 0,$$

$$a, b \in I$$

Therefore, $\frac{a}{b} - 3 = 2\sqrt{5}$

$$\Rightarrow \frac{a-3b}{b} = 2\sqrt{5}$$

$$\Rightarrow \frac{a-3b}{2b} = \sqrt{5} \Rightarrow \frac{a}{2b} - \frac{3}{2} = \sqrt{5}$$

Since a and b are integers, we get $\frac{a}{2b} - \frac{3}{2}$ is

rational, and so $\frac{a-3b}{2b} = \sqrt{5}$ is rational.

But this contradicts the fact that $\sqrt{5}$ is irrational. This contradiction has arisen because of our incorrect assumption that $3 + 2\sqrt{5}$ is rational.

So, we conclude that $3 + 2\sqrt{5}$ is irrational.

OR

$$l = 850$$

$$b = 625$$

$$h = 475$$

$$\text{longest rod} = \text{HCF}(850, 625, 475)$$

$$= 5^2 = 25 \text{ cm}$$

24. Yes,

$$\text{Here, } \frac{a_1}{a_2} = \frac{2a}{4a} = \frac{1}{2}, \frac{b_1}{b_2} = \frac{b}{2b} = \frac{1}{2}, \frac{c_1}{c_2} = \frac{-a}{-2a} = \frac{1}{2}$$

$$\therefore \frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

\therefore The given system of equations is consistent.

25. We have, the total number of possible outcomes associated with the random experiment of throwing a die is 6 (i.e. 1, 2, 3, 4, 5, 6).

(i) Let E denotes the event of getting a prime number.

So, favourable number of outcomes = 3 (i.e., 2, 3, 5)

$$\therefore P(E) = \frac{3}{6} = \frac{1}{2}$$

(ii) Let E be the event of getting a number lying between 2 and 6.

∴ Favourable number of elementary events (outcomes) = 3 (i.e., 3, 4, 5)

$$\therefore P(E) = \frac{3}{6} = \frac{1}{2}$$

26. $S_5 + S_7 = 167$

$$\frac{5}{2}(2a + 4d) + \frac{7}{2}(2a + 6d) = 167$$

$$\Rightarrow 24a + 62d = 334$$

$$\text{or } 12a + 31d = 167 \quad \dots(i)$$

$$S_{10} = 235$$

$$\Rightarrow 5(2a + 9d) = 235$$

$$\text{or } 2a + 9d = 47 \quad \dots(ii)$$

Solving (i) and (ii), we get

$$a = 1, d = 5$$

Here A.P. = 1, 6, 11,

OR

$$\text{Here, } a = 27, d = -3, S_n = 0$$

$$\therefore \frac{n}{2}[54 + (n-1)(-3)] = 0$$

$$\Rightarrow n = 19$$

SECTION-IV

27. LHS $\sqrt{\frac{1+\sin\theta}{1-\sin\theta}} + \sqrt{\frac{1-\sin\theta}{1+\sin\theta}}$

$$= \sqrt{\frac{1+\sin\theta}{1-\sin\theta} \times \frac{1+\sin\theta}{1+\sin\theta}} + \sqrt{\frac{1-\sin\theta}{1+\sin\theta} \times \frac{1-\sin\theta}{1-\sin\theta}}$$

$$= \sqrt{\frac{(1+\sin\theta)^2}{1-\sin^2\theta}} + \sqrt{\frac{(1-\sin\theta)^2}{1-\sin^2\theta}}$$

$$= \sqrt{\frac{(1+\sin\theta)^2}{\cos^2\theta}} + \sqrt{\frac{(1-\sin\theta)^2}{\cos^2\theta}}$$

$$= \frac{1+\sin\theta}{\cos\theta} + \frac{1-\sin\theta}{\cos\theta}$$

$$= \frac{2}{\cos\theta} = 2\sec\theta = \text{RHS}$$

Hence proved

OR

We have,

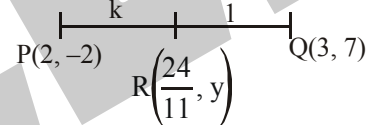
$$\text{LHS} = \frac{\cos^2\theta}{1-\tan\theta} + \frac{\sin^3\theta}{\sin\theta - \cos\theta}$$

$$\Rightarrow \text{LHS} = \frac{\cos^3\theta}{\cos\theta - \sin\theta} - \frac{\sin^3\theta}{\cos\theta - \sin\theta}$$

$$\Rightarrow \text{LHS} = \frac{\cos^3\theta - \sin^3\theta}{\cos\theta - \sin\theta}$$

$$\Rightarrow \text{LHS} = \frac{(\cos\theta - \sin\theta)(\cos^2\theta + \sin^2\theta + \cos\theta\sin\theta)}{\cos\theta - \sin\theta}$$

$$\Rightarrow \text{LHS} = 1 + \sin\theta \cos\theta = \text{RHS}$$

28. 

Let R divides PQ in ratio k : 1

By section formula co-ordinates of 'R' are

$$\frac{m_1x_2 + m_2x_1}{m_1 + m_2}; \frac{m_1y_2 + m_2y_1}{m_1 + m_2}$$

$$\left(\frac{3k+2}{k+1}; \frac{7k-2}{k+1} \right)$$

$$\frac{3k+2}{k+1} = \frac{24}{11}$$

$$33k + 22 = 24k + 24$$

$$9k = 2$$

$$k = \frac{2}{9} \quad \text{Ratio} \rightarrow 2 : 9$$

$$y = \frac{7k-2}{k+1} \Rightarrow \frac{7 \times \frac{2}{9} - 2}{\frac{2}{9} + 1}$$

$$y = \frac{-4}{11}$$

29. We have, $2x + 3y = 7$ (i)

$(a - b)x + (a + b)y = 3a + b - 2$ (ii)

Here, $a_1 = 2, b_1 = 3, c_1 = 7$

and $a_2 = a - b, b_2 = a + b, c_2 = 3a + b - 2$

For infinite number solutions, we have

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2} \Rightarrow \frac{2}{a-b} = \frac{3}{a+b} = \frac{7}{3a+b-2}$$

Now, $\frac{2}{a-b} = \frac{3}{a+b}$

$\Rightarrow 2a + 2b = 3a - 3b$

$\Rightarrow 2a - 3a = -3b - 2b$

$\Rightarrow -a = -5b$

$\therefore a = 5b$ (iii)

Again, we have

$$\frac{3}{a+b} = \frac{7}{3a+b-2} \Rightarrow 9a + 3b - 6 = 7a + 7b$$

$\Rightarrow 9a - 7a + 3b - 7b - 6 = 0$

$\Rightarrow 2a - 4b - 6 = 0 \Rightarrow 2a - 4b = 6$

$\Rightarrow a - 2b = 3$ (iv)

Putting $a = 5b$ in equation (iv), we get

$5b - 2b = 3$ or $3b = 3$ i.e., $b = \frac{3}{3} = 1$

Putting the value of b in equation (iii), we get $a = 5(1) = 5$

Hence, the given system of equations will have an infinite number of solutions for

$a = 5$ and $b = 1$.

OR

Given equations

$$\frac{2}{x} + \frac{3}{y} = 13 \quad \text{.....(1)}$$

$$\frac{5}{x} - \frac{4}{y} = -2 \quad \text{.....(2)}$$

Let $\frac{1}{x} = u, \frac{1}{y} = v$

From (1) and (2)

$2u + 3v = 13$ (3)

$5u - 4v = -2$ (4)

Multiplying equation (3) from 5 and equation (4) by 2 and subtract them

$10u + 15v = 65$

$10u - 8v = -4$

$\underline{- \quad + \quad +}$

$23v = 69$

$v = 3$

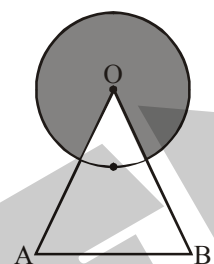
From (3) $2u + 3.3 = 13$

$2u = 4$

$u = 2$

Thus, $x = \frac{1}{2}, y = \frac{1}{3}$

30.



$\angle AOB = 60^\circ$

Area of shaded region = Area of major sector

$$= \frac{300}{360} \times \frac{22}{7} \times (6)^2 = 94.29 \text{ cm}^2$$

31. Let $\sqrt{2}$ be rational

i.e $\sqrt{2} = \frac{p}{q}$ ($q \neq 0, p$ and q are co-prime)

Squaring $2 = \frac{p^2}{q^2}$

$\therefore q^2 = \frac{p^2}{2}$ (1)

If 2 divides p^2 ; then 2 divides p i.e.

2 is factor of p

Let $p = 2k$ (2)

Putting value of p from (2) in (1)

$$q^2 = \frac{4k^2}{2}$$

$$k^2 = \frac{q^2}{2}$$

If 2 divides q^2 then 2 divides q i.e. 2 is factor of q .

\Rightarrow 2 is common factor of p and q

Which is contrary to our assumption

Hence $\sqrt{2}$ is irrational.

32. We observe that the class 12 – 15 has maximum frequency. Therefore, this is the modal class.

We have,

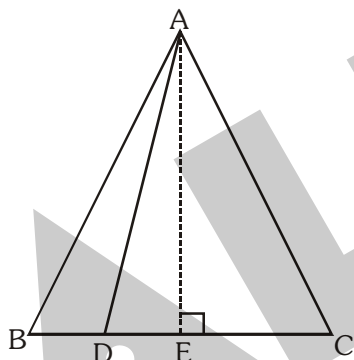
$$l = 12, h = 3, f = 23, f_1 = 10 \text{ and } f_2 = 21$$

$$\therefore \text{Mode} = l + \frac{f - f_1}{2f - f_1 - f_2} \times h$$

$$\Rightarrow \text{Mode} = 12 + \frac{23 - 10}{46 - 10 - 21} \times 3$$

$$\Rightarrow \text{Mode} = 12 + \frac{13}{15} \times 3 = 12 + \frac{13}{5} = 14.6$$

33. Draw $AE \perp BC$



In ΔAEB and ΔAEC , we have

$$AB = AC$$

$$AE = AE \quad [\text{Common}]$$

$$\angle B = \angle C \quad [\because AB = AC]$$

$$\angle AEB = \angle AEC \quad [\text{Each } 90^\circ]$$

$\therefore \Delta AEB \cong \Delta AEC$ [by AAS congruence]

$\Rightarrow BE = CE$ [by cpct]

Since ΔAED and ΔABE are right triangles right angled at E. Therefore,

$$AD^2 = AE^2 + DE^2 \text{ and } AB^2 = AE^2 + BE^2$$

$$\Rightarrow AB^2 - AD^2 = BE^2 - DE^2$$

$$\Rightarrow AB^2 - AD^2 = (BE + DE)(BE - DE)$$

$$\Rightarrow AB^2 - AD^2 = (CE + DE)(BE - DE)$$

$$\Rightarrow AB^2 - AD^2 = CD \cdot BD$$

$$\Rightarrow AB^2 - AD^2 = BD \cdot CD$$

Hence proved

SECTION-V

$$\begin{aligned} 34. \text{ LHS} &= \frac{\tan A}{1 - \frac{1}{\tan A}} + \frac{1}{\tan A(1 - \tan A)} \\ &= \frac{\tan^2 A}{\tan A - 1} - \frac{1}{\tan A(\tan A - 1)} \\ &= \frac{\tan^3 A - 1}{\tan A(\tan A - 1)} \\ &\text{using } a^3 - b^3 = (a - b)(a^2 + ab + b^2) \\ &= \frac{(\tan A - 1)(\tan A + 1 + \tan^2 A)}{\tan A(\tan A - 1)} \end{aligned}$$

$$\frac{\tan A + 1 + \tan^2 A}{\tan A}$$

$$\begin{aligned} &= \frac{1}{\tan A} + \frac{\tan^2 A}{\tan A} + \frac{\tan A}{\tan A} \\ &= \cot A + \tan A + 1 \end{aligned}$$

Hence proved

$$\begin{aligned} \text{Now } 1 + \frac{\sin A}{\cos A} + \frac{\cos A}{\sin A} \\ &= 1 + \frac{\sin^2 A + \cos^2 A}{\cos A \sin A} \\ &= 1 + \frac{1}{\cos A \sin A} \end{aligned}$$

$$= 1 + \sec A \operatorname{cosec} A$$

Hence proved

OR

$$\begin{aligned} m^2 - n^2 &= (\tan \theta + \sin \theta)^2 - (\tan \theta - \sin \theta)^2 \\ &= \tan^2 \theta + \sin^2 \theta + 2 \tan \theta \sin \theta - \tan^2 \theta \\ &\quad - \sin^2 \theta + 2 \tan \theta \sin \theta \\ m^2 - n^2 &= 4 \tan \theta \sin \theta \end{aligned}$$

$$\therefore (m^2 - n^2)^2 = 16 \tan^2 \theta \sin^2 \theta \quad \dots\dots(1)$$

$$\begin{aligned} \text{Now, } 16mn &= 16(\tan \theta + \sin \theta)(\tan \theta - \sin \theta) \\ &= 16[\tan^2 \theta - \sin^2 \theta] \end{aligned}$$

$$= 16 \left[\frac{\sin^2 \theta}{\cos^2 \theta} - \sin^2 \theta \right]$$

$$= 16 \sin^2 \theta \left[\frac{1}{\cos^2 \theta} - 1 \right]$$

$$= 16 \sin^2 \theta [\sec^2 \theta - 1]$$

$$16mn = 16 \sin^2 \theta \tan^2 \theta \quad \dots\dots(2)$$

$$\therefore (m^2 - n^2)^2 = 16mn$$

35. In $\triangle ABD$; $\angle B = 90^\circ$

$$\therefore AD^2 = AB^2 + BD^2$$

$$AD^2 = AB^2 + \left(\frac{BC}{2}\right)^2$$

$$AD^2 = AB^2 + \frac{BC^2}{4} \quad \dots\dots(1)$$

In $\triangle BEC$

$$CE^2 = BC^2 + BE^2$$

$$CE^2 = BC^2 + \left(\frac{AB}{2}\right)^2$$

$$CE^2 = BC^2 + \frac{AB^2}{4} \quad \dots\dots(2)$$

equation (1) and (2)

$$AD^2 + CE^2 = AB^2 + \frac{BC^2}{4} + BC^2 + \frac{AB^2}{4}$$

$$AD^2 + CE^2 = \frac{5}{4}(AB^2 + BC^2)$$

$$AD^2 + CE^2 = \frac{5}{4}AC^2 \quad [AC^2 = AB^2 + BC^2]$$

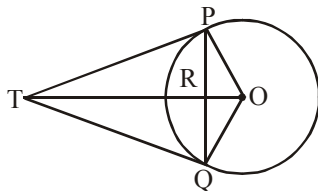
$$\left(\frac{3\sqrt{5}}{2}\right)^2 + CE^2 = \frac{5}{4} \times 25$$

$$CE^2 = \frac{125}{4} - \frac{45}{4}$$

$$CE^2 = 20$$

$$CE = \sqrt{20} = 2\sqrt{5} \text{ cm}$$

36.



Given $PQ = 16$ cm

Radius = 10 cm

To find : TP

Solution : Join OP and OQ

In $\triangle OTP$ and $\triangle OTQ$

$$OP = OQ \quad \text{(Radius)}$$

$$TP = TQ \quad \text{(Tangents from external point)}$$

$$OT = OT \quad \text{(common)}$$

$$\therefore \triangle OPT \cong \triangle OQT \quad \text{(SSS)}$$

$$\angle POT = \angle QOT \quad \text{(cpct) } \dots\dots(i)$$

In $\triangle OPR$ and $\triangle OQR$

$$OP = OQ$$

$$\angle POR = \angle QOP \quad \text{(from (i))}$$

$$OR = OR$$

$$\triangle OPR \cong \triangle OQR \quad \text{(SAS)}$$

$$\text{So, } PR = RQ = \frac{1}{2} \times 16 = 8 \text{ cm } \dots\dots(ii)$$

$$\angle ORP = \angle ORQ = 90^\circ \quad \dots\dots(iii)$$

(By cpct)

In $\triangle OPR$

$$OR^2 = OP^2 - PR^2$$

$$OR^2 = 100 - 64$$

$$OR = 6$$

$$\text{In } \triangle TRP \quad TR^2 = TP^2 - 64 \quad \dots\dots(iv)$$

In $\triangle TOP$

$$OT^2 = TP^2 + (10)^2$$

$$(TR + OR)^2 = TP^2 + 100$$

$$(TR + 6)^2 = TP^2 + 100$$

$$TR^2 + 12TR + 36 = TP^2 + 100$$

$$TP^2 - 64 + 12TR + 36 = TP^2 + 100$$

(using (iv))

$$12TR = 128$$

$$TR = \frac{32}{3} \text{ cm}$$

$$\text{from (iv) } \left(\frac{32}{3}\right)^2 = TP^2 - 64$$

$$TP^2 = \frac{1024}{9} + 64$$

$$TP^2 = \frac{1024 + 576}{9} = \frac{1600}{9}$$

$$TP = \frac{40}{3} \text{ cm}$$