NSEJS-2019 (IJSO STAGE-I)

Date of Examination : 17th November, 2019

PAPER CODE - 54

SOLUTIONS

1. A group of students was studying development of an organism under controlled laboratory conditions. Following observations were made by them.

i. The larvae had a rod-like supporting structure that separated the nervous system and the gut.

ii. A prominent central cavity was present in the transverse section of the part of the nervous system of the larvae; while the adults had cerebral ganglia as the main component of the nervous system.

iii. The eyes were prominently seen in larvae.

iv. The tails were absent in the adults, which the larvae had.

v. A lot of phagocytic activity was observed before conversion of larvae into adults.

vi. The adults had a cuticular exoskeleton.

The organism under study must be belonging to:

(a) Amphibia (b) Pisces (c) Protochordata (d) Arthropoda

Ans. **(c)**

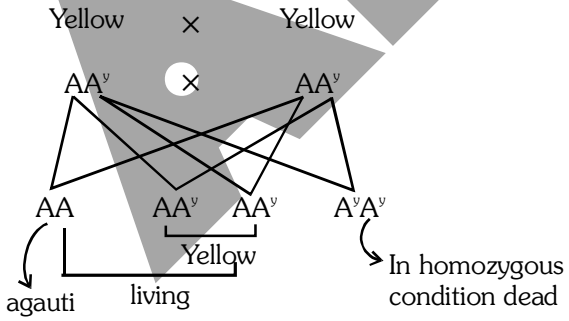
Sol. All the observations are of protochordata (Protochordata include urochordata and cephalochordata, urochordata has retrogressive metamorphosis while cephalochordata has progressive metamorphosis)

2. In case of mice coat colour, two genes are responsible for colour of the hair. Gene 'A' is responsible for distribution of pigments on shaft of hair. Wild type allele of 'A' produces a yellow band on dark hair shaft (agouti), whereas recessive allele produces no yellow band. There is another allele of A, known as A^Y, which is embryonic lethal in homozygous condition only. In an experiment, two yellow mice were crossed to obtain a progeny of 6 pups. What would be the most probable number of agouti mice among them?

(a) 0 (b) 2 (c) 4 (d) None of the above

Ans. **(b)**

Sol.



Probability of agouti is 1/3

out of 3 → 1 agouti

so out of 6 → 2 agouti

3. A stain was developed by a group of scientists to stain a particular cell organelle. The stain was tested on various tissues derived from an autopsy sample from a mammal. The organelles were counted. The results showed maximum number of the organelles in cells of brain, lesser in cells of heart, least in mature sperms and absent in erythrocytes. Identify the organelles from following options.

(a) Nissl bodies (b) Mitochondria (c) Golgi bodies (d) Endoplasmic reticulum

Ans. (b)

Sol. The organelle which is obtained after stain is mitochondria which is abudantly present in brain cell lesser in heart, least in mature sperms and completely absent in erythrocytes.

4. Pinus sylvestris grows at low temperatures in Russia. The plant survives under such freezing conditions due to the presence of:

(a) Saturated lipids in plasma membrane (b) Glycoproteins in plasma membrane

(c) Glycolipids in plasma membrane (d) Polyunsaturated lipids in plasma membrane*

Ans. (d)

Sol. The plants which grows at low temperature basicaly have unsaturated lipid in their plasma membrane. For maintaining flexibility of plasma membrane.

5. In an experimental setup, certain pathogen caused a disease in primates with nasal congestion, sore throat and fever being the common symptoms. The scientists injected an extract from blue-green mold as the first line of action. However, the symptoms did not subside. The possible causative agents of the disease were listed out as follows.

i. a virus ii. a fungus

iii. a conjugation deficient bacterium iv. a tapeworm

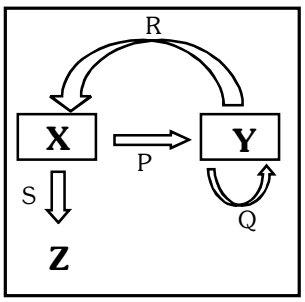
Choose the correct option from the following that indicate the pathogen.

(a) i, ii (b) i, iii (c) ii, iv (d) iii only

Ans. (b)

Sol. Nasal congestion also called sinusitis has sore throat and fever which is caused by virus, conjugation deficient bacterium has no role in it. (Conjugation in bacteria is a process in which plasmid are transferred by themselves alone or along with other DNA element from one cell to another cell through conjugation tube)

6. A process is represented in the adjacent figure. The arrows indicate the flow of a biochemical reaction. The arrowhead points to the product, while the base of the arrow indicates the template biomolecule. What do P, Q, R, and S represent?



(a) P : Replication, Q : Translation, R : Transcription, S : Reverse Transcription

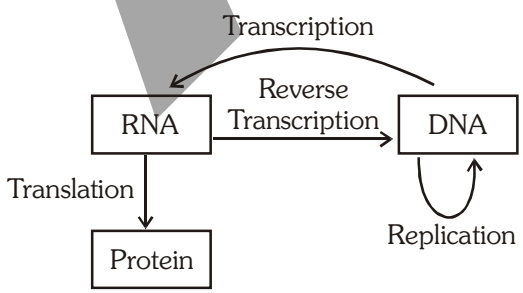
(b) P : Transcription, Q : Replication, R: Reverse Transcription, S : Translation

(c) P : Reverse Transcription, Q : Replication, R : Translation, S : Transcription

(d) P : Reverse Transcription, Q : Replication, R : Transcription, S : Translation

Ans. (d)

Sol.



7. The whooping cranes were on the verge of extinction with only 21 individuals in wild in 1941. After conservation measures, the cranes are now included in the endangered category by IUCN. The highlight of the conservation efforts is the reintroduction of the whooping cranes in wild. This was possible due to raising of the young cranes in absence of their parents by biologists dressed in crane costumes. Aircraft Guided bird migration technique was used for teaching the captive-bred cranes to follow the scientists to learn the migratory route. What type of animal behaviour might be responsible for these captive-bred cranes to follow the crane costume dressed scientists?

(a) Cognitive learning (b) Habituation (c) Operant conditioning (d) Genetic Imprinting

Ans. (d)

Sol. Imprinting : A type of behavior that includes both learned and innate components is called imprinting. it is become an important component of efforts to save endangered species.

8. A 4 μm long bacterial cell was magnified and drawn to a dimension of 6 cm. How many times has it been magnified?

(a) 1.5×10^3 (b) 15×10^4 (c) 1.5×10^4 (d) 1.5

Ans. (c)

Sol. 4μm long bacterial cell $= 4 \times 10^{-6}$ m

magnification to a dimension of 6 cm $= 6 \times 10^{-2}$ m

$$\text{magnification} = \frac{\text{magnifying dimension}}{\text{actual dimention}} = \frac{6\times10^{-2}}{4\times10^{-6}} = 1.5 \times 10^4$$

9. In the baking industry, when the dough is prepared, various ingredients are mixed together with the flour. At one instance, the dough was fermented, but failed to rise sufficiently during the baking process. Choose the correct cause(s) from following possibilities.

i. The salt was mixed before the fermentation process was completed

ii. The sugar was added in excess

iii. Yeast granules were not activated prior to mixing with the flour.

(a) i, iii (b) iii only (c) i, ii, iii (d) i, ii

Ans. (a)

Sol. In process of fermentation, if the dough failed to rise sufficiently during backing process, this might be due to inactivation of yeast. Salt addition to the better can be done immidiately after fermenting.

10. Given below are four statements.

1. Prokaryotic cells are unicellular while eukaryotes are multicellular.

II. Histones are present in eukaryotes and absent in prokaryotes.

III. The nucleoid contains the genetic material in prokaryotes and eukaryotes.

IV. Prokaryotic flagellum is composed of flagellin while eukaryotic flagellum is composed of tubulin.

Identify which amongst these are false.

(a) I and II (b) III and IV (c) II and III (d) I and III

Ans. (d)

Sol. because prokaryotic cell are unicellular but eukaryotic are not only multicellular, it may be unicellular also. Only prokaryotic genetic material is called nucleoid.

11. The students of a college were working on regeneration using Planaria (Platyhelminthes) and Asterias (Echinodermata). Planaria was cut in three pieces, namely, a piece with head, with tail and the middle piece. Asterias (bearing five arms) was cut in such a way that after separation, six pieces were obtained, namely, an arm with a portion of the central disc, four pieces cut from tips of each of the remaining arms and the remaining body. The animals were allowed to regenerate completely.

How many Planaria and Asterias respectively will be obtained after the completion of regeneration in both?

(a) 1, 1 (b)3,2 (c) 3, 6 (d)l,2

Ans. (b)

Sol. Planaria can be cut into pieces, and each piece can regenerate into a complete organism over the course of a few weeks.

Most species of sea stars must split part of their central disc along with a limb for regeneration to occur. It is very unlikely that a severed limb will be able to regenerate into a full-grown starfish unless it is already attached to at least a portion of the central disc.

12. Fecundity in animal world is the maximum possible ability of an individual to produce offsprings during its entire lifetime. Following factors were checked for their effect on fecundity of different animal models.

i. Availability of food during breeding season

ii. Mode of fertilization

iii. Population density

Which of these factor(s) can regulate fecundity?

(a) i, ii (b) ii, iii (c) i, ii, iii (d) None of the above

Ans. (c)

Sol. Fecundity means maximum possible ability of an individual to produce offsprings during life time.

Availability of food, mode of fertilization and population density regulate the fecundity.

13. In case of peppered moths, pale and dark moths are observed. Pale variety is known to be the wild type variety. During industrial revolution, industrial melanism led to prevalence of dark variety around the cities and pale variety continued to be in majority in areas away from the industries.

After enforcement of regulations for controlling pollution, reappearance of pale moths in majority was observed around cities again. Driving force(s) for these adaptive changes is/are:

i. Increased pollution around industries

ii. A stable transposition of a gene in moths

iii. Limitations of the vision of birds to differentiate dark moths on darkened barks and pale moths in presence of lichens

iv. Ability of lichens to grow on barks in less polluted areas only.

(a) i, iv (b) i, iii, iv (c) i, ii (d) i, ii, iii and iv

Ans. (b)

Sol. It is the example of progressive/ directional Natural selection, in which only one extremity is selected, when environment condtion change it is shited towards another extremity, pale moth is selected after controlling of pollutions and its reappearance observed in cities again.

14. Four different human body fluid samples were subjected to quantification of hydrogen ion concentration. mEq/L is the unit of measurement for hydrogen ion concentration. The results of the experiment were as follows:

Sample A: 1.6×10^{2} units Sample B: 4.5×10^{-5} units

Sample C: 1×10^{-3} units Sample D: 3×10^{-2} units

Identify the samples in sequence from A to D.

(a) Gastric HCl, Venous blood. Intracellular Fluid, Urine

(b) Venous blood, Intracellular Fluid, Gastric HCl, Urine

(c) Urine, Gastric HCl, Venous blood, Intracellular Fluid

(d) Intracellular Fluid, Urine, Gastric HCl, Venous blood

Ans. (a)

Sol. $PH = -\log_{10}[H^+]$

According to this formula correct answer is (a)

15. Any damage or injury to a particular area causes nociceptors to release some chemicals, which carry the signal to the higher centres in the nervous system for the processing and a subsequent action. However, there is a difference in the way in which the stimulus is received which is related to the acuity of the detection. Fingertips are more sensitive as compared to the forearm. Following reasons for the observed phenomenon were suggested.

i. The receptive fields in the fingertip are smaller

ii. The number of nociceptors per receptive field in the forearm is lesser

iii. The amount of prostaglandins released by the nociceptors per receptive field is more in fingertips

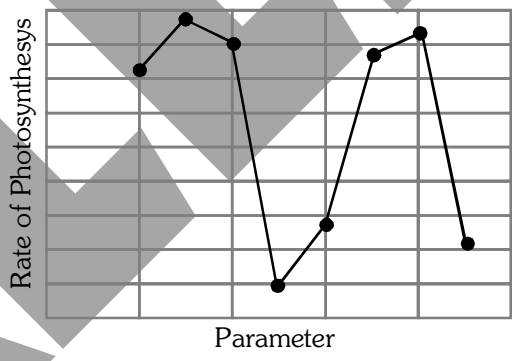
The most probable reason(s) for this may be:

(a) i (b) i, iii (c) ii, iii (d) i, ii, iii

Ans. (d)

Sol. Due to any external stimulus (injury) nociceptors release prostaglandins which is more in fingertips per receptive field that's why fingertips are more sensitive as compared to forearm

16. Rate of photosynthesis in hydrophytes depends on various parameters. The adjacent graph shows the effect of one parameter (while keeping all the others constant) on the rate of photosynthesis. Rate of photosynthesis is plotted on Y axis. Identify the parameter which is plotted along X axis:



(a) light intensity (b) wavelength (c) temperature (d) CO_2 concentration

Ans. (b)

Sol. According to figure wavelength of light affect the photosynthesis.

17. An organism has 27 pairs of homologous chromosomes. In each daughter cell after completion of mitosis and in each gamete after completion of meiosis II, _____ and _____ chromosomes would be present respectively.

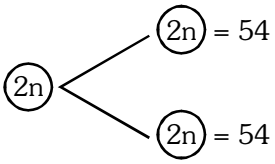
(a) 27 and 27 (b) 54 and 27 (c) 108 and 54 (d) 54 and 108

Ans. (b)

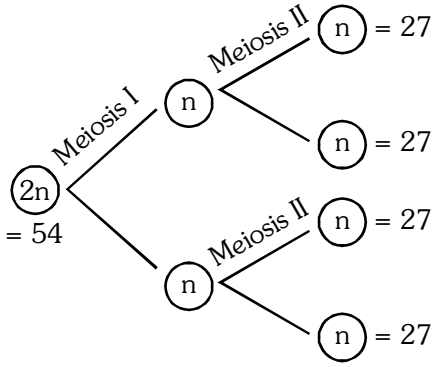
Sol. A cell have 27 pairs of chromosome.

2n = 54 chromosomes.

In mitosis :



In meiosis II :



So ans is 54 and 27 respectively.

18. Gymnosperms are called naked seed bearing plants' because they lack:

(a) Male gamete (b) Ovule (c) Ovary (d) Seeds

Ans. **(c)**

Sol. Gymnosperm are called 'naked seed bearing plant' because they lacks ovary.

19. Rahul sprayed a chemical 'X' on a plant with rosette habit. After few days, he found the internodal distances to have increased suddenly. The chemical 'X' might be:

(a) Ethylene (b) Abscisic acid (c) Auxin (d) Gibberellic acid

Ans. **(d)**

Sol. Gibberellic acid initiate the internodal distance.

20. On a study tour, plants with leathery leaves with thick cuticle, vivipary, salt glands, apogeotropic roots, and stomata limited to abaxial surface were observed. The plants might be:

(a) Bromeliads (b) Cycads (c) Mangroves (d) None of the above

Ans. **(c)**

Sol. Mangroves plant have the properties of thick cuticle, vivipary, apogeotropic root and so on.

21. An element Y is a white translucent solid at room temperature and exhibits various allotropic forms. Some compounds of element Y find application in agriculatural industry. Y forms two solid oxides which dissolve in water to form comparatively weak acids. The element Y is :

(a) Sulphur (b) Nitrogen (c) Phosphrous (d) Carbon

Ans. **(c)**

Sol. $_{15}P$

Allotropic forms white P, red P, black P, P_2

fertilizers : Ammonium phosphate

$$P_4O_6 + 6H_2O \longrightarrow 4H_3PO_3$$

$$P_4O_{10} + 6H_2O \longrightarrow 4H_3PO_4$$

22. How many sigma bonds are present between any two carbon atoms in fullerenes?

(a) 1 (b) 2 (c) 3 (d) 4

Ans. **(a)**

Sol. Number of sigma bonds between any two atoms will be equal to one.

23. Four gram of mixture of calcium carbonte and sand is treated with excess of HCl and 0.880 g of carbon-di-oxide is produced. What is the percentage of calcium carbonate in original mixture?

(a) 40% (b) 50% (c) 55% (d) 45%

Ans. **(b)**

Sol. $CaCO_3 + 2HCl \longrightarrow CaCl_2 + H_2O + CO_2$

$CO_2 = \text{moles} = \frac{0.88}{44} = 0.02 \text{ moles}$

$CaCO_3$

amount $= 0.02 \times 100 = 2g$

% of $CaCO_3$ in the mixture is, $\frac{2}{4} \times 100 = 50\%$

24. A student was studying reactions of metals with dilute NaOH at room temperature. The student took dilute NaOH in four different test tubes and added Copper powder to test tube A, Zinc dust to test tube B, Aluminium powder to test tube C and Iron powder to test tube D and observed effervescence in.

(a) Test tubes A & B (b) Test tubes B & C (c) Test tubes C & D (d) Test tubes A & D

Ans. **(b)**

Sol. For the given infomation,

Zn and Al can only react will aqeous NaOH because of their amphoteric nature.

$Zn + 2NaOH \longrightarrow Na_2ZnO_2 + H_2\uparrow$

$Al + NaOH + H_2O \longrightarrow NaAlO_2 + 1.5\, H_2\uparrow$

25. A magician performed following act: He dipped Rs. 50 note in a 50% solution of alcohol in water and held it on the burning flame, but the note did not burn. The reason behind this is-

(a) The alcohol kept on dousing the fire

(b) Air required for burning was not available

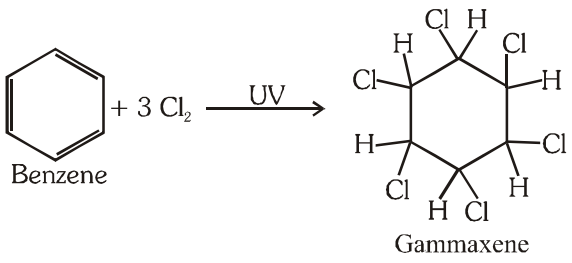
(c) The Rs 50 note failed to reach ignition temperature

(d) The Rs. 50 note is fire proof

Ans. **(c)**

Sol. Conceptual question

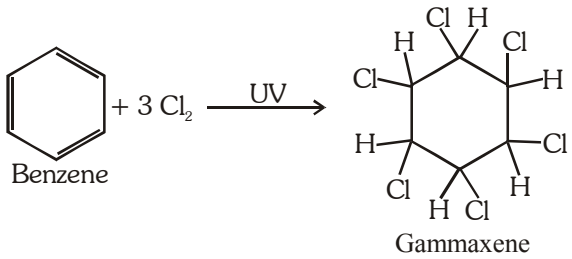
26. Gammaxene insecticide powder is prepared by the reaction given in the adjacent box. If 78 g of benzene when reacted with 106.5 g of chlorine, how much Gammaxene would be formed?



(a) 140 g (b) 154.5 g (c) 145.5 g (d) 160 g

Ans. **(c)**

Sol.



78g 106.5 g (L.R.)

Amount of gammaxene formed is, $\frac{291}{3\times 71} \times 106.5 = 145.5\,g$

27. Arrange following solutions in increasing hydronium ion concentration. The solutions are :

(P) 0.1 M HCl

(Q) 0.1 M H_2SO_4

(R) 0.001 M NH_4OH

(S) 0.001 M $Ca(OH)_2$

(a) P > Q > R > S (b) Q > P > S > R (c) S > R > Q > P (d) S > R > P > Q

Ans. (Bonus)

Sol. Correct option should be Q > P > R > S

28. A zinc rod was dipped in 100 cm3 of 1M copper chloride solution. After certain time the molarity of Cu^{2+} ions in the solution was found to be 0.8 M. If the weight of zinc rod, 20 g, then the molarity of chloride ions is ____.

(a) 2M (b)1.5M (c) 1 M (d) 0.5 M

Ans. (a)

Sol. $CuCl_2\,(aq) \longrightarrow Cu^{2+}(aq) + 2Cl^-\,(aq)$

1M 1M 2M

the actual reaction is, $Cu^{+2} + Zn \longrightarrow Zn^{+2} + Cu$.

(There will be no change in concentration of Cl^- ions)

29. Which of the following polymeric material will be ideal for remolding?

(a) Polythene and Melamine (b) Polyvinyl chloride and Polythene

(c) Melamine and Bakelite (d) Bakelite and Polyvinyl chloride

Ans. (b)

Sol. Polyvinyl chloride and polyethene are thermoplastics.

30. When four dilute solutions of (I) vinegar, (II) common salt, (III) caustic soda and (IV) baking soda are tested with universal indicator which will be the correct observation

(a) I- Green, II - Violet, III - Blue, IV - Red

(b) I - Green, II - Blue, III -Violet IV- Red

(c) I - Red, H - Green, III - Violet, IV - Blue

(d) I-Red, II- Violet, III - Green, IV - Blue

Ans. (c)

Sol. Vinegar – acidic, NaCl – Netural, NaOH - Strong Base, Baking Soda-weak base.

31. Substance X is white crystalline solid which melts after 10 seconds on burner flame. It is soluble in water and insoluble in CCl_4 It is a poor conductor of electricity in molten state as well as in the form of aqueous solution, hence we conclude that substance X is

(a) an ionic compound (b) a non polar covalent compound

(c) a polar covalent compound (d) a pure element

Ans. (c)

Sol. Informative question.

32. In a beaker 50 ml of a normal HC1 solution was taken and NH_3 gas was passed through it for some time. The contents of the beaker were then titrated, which required 60 ml of semi normal NaOH solution. How much ammonia was passed through the beaker?

(a) 0.85 g (b) 0.34 g (c)0.51 g (d) 0.4 g

Ans. (b)

Sol. HCl taken = 1×50 = 50 m.xmol

NaOH consumed = $\frac{1}{2} \times 60 = 30$ m.mol

NH_3 Reacted = 20 m.mol.

Amount of NH_3 passed is, $\frac{20 \times 17}{1000}$

W = 0.34 g

33. Which is the correct order of metals with reference to their melting point in increasing order?

(a) Hg, Ga, Li, Ca (b) Ca, Li, Ga, Hg (c) Hg, Li, Ga, Ca (d) Hg, Ga, Ca, Li

Ans. (a)

Sol.

Element	Melting point
Hg	–38.83 °C
Ga	29.76 °C
Li	180.5 °C
Ca	842 °C

34. Sodium tungstate has formula Na_2WO_4, lead phosphate has formula $Pb_3(PO_4)_2$, formula for lead tungstate should be:

(a) $PbWO_4$ (b) $Pb_2(WO_4)_3$ (c) $Pb_3(WO_4)_2$ (d) $Pb_3(WO_4)_4$

Ans. (a)

Sol. $Na_2WO_4 \longrightarrow WO_4^{2-}, Na^+$

$Pb_2(PO_4)_2 \longrightarrow Pb^{2+}, PO_4^{3-}$

Hence, the compound will be Pb (WO_4)

35. What is the ratio of reducing agent to oxidizing agent, if the following reaction is correctly balanced?

$NH_3 + O_2 \longrightarrow NO + H_2O$

(a) 4 : 5 (b) 5 : 4 (c) 5 : 3 (d) 3 : 5

Ans. (a)

Sol. Balanced Equation is

$4\,NH_3 + 5O_2 \longrightarrow 4NO + 6H_2O$

Ratio of Reduing agent to Oxidizing agent is 4 : 5.

36. Which of the following is iso-structural with CO_2?

(a) NO_2 (b) N_2O_4 (c) NO (d) N_2O

Ans. (d)

Sol. O = C = O

$$:\overset{\ominus}{N} = \overset{+}{N} = O:$$

37. In one litre of pure water, 44.4 g of calcium chloride is dissolved. The number of ions in one mL of the resultant solution is :

(a) 7.23×10^{23} (b) 7.23×10^{20} (c) 4.82×10^{23} (d) 4.82×10^{20}

Ans. (b)

Sol. Molarity $= \frac{44.4}{111} \times \frac{1}{1} = 0.4$ molar

$CaCl_2(aq) \longrightarrow Ca^{2+}(aq) + 2Cl^-(aq)$

0.4 m mol 0.4 m mol 2×0.4 m mol

Ions $= 1.2 \times 10^{-3} \times 6.023 \times 10^{23} = 7.23 \times 10^{20}$

38. Which of the following species is / are isoelectronic with Neon?

(i) N^{3-} (ii) Mg^{2+} (iii) K^+ (iv) Ca^{2+}

(a) only (iv) (b) only (ii) (c) both (i) and (ii) (d) both (i) and (iii)

Ans. (c)

Sol.

Species	No. of electrons
Ne	$10e^-$
N^{3-}	$10e^-$
Mg^{2+}	$10e^-$

39. Which of the following gases will have equal volume at STP, if the weight of gases is 14.0 g?

(i) N_2O (ii) NO_2 (iii) N_2 (iv) CO

(a) (i) & (ii) (b) (ii) & (iii) (c) (i) & (iii) (d) (iii) & (iv)

Ans. (d)

Sol. For the given amount of gas (14g) at S.T.P. as volume is same, molecular mass should be same.

Molecular mass : $N_2 = 28$; $CO = 28$

40. Which of the following are not ionic?

(i) $AlCl_3$ (ii) $CaCl_2$ (iii) $MgCl_2$ (iv) LiCl

(a) (i) and (iv) (b) (i) and (ii) (c) (ii) and (iii) (d) (iii) and (iv)

Ans. (a, d)

Sol. $AlCl_3$, $MgCl_2$, LiCl are covalent in nature.

41. Apples dropping from apple trees were observed by many many people before Newton. But why they fall, was explained by Isaac Newton postulating the law of universal gravitation. Which of the following was explained by Isaac Newton postulating statements best describes the situation.

(a) The force of gravity acts only on the apple

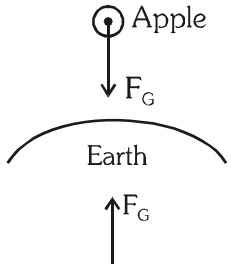
(b) The apple is attracted towards the surface of the earth

(c) Both earth and apple experience the same force of attraction towards each other

(d) Apple falls due to earth's gravity and hence only (a) is true and (c) is absurd

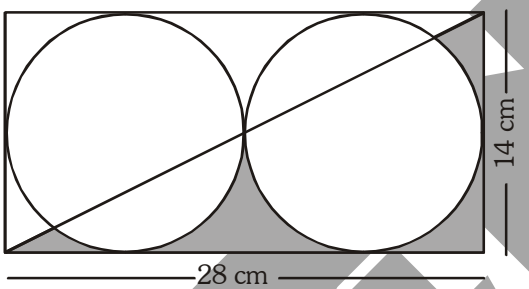
Ans. **(c)**

Sol. Both earth and apple will attract each other with force



$$F_G = \frac{G m_1 m_2}{r^2}$$

42. A rectangular metal plate, shown in the adjacent figure has a charge of 420 μC assumed to be uniformly distributed over it. Then how much is the charge over the shaded area? No part of metal plate is cut. (Circles and the diagonal are shown for clarity only. $\pi = 22/7$)



(a) 45 μC (b) 450 μC (c) 15 μC (d) 150 μC

Ans. **(a)**

Sol. Area of shaded region $= \frac{14\times 28}{2} - \pi r^2 = \frac{14\times 28}{2} - \frac{22}{7}\times 7^2 = 42\ cm^2$

$\therefore$ charge of shaded area $= 42 \times \frac{420}{14\times 28}\ \mu C = 45\ \mu C$

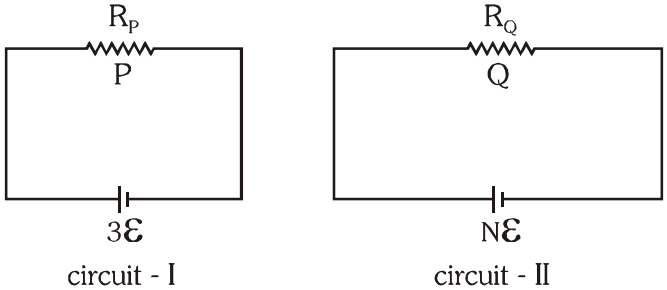
43. A piece of wire P and three identical cells are connected in series. An amount of heat is generated in a certain time interval in the wire due to passage of current. Now the circuit is modified by replacing P with another wire Q and N identical cells, all connected in series. Q is four times longer in length than P. The wire P and Q are of same material and have the same diameter. If the heat generated in second situation is also same as before in the same time interval, then find N.

(a) 4 (b) 6 (c) 16 (d) 36

Ans. **(b)**

Sol. Given ;

Power in circuit - I = Power in circuit - II



circuit - I circuit - II

$\Rightarrow \quad \frac{9\mathcal{E}^2}{R_P} = \frac{N^2\mathcal{E}^2}{R_Q} \qquad \Rightarrow \quad \frac{9}{R_P} = \frac{N^2}{4R_P} \qquad \left[\because \ell_Q = 4\ell_P \Rightarrow R_Q = 4R_P\right]$

$\Rightarrow \quad N^2 = 36 \qquad \Rightarrow \quad \boxed{N = 6}$

44. A piece of ice is floating in water at 4° C in a beaker. When the ice melts completely, the water level in the beaker will

(a) rise (b) fall (c) remains unchanged (d) unpredictable

Ans. (a)

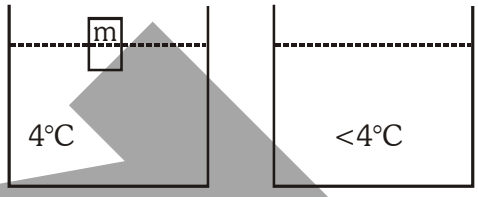
Sol. Since ice at 0° melts completely in water at 4°C.

Equilibrium temperature will be slightly less than 4°C.

Density of water is maximum at 4°C.

$\therefore$ Hence volume of water increases as ice melts completely.

So water level will rise.



Mathematically

$$V_1 = \frac{m}{\rho_w} \qquad \ldots. (1)$$

$$V_2 = \frac{m}{\rho'_w} \qquad \ldots. (2)$$

$\because \quad \rho'_w < \rho_w$

$\therefore \quad V_2 > V_1$

Hence water level rises.

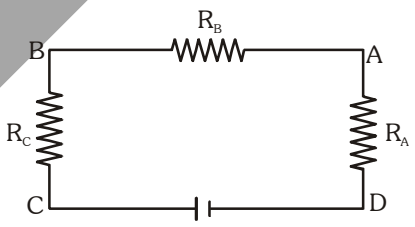
45. In the adjacent circuit, the voltages across AD, BD and CD are 2 V, 6 V and 8 V respectively. If resistance $R_A = 1$ kΩ, then the values of resistances R_B and R_C are _____ and _____ respectively.

(a) 4 kΩ and 6 kΩ

(b) 2 kΩ and 1 kΩ

(c) 1 kΩ and 2 kΩ

(d) data insufficient as battery voltage is not given



Ans. (b)

Sol. Given; $V_{AD} = 2V$

$\Rightarrow \quad i^2 R_A = 2 \qquad \ldots. (1)$

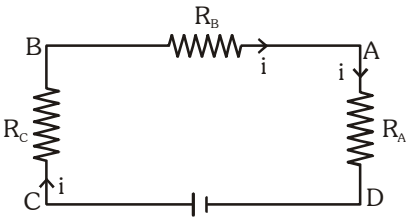
Also, $V_{BD} = 6V$

$\Rightarrow \quad i^2(R_A + R_B) = 6 \qquad \ldots. (2)$

and, $V_{CD} = 8$

$\Rightarrow \quad i^2(R_A + R_B + R_C) = 8 \qquad \ldots. (3)$

Dividing (2) by (1)

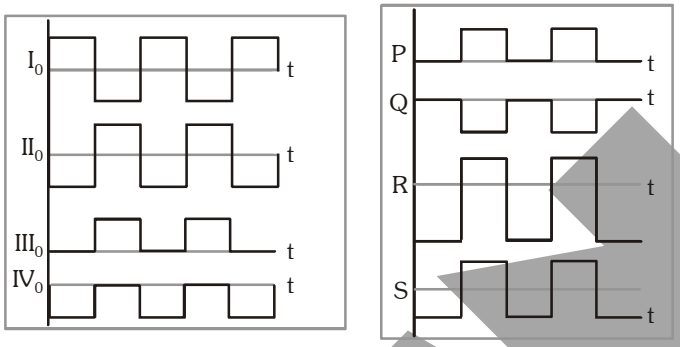


$$\frac{R_A + R_B}{R_A} = 3 \qquad \Rightarrow \quad R_B = 2R_A \Rightarrow \boxed{R_B = 2k\Omega}$$

Dividing (3) by (1) we get $\frac{R_A + R_B + R_C}{R_A} = 4$

$\Rightarrow 3R_A + R_C = 4R_A \quad \Rightarrow R_C = R_A \quad \Rightarrow \boxed{R_C = 1k\Omega}$

46. Some waveforms among I, II, III and IV superpose (add graphically) to produce the waveforms P, Q, R and S. Among the following, match the pairs that give the correct combinations:



Resultant	Superposition of
P	(K) III and IV
Q	(L) II and IV
R	(M) I, II and III
S	(N) I and IV
	(O) II and III

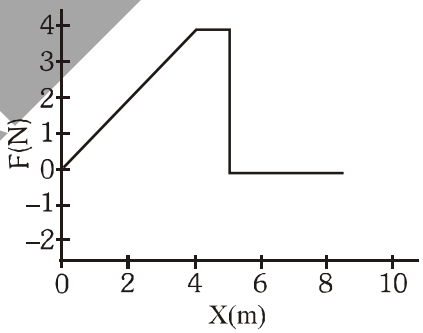
(a) P ↔ O, Q ↔ N, R ↔ L, S ↔ M (b) P ↔ M, Q ↔ N, R ↔ L, S ↔ K

(c) P ↔ M, Q ↔ N, R ↔ K, S ↔ L (d) P ↔ O, Q ↔ M, R ↔ L, S ↔ K

Ans. **(b)**

Sol. By observation option (b) should be correct.

47. Refer to the adjacent figure. A variable force F is applied to a body of mass 6 kg at rest. The body moves along x - axis as shown. The speed of the body at x = 5 m and x = 6 m is ____ and ____ respectively.



(a) 0 m/s, 0 m/s (b) 0 m/s, 2 m/s (c) 2 m/s, 2 m/s (d) 2 m/s, 4 m/s

Ans. **(c)**

Sol. area under curve given us the work done.

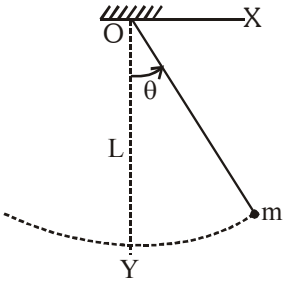
(area under curve from t = 0 to t = 5) = (area under curve from t = 0 to t = 6)

$\therefore$ area $= \frac{1}{2} \times 4 \times 4 + 1 \times 4 = 12$ sq. units

$\therefore$ work done $= \Delta K$

$\Rightarrow\ 12 = \frac{1}{2} \times 6 \times v^2$ $\Rightarrow\ \boxed{v = 2m/s}$ at both x = 5 m & x = 6 m

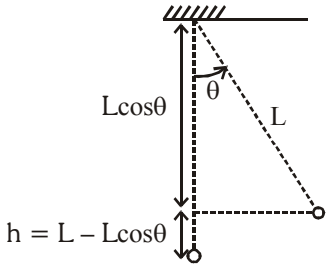
48. A rigid body of mass m is suspended from point O using an inextensible string of length L When it is displaced through an angle θ, what is the change in the potential energy of the mass? (Refer adjacent figure.)



(a) mg L (1 – cosθ) (b) mg L (cos θ – 1) (c) mg L cos θ (d) mg L (1 – sin θ)

Ans. (a)

Sol.



$\Delta U = mgh = mgL(1 - \cos\theta)$

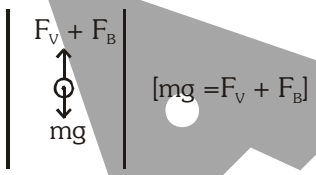
49. Consider the motion of a small spherical steel body of mass m, falling freely through a long column of a fluid that opposes its motion with a force proportional to its speed. Initially the body moves down fast, but after some time attains a constant velocity known as terminal velocity. If weight mg, opposing force (F_v) and buoyant force (F_b) act on the body, then the correct equation relating these forces, after the terminal velocity is reached, is:

(a) $mg + F_v = F_b$ (b) $mg = F_v - F_b$ (c) $mg = F_v + F_b$ (d) None of these

Ans. (c)

Sol. To acquire terminal velocity

Net force on ball should be zero so



50. At any instant of time, the total energy (E) of a simple pendulum is equal to the sum of its kinetic energy $\left(\frac{1}{2}mv^2\right)$ and potential energy $\left(\frac{1}{2}kx^2\right)$, where, m is the mass, v is the velocity, x is the displacement of the bob and k is a constant for the pendulum. The amplitude of oscillation of the pendulum is 10 cm and its total energy is 4 mJ. Find k

(a) 1.8 Nm^{-1} (b) 0.8 Nm^{-1} (c) 0.5 Nm^{-1} (d) data insufficient

Ans. (b)

Sol. $E = \frac{1}{2}kA^2 = 4\times10^{-3}J$

$$\frac{1}{2}\times k\times\left(10^{-1}\right)^2 = 4\times10^{-3}$$

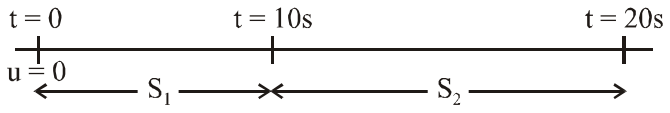
$$k = 8\times10^{-1} = 0.8\,Nm^{-1}$$

51. A particle experiences constant acceleration for 20 s after starting from rest. If it travels a distance S_1 in the first 10 s and distance S_2 in the next 10 s, the relation between S_1 and S_2 is:

(a) $S_2 = 3S_1$ (b) $S_1 = 3S_2$ (c) $S_2 = 2S_1$ (d) $S_1 = 10S_2$

Ans. (a)

Sol.



$$S_1 = \frac{1}{2}a\,(10)^2 = \frac{1}{2}a\,100 = 50a$$

$$S_2 = [\frac{1}{2}a\ (20)^2] - [50a] = 150a$$

$$\boxed{S_2 = 3S_1}$$

52. When a charged particle with charge q and mass m enters uniform magnetic field B with velocity v at right angles to B the force on the moving particle is given by qvB. This force acts as the centripetal force making the charged particle go in a uniform circular motion with radius $r = \frac{mv}{Bq}$. Now if a hydrogen ion and a deuterium ion enter the magnetic field with velocities in the ratio 2:1 respectively, then the ratio of their radii will be _____.

(a) 1 : 2 (b) 2 : 1 (c) 1 : 4 (d) 1 : 1

Ans. (d)

Sol. $r = \frac{mv}{Bq}$

For Hydrogen ion $r_H = \frac{m_H v_H}{Bq_H}$ .....(1)

For Deuterium ion $r_d = \frac{m_d v_d}{Bq_d}$ .....(2)

$$\frac{(1)}{(2)} \Rightarrow \frac{r_H}{r_d} = \frac{m_H}{m_d}\frac{v_H}{v_d}\times\frac{q_d}{q_H} = \frac{1}{2}\times\frac{2}{1}\times\frac{1}{1} = \frac{1}{1}$$

53. A physics teacher and his family are travelling in a car on a highway during a severe lightning storm. Choose the correct option:

(a) Safest place will be inside the car as the charges due to lightning tend to remain on the metal sheet / skin of the vehicle if struck by lightning.

(b) It's too dangerous to be inside the car. As the car has a metal body the charges tend to accumulate on the surface and will generate a strong electric field inside the car.

(c) Safest place is under a tree. It's better to get drenched under a tree as the wet tree will provide a path to the charges for earthing.

(d) It is safer to exit the car and stand on open ground.

Ans. (a)

Sol. By the concept of electrostatic shielding.

54. The radius of curvature of a convex mirror is 'x'. The distance of an object from focus of this mirror is 'y'. Then what is the distance of image from the focus?

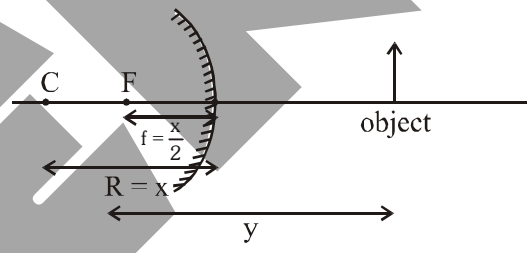
(a) $y^2/4x$ (b) x^2/y (c) $x^2/4y$ (d) $4y^2/x$

Ans. (c)

Sol. By Newton's formula

$f^2 = x_1x_2$ where $x_1 \rightarrow$ Object distance from focus

$x_2 \rightarrow$ Image distance from focus.

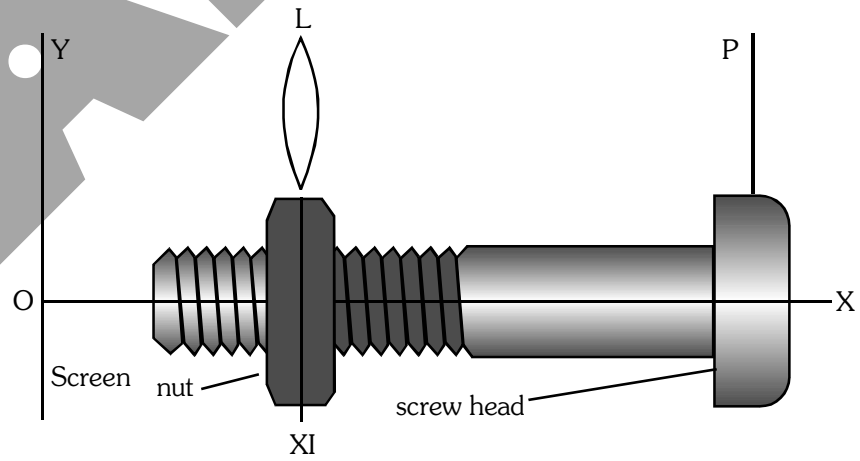


$$\left(\frac{x}{2}\right)^2 = yx_2$$

$$\frac{x^2}{4} = yx_2$$

$$x_2 = \frac{x^2}{4y}$$

55. In a screw-nut assembly (shown below) the nut is held fixed in its position and the screw is allowed to roatate inside it A convex lens (L) of focal length 6.0 cm is fixed on the nut. An object pin (P) is attached to the screw head. The image of the object is observed on a screen Y. When the screw head is rotated through one rotation, the linear distance moved by the screw tip is 1.0 mm. The observations are made only when the image is obtained in the same orientation on the screen. At a certain position of P, the image formed is three times magnified as that of the pin height. Through how many turns should the screw head be rotated so that the image is two times magnified?



(a) 8 (b) 10 (c) 12 (d) 14

Ans. (b)

Sol. For convex lense

$$m = \frac{f}{f+u}$$

For I[st] image $\quad (-3) = \frac{6}{6+u_1}$

$$-18 - 3u_1 = 6$$
$$3u_1 = -24$$
$$u_1 = -8 \text{ c.m.}$$

For II[nd] image $\quad (-2) = \frac{6}{6+u_2}$

$$\Rightarrow u_2 = -9 \text{ cm}$$

$|\Delta u| = 1$ c.m. $= 10$ mm

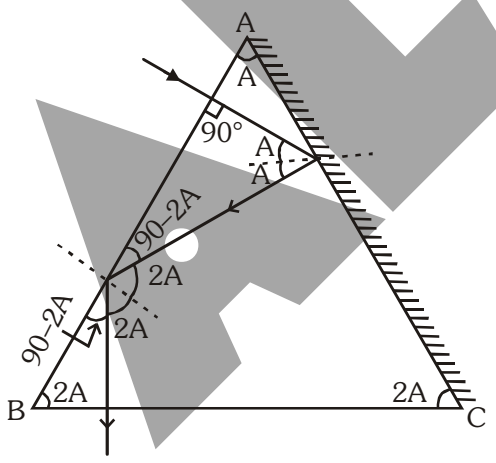
So no. of roatations $= \frac{10\text{mm}}{1\text{mm}} = 10$ rotations

56. The triangular face of a crown glass prism ABC is isosceles. Length AB = length AC and the rectangular face with edge AC is silvered. A ray of light is incident normally on rectangular face with edge AB. It undergoes reflections at AC and AB internally and it emerges normally through the rectangular base with edge BC. Then angle BAC of the prism is _____.

(a) 24° (b) 30° (c) 36° (d) 42°

Ans. (c)

Sol.



since $\angle B = \angle C$

and by geometry $\angle B = 2\angle A$

so $\angle A + \angle B + \angle C = 180°$

$A + 2A + 2A = 180°$

$5A = 180°$

$$A = \frac{180°}{5} = 36°$$

57. A sound wave is produced by a vibrating metallic string stretched between its ends. Four statements are given below. Some of them are correct.

(P) Sound wave is produced inside the string.

(Q) Sound wave in the string is transverse.

(R) Wavelength of the sound wave in surrounding air is equal to the wavelength of the transverse wave on the string.

(S) Loudness of sound is proportional to the square of the amplitude of the vibrating string.

Choose the correct option.

(a) P (b) R and S (c) P and Q (d) S

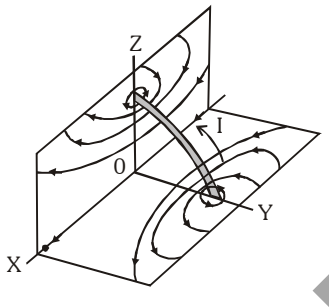
Ans. (d)

Sol. Since sound wave produced by the string will be longitudinal in nature.

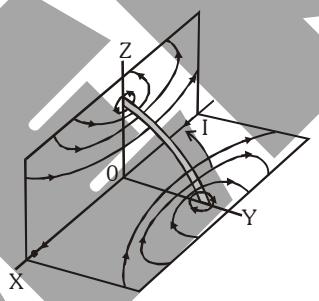
The loudness of sound depends on the amplitude of vibration of the vibrating object.

58. A conductor in the form of a circular loop is carrying current I. The direction of the current is as shown. Then which figure represents the correct direction of magnetic field lines on the surfaces of the planes XY and XZ. (Consider those surfaces of the XY and XZ planes which are seen in the figure.)

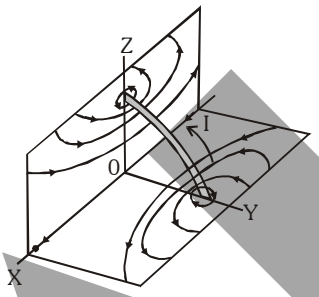
(a)



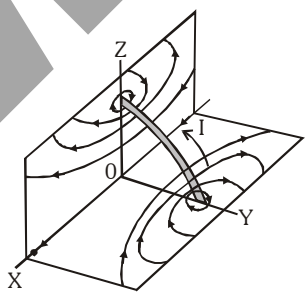
(b)



(c)



(d)



Ans. (a)

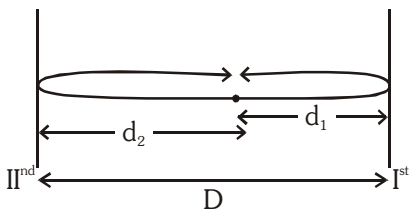
Sol. By right hand thumb rule option (a) is correct.

59. A school is located between two cliffs. When the metal bell is struck by school attendant, first echo is heard by him after 2.4 s and second echo follows after 2.0 s for him at the same position near the bell. If the velocity of sound in air is 340 ms^{-1} at the temperature of the surroundings, then the distance between the cliffs is approximately _____.

(a) 0.488 km (b) 0.751 km (c) 1.16 km (d) 1.41 km

Ans. (c)

Sol.



For echo from right sided cliff

$$t_1 = \frac{2d_1}{v} \qquad \Rightarrow d_1 = \frac{v \times t_1}{2} = \frac{340 \times 2.4}{2}$$

$$= 170 \times 2.4$$

$$= 408\text{m}$$

For echo from left cliff

$$t_2 = \frac{2d_2}{v} \qquad \Rightarrow d_2 = \frac{v \times t_2}{2} = \frac{340 \times (4.4)}{2}$$

$$= 170 \times 4.4$$

$$= 748 \text{ m}$$

$$D = d_1 + d_2 = 408 + 748 = 1156 \text{ m}$$

$$= 1.156 \text{ m}$$

$$= 1.16 \text{ k.m.}$$

60. A new linear scale of temperature measurement is to be designed. It is called a 'Z scale' on which the freezing and boiling points of water are 20 Z and 220 Z respectively. What will be the temperature shown on the 'Z scale' corresponding to a temperature of 20° C on the Celsius scale?

(a) 10 Z (b) 20 Z (c) 40 Z (d) 60 Z

Ans. (d)

Sol. $\frac{C-0}{100} = \frac{Z-20}{220-20}$

$$\frac{C}{100} = \frac{Z-20}{200}$$

$z = 20 + 2C = 20 + 2\times(20) = 60$

61. Let α and β the roots of $x^2 - 5x + 3 = 0$ with $\alpha > \beta$. If $a_n = \alpha^n - \beta^n$ for $n \geq 1$ the the value of $\frac{3a_6 + a_8}{a_7}$ is

(a) 2 (b) 3 (c) 4 (d) 5

Ans. (d)

Sol. $x^2 - 5x + 3 = 0$

$\Rightarrow x^2 + 3 = 5x$

then, $\alpha^2 + 3 = 5\alpha$

$\Rightarrow \alpha^8 + 3x^6 = 5\alpha^7$ .....(1)

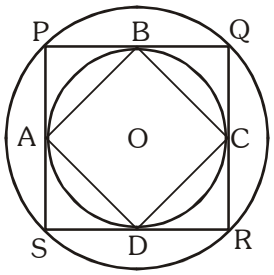
and $\beta^8 + 3\beta^6 = 5\beta^7$ .....(2)

Subtracting equation (2) from equation (1)

$3(\alpha^6 - \beta^6) + \alpha^8 - \beta^8 = 5(\alpha^7 - \beta^7)$

$$\Rightarrow \frac{3a_6 + a_8}{a_7} = 5$$

62. In the given figure, two concentric circles are shown with centre O. PQRS is a square inscribed in the outer circle. It also circumscribes the inner circle, touching it at points B, C, D and A. What is the ratio of the perimeter of the outer circle to that of quadrilateral ABCD?



(a) $\frac{\pi}{4}$ (b) $\frac{3\pi}{2}$ (c) $\frac{\pi}{2}$ (d) π

Ans. (c)

Sol. Let radius of inner circle be r, then AB = $\sqrt{2}$r

$\Rightarrow$ AC = 2r = PQ = QR

so, PR = $\sqrt{PQ^2 + QR^2} = 2\sqrt{PQ} = 2\sqrt{2}r$

$\Rightarrow$ OR = $\frac{PR}{2} = \sqrt{2}r$

Required Ratio is $2\pi(\sqrt{2}\,r) : 4\sqrt{2}\,r$

$\Rightarrow \pi : 2$

63. In rectangle ABCD, AB=5 and BC=3. Points F and G are on the line segment CD so that DF = 1 and GC=2. Lines AF and BG intersect at E. What is the area of ΔAEB?

(a) 10 sq. units (b) 15/2 sq. units (c) 25/2 sq. units (d) 20 sq. units

Ans. (c)

Sol. $\Delta EFG \sim \Delta EAB$

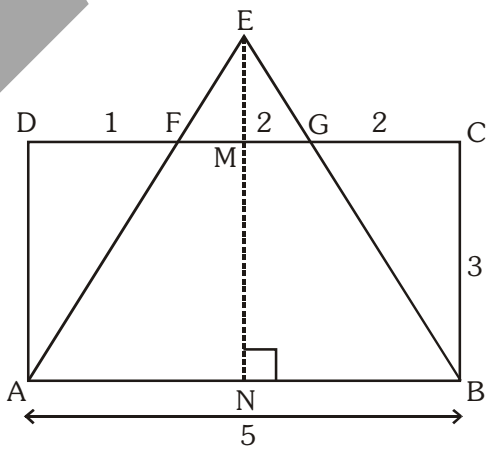
$\Rightarrow \frac{EM}{EN} = \frac{2}{5}$

$\Rightarrow \frac{EN - MN}{EN} = \frac{2}{5}$

$\Rightarrow 1 - \frac{3}{EN} = \frac{2}{5}$

$\Rightarrow EN = 5$

ar (ΔABE) = $\frac{1}{2} \times 5 \times 5 = \frac{25}{2}$ sq. units



64. The number of triples (x,y,z) such that any one of these numbers is added to the product of the other two, the result is 2, is

(a) 1 (b) 2 (c) 4 (d) infinitely many

Ans. (b)

Sol. $xy + z = 2$ .....(1)

$yz + x = 2$ .....(2)

$zx + y = 2$ .....(3)

equation (1) – equation (2)

$\Rightarrow$ $y(x-z)+z-x=0$

$\Rightarrow$ $(x-z)(y-1)=0$

Either $x=z$ or $y=1$

Case-1 $x=z$

$\Rightarrow$ $x(y+1)=2$ .....(4) (from equation 1)

$\Rightarrow$ $x^2+y=2 \Rightarrow x^2=2-y$ .....(5) (from equation 3)

and equation (4)/(5) $\Rightarrow \dfrac{x^2(y+1)^2}{x^2}=\dfrac{4}{2-y}$

$\Rightarrow (y^2+2y+1)(z-y)=y$

$y^2-3y+2=0$

$\Rightarrow$ $(y-1)^2(y+2)=0$

When $y=1$ we get $x=z=1$

and when $y=-2$ we get $x=z=-2$

Case-2 $y=1$

$x+z=2$ (from equation 1)

$xz=1$ (from equation 3)

On solving these equation we get $x=z=1$

Therefore possible triples are

(1, 1, 1) & (–2, –2, –2)

65. What is the product of all the roots of the equation $\sqrt{5|x|+8}=\sqrt{x^2-16}$?

(a) – 64 (b) – 24 (c) 576 (d) 24

Ans. **(a)**

Sol. $\sqrt{5|x|+8}=\sqrt{x^2-16}$

$\Rightarrow$ $5|x|+8=x^2-16$

$\Rightarrow$ $|x|^2-5|x|-24=0$

$\Rightarrow$ $(|x|-8)(|x|+3)=0$

$\Rightarrow$ $|x|=8$ As $|x| \neq -3$

$\Rightarrow$ $x=\pm 8$

$\therefore$ Products of the roots = –64

66. How many positive integers N give a remainder 8 when 2008 is divided by N.

(a) 12 (b) 13 (c)14 (d) 15

Ans. **(d)**

Sol. Given : $2008 \equiv 8 \pmod N$

$\Rightarrow$ $2008=NK+8, K \in N$

$\Rightarrow$ $NK=2^4 \times 5^3$

$\therefore$ Number of factors = 20

Leaving factors 1, 2, 4, 5, 8 as divisor > remainder

So number of required positive integers = 20 – 5 = 15

67. LCM of two numbers is 5775. Which of the following cannot be their HCF?

(a) 175 (b) 231 (c) 385 (d) 455

Ans. (d)

Sol. Let a and b be the two number

The, LCM (a, b) = $5775 = 5^2 \times 3 \times 7 \times 11$

Here, possible products are 231, 175, 385

$\therefore$ 455 is not possible.

68. If a, b, c are distinct real numbers such that $a + \frac{1}{b} = b + \frac{1}{c} = c + \frac{1}{a}$ evaluate abc.

(a) $\pm\sqrt{2}$ (b) $\sqrt{2} - 1$ (c) $\sqrt{3}$ (d) ± 1

Ans. (d)

Sol. $a + \frac{1}{b} = b + \frac{1}{c} = c + \frac{1}{a}$

Then, $a - b = \frac{1}{c} - \frac{1}{b} = \frac{b-c}{bc}$

$b - c = \frac{1}{a} - \frac{1}{c} = \frac{c-a}{ac}$

$c - a = \frac{1}{b} - \frac{1}{a} = \frac{a-b}{ab}$

$\Rightarrow (a-b)(b-c)(c-a) = \frac{(a-b)(b-c)(c-a)}{a^2b^2c^2}$

$\Rightarrow (a-b)(b-c)(c-a)\left[1 - \frac{1}{a^2b^2c^2}\right] = 0$

So, either $(a-b)(b-c)(c-a) = 0$ or $1 - \frac{1}{(abc)^2} = 0$

But $a = b$, $b = c$, $c = a$ is not possible as a, b, c are distinct.

So, $a^2b^2c^2 = 1$

$\therefore abc = \pm 1$

69. Mr. X with his eight children of different ages is on a family trip. His oldest child, who is 9 years old saw a license plate with a 4-digit number in which each of two digits appear two times. "Look daddy" she exclaims. "That number is evenly divisble by the age of each of us kids! "That's right," replies Mr. X, "and the last two digits just happen to be my age" Which of the following is not the age of one of Mr. X's children?

(a) 4 (b) 5 (c) 6 (d) 7

Ans. (b)

Sol. age of oldest child = 9 years

$\therefore$ age of 7 younger childs $\in \{1, 2, ..., 8\}$

The 4 digit number is divisible by all the ages of 8 childrens. And in {1, 2, ..., 9}, there are 4 even numbers. So atleast 3 childs will be even aged, so that 4 digit number will be surely divisible by all even number so option (a) 4 and (c) 6 will be neglected.

Let's check option (b) i.e., 5.

If none of children has age 5, then smallest number divisible by rest of the numbers is $9 \times 8 \times 7 = 504$ and checking its multiples we get $504 \times 11 = 5544$ a four digit number with same two digits.

So, age of X = 44

age of children = 1, 2, 3, 4, 6, 7, 8, 9.

So answer is option (b) 5.

70. How many numbers lie between 11 and 1111 which divided by 9 leave a remainder 6 and when divided by 21 leave a remainder 12?

(a) 18 (b) 28 (c) 8 (d) None of these

Ans. (a)

Sol. 33 is one of the number that satisfies the condition

Required number will be $k \times LCM (21, 9) + 33$

$\Rightarrow 63k + 33$

$\Rightarrow 11 < 63k + 33 < 1111$

$$\frac{-22}{63} < k < \frac{1078}{63}$$

k = 0, 1, 2, ... 17

18 numbers

71. If the equation $(a^2 - 5a + 6)x^2 + (a^2 - 3a + 2)x + (a^2 - 4) = 0$ has more than two roots, then the value of a is

(a) 2 (b) 3 (c) 1 (d) None of these

Ans. (a)

Sol. As the quadratic equation has more than 2 roots, so it is an identity.

$\therefore \quad a^2 - 5a + 6 = 0$

$(a - 2)(a - 3) = 0$

$\Rightarrow a = 2, a = 3$

$a^2 - 3a + 2 = 0$

$(a - 2(a + 2) = 0$

$\Rightarrow a = 2, a = 1$

$a^2 - 4 = 0$

$(a - 2)(a + 2) = 0$

$\Rightarrow a = 2, a = -2$

The common value in all three equations is a = 2

$\therefore \quad a = 2$

72. The sum of the roots of $\frac{1}{x+a}+\frac{1}{x+b}=\frac{1}{c}$ is zero. The product of roots is

(a) 0 (b) $\frac{a+b}{2}$ (c) $-\frac{1}{2}(a^2+b^2)$ (d) $2(a^2+b^2)$

Ans. (c)

Sol. $\frac{1}{x+a}+\frac{1}{x+b}=\frac{1}{c}$

$\Rightarrow \frac{1}{x+a}=\frac{1}{c}-\frac{1}{x+b}$

$\Rightarrow \frac{1}{x+a}=\frac{x+b-c}{c(x+b)}$

$(x + a)(x + b - c) = (cx + bc)$

$x^2 + bx - cx + ax + ab - ac = (cx + bc)$

$x^2 + (b + a - 2c)x + ab - ac - bc = 0$

Sum of roots = 0

$\therefore$ $b + a = 2c$

Product of roots

$\Rightarrow ab - (a + b)c$

$ab - (a + b)\left(\frac{a+b}{2}\right)$

$\frac{2ab - a^2 - b^2 - 2ab}{2} \Rightarrow \frac{-1}{2}(a^2+b^2)$

73. The solution of the equation $1 + 4 + 7 + ... + x = 925$ is

(a) 73 (b) 76 (c) 70 (d) 74

Ans. (a)

Sol. $1 + 4 + 7 + ... + x = 925$

$a = 1, d = 3$

$\frac{n}{2}[2(1) + (n-1)3] = 925$

$n[2 + 3n - 3] = 1850$

$n[3n - 1] = 1850$

$3n^2 - n - 1850 = 0$

$3n^2 - 75n + 74n - 1850 = 0$

$3n(n - 25) + 74(n - 25) = 0$

$n = 25$

$\therefore$ $x = T_{25} = 1 + (25 - 1)3 = 1 + 72 = 73$

74. Two unbiased dice are rolled. What is the probability of getting a sum which is neither 7 nor 11?

(a) 7/9 (b) 7/18 (c) 2/9 (d) 11/18

Ans. (a)

Sol. For sum = 7, Favourable cases = (1, 6); (2, 5); (3, 4); (4, 3); (5, 2); (6, 1)

For sum = 11, Favourable cases = (6, 5); (5, 6)

Total events = 36

Let A = event of getting sum 7, B = event of getting sum 11.

$P(A) = \frac{6}{36} = \frac{1}{6}$ $\quad$ $P(B) = \frac{2}{36} = \frac{1}{18}$

$P(A \cup B) = \frac{1}{6} + \frac{1}{18} = \frac{4}{18} = \frac{2}{9}$

$P(\overline{A \cup B}) = 1 - \frac{2}{9} = \frac{7}{9}$.

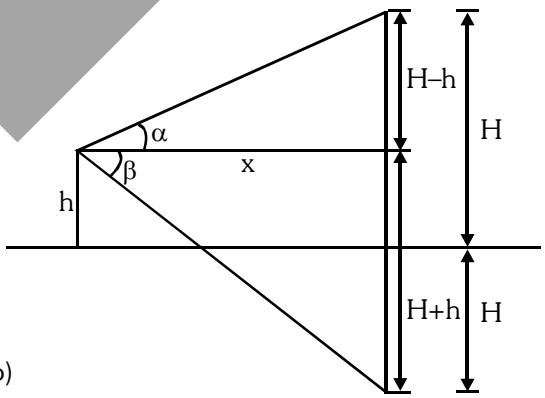
75. An observer standing at the top of a tower, find that the angle of elevation of a red bulb on the top of a light house of height H is α. Further, he finds that the angle of depression of reflection of the bulb in the ocean is β. Therefore, the height of the tower is

(a) $\frac{H(\tan\beta - \tan\alpha)}{(\tan\beta + \tan\alpha)}$ $\quad$ (b) $\frac{H\sin(\beta - \alpha)}{\cos(\alpha + \beta)}$ $\quad$ (c) $\frac{H(\cos\alpha - \cos\beta)}{(\cot\alpha + \cot\beta)}$ $\quad$ (d) H

Ans. (a)

Sol. $\tan\alpha = \frac{H - h}{x}$ .....(1)

$\tan\beta = \frac{H + h}{x}$ .....(2)



Equation (1)/(2) $\Rightarrow \frac{\tan\alpha}{\tan\beta} = \frac{H - h}{H + h}$

$\Rightarrow \frac{\tan\alpha + \tan\beta}{\tan\alpha - \tan\beta} = \frac{2H}{-2h}$ (Using componendo and dividendo)

$\Rightarrow h = H\left(\frac{\tan\beta - \tan\alpha}{\tan\beta + \tan\alpha}\right)$

76. In the convex quadrilateral ABCD, the diagonals AC and BD meet at O and the measure of angle AOB is 30°. If the areas of triangle AOB, BOC COD and AOD are 1, 2, 8 and 4 square units respectively, what is the product of the lengths of the diagonals AC and DB in sq. units?

(a) 60 $\quad$ (b) 56 $\quad$ (c) 54 $\quad$ (d) 64

Ans. (a)

Sol. $\frac{AO}{OC} = \frac{ar(\Delta AOD)}{ar(\Delta DOC)} = \frac{1}{2}$

$\Rightarrow$ AC = AO + OC = 3AO

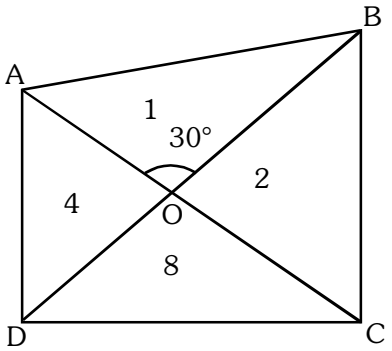
Also $\frac{BO}{OD} = \frac{ar(\Delta BOC)}{ar(\Delta COD)} = \frac{2}{8} = \frac{1}{4}$

$\Rightarrow$ BD = BO + OD = 5 BO

$ar(\Delta AOB) = \frac{1}{2} AO \times BO \times \sin 30° = 1$

$\Rightarrow$ AO × BO = 4

$\Rightarrow$ AC × BD = 3AO × 5BO = 3 × 5 × 4 = 60



77. If $\tan\theta + \sec\theta = 1.5$, then value of $\sin\theta$ is

(a) $\frac{5}{13}$ (b) $\frac{12}{13}$ (c) $\frac{3}{5}$ (d) $\frac{2}{3}$

Ans. (a)

Sol. Given $\sec\theta + \tan\theta = 1.5 = \frac{3}{2}$ and $\sec^2\theta - \tan^2\theta = 1$

$\Rightarrow (\sec\theta - \tan\theta)(\sec\theta + \tan\theta) = 1$

$= \sec\theta - \tan\theta = \frac{2}{3}$

$\therefore \sec\theta = \frac{13}{12}$

So, $\boxed{\sin\theta = \frac{5}{13}}$

78. If $\sin^2x + \sin^2y + \sin^2z = 0$, then which of the following is NOT a possible value of cos x + cosy + cosz ?

(a) 3 (b) – 3 (c) – 1 (d) – 2

Ans. (d)

Sol. Given $\sin^2x + \sin^2y + \sin^2z = 0$

$\Rightarrow \sin x = \sin y = \sin z = 0$

$\therefore$ x, y, z $\in n\pi$ where n $\in$ z

So, possible values of cos x + cos y + cos z = 3, – 3, –1, 1

Hence –2 is not possible.

79. Find the remainder when x^{51} is divided by $x^2 - 3x + 2$.

(a) x (b) $(2^{51} - 2)x + 2 - 2^{51}$ (c) $(2^{51} - 1)x + 2 - 2^{51}$ (d) 0

Ans. (c)

Sol. Let the remainder be Ax + B

Then, By Remainder Theorem,

$x^{51} = (x^2 - 3x + 2) Q(x) + Ax + B$

When x = 1, A + B = 1 .....(1)

and when x = 2, 2A + B = 2^{51} .....(2)

Solving (1) and (2)

A = $2^{51} - 1$ and B = $2 - 2^{51}$

$\therefore$ Remainder is $(2^{51} - 1)x + (2 - 2^{51})$

80. In an equilateral triangle, three coins of radii 1 unit each are kept so that they touch each other and also sides of the triangle. The area of triangle ABC (in sq. units) is

(a) $4+2\sqrt{3}$ (b) $4\sqrt{3}+6$ (c) $12+\frac{7\sqrt{3}}{4}$ (d) $3+\frac{7\sqrt{3}}{4}$

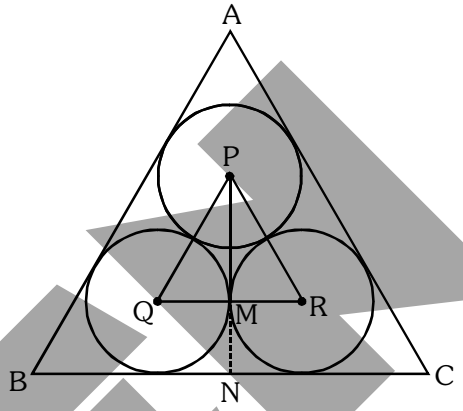
Ans. (b)

Sol. Here, PQ = QR = PR = 2

Then, PM = $\sqrt{3}$

As G is the centroid of ΔABC and ΔPQR

$\therefore$ GM = $\frac{1}{3}(\sqrt{3}) = \frac{1}{\sqrt{3}}$ and GN = $1 + \frac{1}{\sqrt{3}}$



Let x se the side of ΔABC then, $\frac{1}{\sqrt{3}}$ AN = GN

$\Rightarrow \frac{1}{\sqrt{3}}\left(\frac{\sqrt{3}x}{2}\right) = \frac{\sqrt{3}+1}{\sqrt{3}}$

$\Rightarrow x = 2(1+\sqrt{3})$

So, Area of $\Delta ABC = \frac{\sqrt{3}}{4}(x^2)$

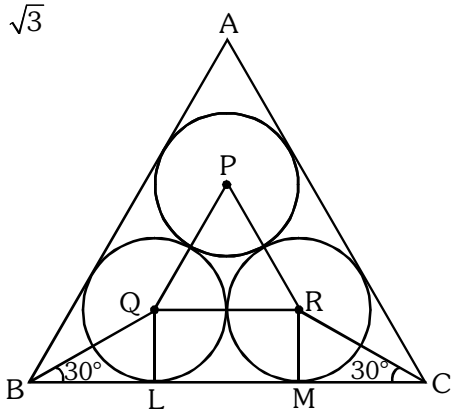
$= \frac{\sqrt{3}}{4}4(1+\sqrt{3})^2$

$= \sqrt{3}(4+2\sqrt{3})$

$= 6 + 4\sqrt{3}$

Alternate Solution

$\frac{QL}{BL} = \tan 30° \Rightarrow \frac{1}{BL} = \frac{1}{\sqrt{3}} \Rightarrow BL = \sqrt{3}$



Similarly CM = $\sqrt{3}$

and LM = QR = 2

BC = $\sqrt{3}+2+\sqrt{3} = 2(1+\sqrt{3})$

So, Area of $\Delta ABC = \frac{\sqrt{3}}{4}(BC^2)$

$= \frac{\sqrt{3}}{4}4(1+\sqrt{3})^2$

$= \sqrt{3}(4+2\sqrt{3}) = 6 + 4\sqrt{3}$