SOLUTION
THE ASSOCIATION OF MATHEMATICS TEACHERS OF INDIA
GAUSS CONTEST - FINAL - PRIMARY
CLASS - V & VI

Instructions:
1. Answer all the questions.
2. Elegant and novel solutions will get extra credits.
3. Diagrams and explanations should be given wherever necessary.
4. Fill in FACE SLIP and your rough working should be in the answer paper itself.
5. Maximum time allowed is THREE hours.
6. All questions carry equal marks.

1. (a) Find the positive integers m, n such that $\frac{1}{m}+\frac{1}{n}=\frac{3}{17}$

(b) Find the positive integers m, n, p such that $\frac{1}{m}+\frac{1}{n}+\frac{1}{p}=\frac{3}{17}$.

Sol. (a) $\frac{1}{m}+\frac{1}{n}=\frac{3}{17}$

$\frac{17}{3m}+\frac{17}{3n} = 1$

$(3m - 17)(3n - 17) = 289 = 1 \times 289$

$3m - 17 = 1,\qquad 3n - 17 = 289$

$3m = 18,\qquad 3n = 306$

$m = 6,\qquad n = 102$

$3m - 17 \neq 17\qquad 3n - 17 \neq 17,\qquad m, n \in z$

$\therefore (m, n) = (6, 102) = (102, 6)$

(b) $\frac{1}{m}+\frac{1}{n}+\frac{1}{p}=\frac{3}{17}$

$m \le n \le p$

$\frac{1}{m} \ge \frac{1}{n} \ge \frac{1}{p}$

$\frac{3}{m} \ge \frac{3}{17} \Rightarrow \frac{1}{m} \ge \frac{1}{17} \Rightarrow m \le 17$

$\frac{1}{m} \le \frac{3}{17} \Rightarrow \frac{17}{3} \le m$

$\therefore 6 \le m \le 17$

for $m = 6$, $\frac{1}{6}+\frac{1}{n}+\frac{1}{p}=\frac{3}{17}$

$\frac{1}{n}+\frac{1}{p}=\frac{1}{102}$

$(n - 102)(p - 102) = 102 \times 102$

Similarly for m = 7, 8, ... 17 equation could be solved.

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2. Find the largest positive integer n such that 3^n divides the 999 digit number 9999...99.

Sol. 9999...9 for 999 times can be written as

$\Rightarrow \dfrac{9(1111.....1,\ 999\text{ times})}{3^n}$

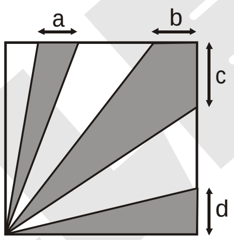
$\Rightarrow \dfrac{9(\text{sum of digit is }999)}{3^n}$

$\Rightarrow \dfrac{9\times9\times111}{3^n}$

$\Rightarrow \dfrac{3^5\times37}{3^n}$

So, maximum possible value of $n = 5$

3. Inside a square of area 36 cm^2, there are shaded regions as shown. The ratio of the shaded area to the unshaded area is 3 : 1. What is the value of $a + b + c + d$ where a, b, c, d are the lengths of the bases of the shaded regions? Further, if three of a,b,c,d are equal integers and one different, then find them.

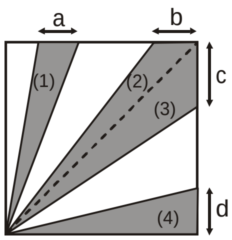


Sol. Total area $= 36\ cm^2$

Ratio of shaded to unshaded $= 3 : 1$

So, shaded area $= 27\ cm^2$

area (1) + area (2) + area (3) + area (4) $= 27\ cm^2$



$\frac{1}{2}\times a\times6+\frac{1}{2}\times b\times6+\frac{1}{2}\times c\times6+\frac{1}{2}\times d\times6 = 27\ cm^2$

$\frac{6}{2}(a+b+c+d) = 27\ cm^2$

$a+b+c+d = 9\ cm^2$

In a,b,c,d if any three are equal then

$3m + n = 9$

if $m = 1$, then $n = 6$ (not feasible)

if $m = 2$, then $n = 3$ (feasible)

So, any three of a,b,c,d is 2 and the other is 3.

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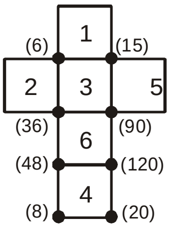
4. Let the six faces of a cube be numbered 1, 2, 3, 4, 5, 6 in such a way that the 3 pairs (1, 6), (2, 5), (3, 4) lie on opposite faces of the cube. At each vertex of the cube, the product of the three numbers on the three faces containing the vertex is written. What is the sum of all the eight numbers written at the eight vertices of the cube?

Sol. Faces of the cube are numbered 1, 2, 3, 4, 5, 6 such that (1,6), (2,5) and (3,4) lie on opposite faces.

If we take few cases

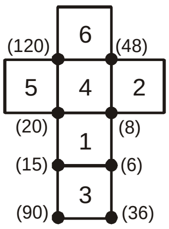
Case-I



Sum of the numbers on vertices

$\Rightarrow$ $6 + 15 + 36 + 90 + 48 + 120 + 8 + 20 = 343$

Case-II

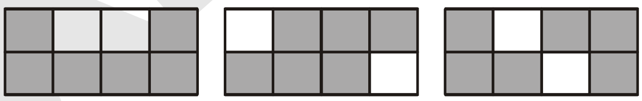


Sum of the numbers on vertices

$\Rightarrow$ $120 + 48 + 20 + 8 + 15 + 6 + 90 + 36 = 343$

Similarly for all possibilities the sum of the products of faces on vertices will remain same. So the sum is 343.

5. Given a 2×4 rectangle with eight cells, find the total number of ways (frames) in which you can shade 75% of the cells. Few such frames are given below.



Sol.

1	2	3	4
5	6	7	8

(1, 2) (1, 3) (1, 4) (1, 5) (1, 6) (1, 7) (1, 8) = 7
(2, 3) (2, 4) (2, 5) (2, 6) (2, 7) (2, 8) = 6
(3, 4) (3, 5) (3, 6) (3, 7) (3, 8) = 5
(4, 5) (4, 6) (4, 7) (4, 8) = 4
(5, 6) (5, 7) (5, 8) = 3
(6, 7) (6, 8) = 2
(7, 8) = 1

Total = 28 ways

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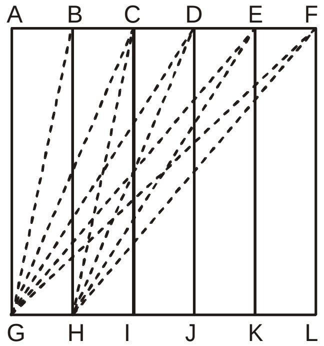
6. A square is divided into 5 identical rectangles as in the figure. Find the sum of the angles $\angle GBH$, $\angle GCH$, $\angle GDH$, $\angle GEH$, $\angle GFH$. Give a valid proof for your answer.



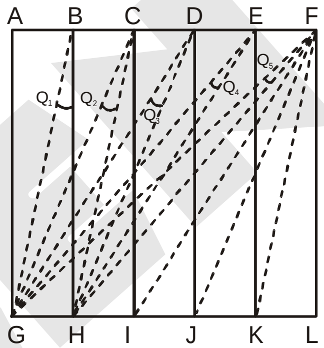
Sol. $\angle GBH = \angle KFL = Q_1$

$\angle GCH = \angle JFK = Q_2$

$\angle GDH = \angle IFJ = Q_3$ (Angles formed by the diagonals of identical Rectangles are equal)

$\angle GEH = \angle HFI = Q_4$

$\angle GFH = Q_5$

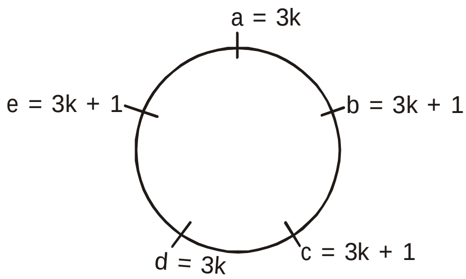


$\therefore$ $Q_1 + Q_2 + Q_3 + Q_4 + Q_5 = 45° = \angle GFL$ (Diagonal of square is angle bisector.)

7. Around a circle five positive integers a, b, c, d, e are written in such a way that the sum of no three or no two adjacent integers is divisible by three. How many of these a, b, c, d, e are divisible by three? Please give proper proof for your answer.

Sol. Let $a = 3k$ then possible value of b,c,d and e, such that the sum of no three or no two adjacent integers is divisible by three are $3k + 1$, $3k + 1$, $3k$ and $3k + 1$.

$\therefore$ Only 2 integers will be divisible by three.



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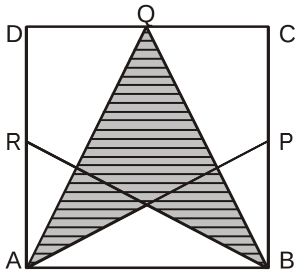
8. Let ABCD be a square with the length of side equal to 12 cm. Points P, Q, R are respectively the midpoints of sides BC, CD, and DA respectively (see figure). Find the area of the shaded region in square cm. Give valid explanation for your steps.



Sol. ar(ABCD) = $12 \times 12 = 144$ cm^2

ar(ΔADQ) = $\frac{1}{2} \times 12 \times 6 = 36$ cm^2

ar(ΔBCQ) = $\frac{1}{2} \times 12 \times 6 = 36$ cm^2

In figure ABPR is a rectangle. OA is the median of ΔARB which divides triangle in two equal areas.

ar(ΔARB) = $\frac{1}{2} \times 12 \times 6 = 36$ cm^2

So, ar(ΔAOB) = 18 cm^2

Area of shaded region = $144 - (36 + 36 + 18)$ cm^2
= $144 - (90)$ cm^2
= 54 cm^2