SOLUTION
THE ASSOCIATION OF MATHEMATICS TEACHERS OF INDIA
GAUSS CONTEST - FINAL - PRIMARY
CLASS - V & VI

Instructions :

1. Answer as many questions as possible.
2. Elegant and novel solutions will get extra credits.
3. Diagrams and explanations should be given wherever necessary.
4. Fill in FACE SLIP and your rough working should be in the answer book.
5. Maximum time allowed is THREE hours.
6. All questions carry equal marks.

1. If $A = \frac{1}{18} + \frac{1}{36} + \frac{1}{60} + \frac{1}{90} + \frac{1}{126} + \frac{1}{168}$,

$B = \frac{1}{5} - \frac{5}{30} + \frac{7}{60} - \frac{9}{100} + \frac{11}{150} + \frac{13}{210}$.

Find the value of 8A + 14 B.

Sol. $A = \frac{1}{18} + \frac{1}{36} + \frac{1}{60} + \frac{1}{90} + \frac{1}{126} + \frac{1}{168}$

$$= \frac{1}{6}\left[\frac{1}{3}+\frac{1}{6}+\frac{1}{10}+\frac{1}{15}+\frac{1}{21}+\frac{1}{28}\right]$$

$$= \frac{1}{6}\left[\frac{1}{3}+\left(\frac{1}{2}-\frac{1}{3}\right)+\frac{1}{3}\left(\frac{1}{2}-\frac{1}{5}\right)+\frac{1}{15}+\frac{1}{4}\left(\frac{1}{3}-\frac{1}{7}\right)+\frac{1}{28}\right]$$

$$= \frac{1}{6}\left[\cancel{\frac{1}{3}}+\frac{1}{2}-\cancel{\frac{1}{3}}+\frac{1}{6}-\cancel{\frac{1}{15}}+\cancel{\frac{1}{15}}+\frac{1}{12}-\cancel{\frac{1}{28}}+\cancel{\frac{1}{28}}\right]$$

$$= \frac{1}{6}\left[\frac{1}{2}+\frac{1}{6}+\frac{1}{12}\right]$$

$$= \frac{1}{6}\left[\frac{6+2+1}{12}\right] = \frac{1}{6}\times\frac{9}{12} = \frac{1}{8}$$

$B = \frac{1}{5} - \frac{5}{30} + \frac{7}{60} - \frac{9}{100} + \frac{11}{150} + \frac{13}{210}$.

$$= \frac{1}{5}\left[1-\frac{5}{6}+\frac{7}{12}-\frac{9}{20}+\frac{11}{30}+\frac{13}{42}\right]$$

$$= \frac{1}{5}\left[1-\frac{5}{6}+\frac{1}{3}+\cancel{\frac{1}{4}}-\cancel{\frac{1}{4}}-\cancel{\frac{1}{5}}+\cancel{\frac{1}{5}}+\frac{1}{6}+\frac{1}{6}+\frac{1}{7}\right]$$

$$= \frac{1}{5}\left[\frac{1}{6}+\frac{1}{3}+\frac{1}{6}+\frac{1}{6}+\frac{1}{7}\right]$$

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$= \frac{1}{5}\left[\frac{7+14+7+7+6}{42}\right] = \frac{1}{5}\times\frac{41}{42}$

So, $8A + 14B = \frac{1}{8} \times 8 + 14 \times \frac{41}{42\times 5}$

$= 1 + \frac{41}{15} = \frac{15+41}{15} = \frac{56}{15}$

2. (a) Given n is the smallest positive integer for which $864 \times n$ is a perfect cube. Find n.

Three strings of lengths 15 m, 42 m and 39 m are to be cut into pieces of equal lengths. If 'm' is the greatest possible length of each piece, find 'm'.

Find the area of the rectangle whose length and breadth are n metres and m metres respectively.

(b) 605 chocolates are distributed equally among some children. The number of chocolates received by each child is 20% of the total number of children. How many chocolates did each child receive?

Sol. (a) (i) $a^3 = 864 \times n$

$\Rightarrow a^3 = \underline{2 \times 2 \times 2} \times \underline{2 \times 2} \times \underline{3 \times 3 \times 3} \times n$

Hence, n = 2.

(ii) In this case, we have to find H.C.F. of 15 m, 42 m and 39 m.

So, $15 = 5 \times 3$

$42 = 14 \times 3$

$39 = 13 \times 3$

m = 3 metres

(iii) Length = n metres and Breadth = m metres

So, Area of Rectangle = $L \times B = (n \times m)\ m^2$.

(b) Let the number of children = n

So, A.T.Q $\Rightarrow \frac{605}{n} = 20\%$ of n

$\Rightarrow \frac{605}{n} = \frac{\cancel{20} \times n}{\cancel{100}^{5}}$

$\Rightarrow n^2 = 605 \times 5$

$\Rightarrow n^2 = 121 \times 25$

$\Rightarrow n = 11 \times 5 = 55$

Hence, each child received $= \frac{605}{55} = 11$ chocolates

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3. John, on his birthday, went to an ice-cream shop along with his five friends. There are three flavours of ice-cream, Butterscotch, Vanilla and Pista. John and his friends do not have any specific choice of a flavour. How many different assortments of the ice-cream can be selected?

Sol. No of assortments of the ice-cream

$\Rightarrow$ $3 \times 3 \times 3 \times 3 \times 3 \times 3$

$\Rightarrow$ $3^6 = 729$

4. A natural number is divisible both by 9 and 4. This number consists of only 4s and 9s. Let 'n' be such a number which is the smallest. Find the last 9 digits of this number.

Sol. Let 4 repeated x times and 9 repeated y times.

$4x + 9y$ = multiple of 9

$4x + 9y = 9n$ [$n \rightarrow$ natural number]

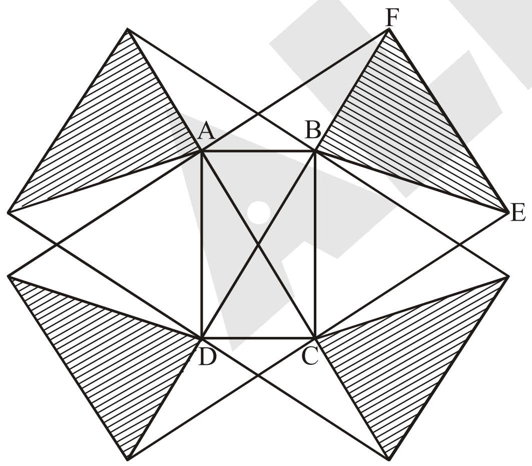
$4x = 9(n - y)$

x should be divisible by 9

Smallest value of x = 9

For the number to be smallest, divisible by 9 & 4 and 4 & 9 as digits should have digit '4' 9 times in it. So number will be 4444444944. Last 9 digits will be 444444944.

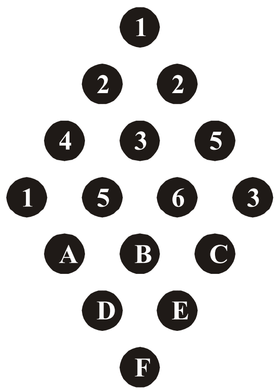
5. (a)



In a figure, ABCD is a rectangle. Square are drawn outwardly on the diagonals. (For example, on AC, the square drawn is ACEF).

The dimensions of the rectangle are 5 cm × 20 cm. Find the total area of the shaded region.

(b)



From the pattern shown in the adjoining figure, find the values of A, B, C, D, E and F. Also compute the value of

$$A^2 + C^2 + F^2 - B - D - E$$

Sol. (a) Area of ΔABC

$\Rightarrow \frac{1}{2} \times h \times AC \quad \Rightarrow \frac{\text{area of ABCD}}{2}$ [where h is height of ΔABC]

$\Rightarrow \frac{1}{2} \times h \times AC \quad \Rightarrow \frac{5 \times 20}{2} \Rightarrow 50$

$\Rightarrow \frac{1}{2} \times h \times AC \quad \Rightarrow 50$ ...(1)

In ΔADC

$$AC = \sqrt{AD^2 + DC^2}$$
$$= \sqrt{(20)^2 + 5^2}$$
$$= \sqrt{400 + 25}$$
$$= \sqrt{425}$$
$$= \sqrt{25 \times 17}$$
$$= 5\sqrt{17} = FE = \text{Side of square}$$

From equation (1)

$$\frac{1}{2} \times h \times AC = 50$$

$$\Rightarrow \frac{1}{2} \times h \times 5\sqrt{17} = 50$$

$$\Rightarrow h = \frac{20}{\sqrt{17}}$$

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Height of $\Delta BFE(H) \Rightarrow 5\sqrt{17} - h \Rightarrow 5\sqrt{17} - \dfrac{20}{\sqrt{17}} = \dfrac{65}{\sqrt{17}}$

area of $(\Delta BFE) \Rightarrow \dfrac{1}{2} \times H \times FE \Rightarrow \dfrac{1}{2} \times \dfrac{65}{\sqrt{17}} \times 5\sqrt{17} \Rightarrow \dfrac{325}{2}$

Shaded Area $= 4 \times$ area $(\Delta BFE) \Rightarrow 4 \times \dfrac{325}{2} \Rightarrow 650\ \text{cm}^2$

(b) $$\begin{array}{ccc} & 1 & \\ 2 & + & 2 \\ & 3 & \end{array} \Rightarrow 1 + 3 = 2 + 2$$

$$\begin{array}{ccc} & 2 & \\ 4 & + & 3 \\ & 5 & \end{array} \Rightarrow 4 + 3 = 5 + 2$$

$$\begin{array}{ccc} & 2 & \\ 3 & + & 5 \\ & 6 & \end{array} \Rightarrow 5 + 3 = 6 + 2$$

$$\begin{array}{ccc} & 4 & \\ 1 & + & 5 \\ & A & \end{array} \Rightarrow 1 + 5 = 4 + A \qquad \Rightarrow A = 2$$

$$\begin{array}{ccc} & 5 & \\ 6 & + & 3 \\ & C & \end{array} \Rightarrow 6 + 3 = 5 + C \qquad \Rightarrow C = 4$$

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$$\begin{array}{ccc} & 3 & \\ 5 & + & 6 \\ & B & \end{array} \Rightarrow 6 + 5 = 3 + B \Rightarrow B = 8$$

$$\begin{array}{ccc} & 6 & \\ 8 & + & 4 \\ & E & \end{array} \Rightarrow 12 = 6 + E \Rightarrow E = 6$$

$$\begin{array}{ccc} & 5 & \\ 2 & + & 8 \\ & D & \end{array} \Rightarrow 10 = 5 + D \Rightarrow D = 5$$

$$\begin{array}{ccc} & 8 & \\ 5 & + & 6 \\ & F & \end{array} \Rightarrow 11 = 8 + F \Rightarrow F = 3$$

$A^2 + C^2 + F^2 - B - D - E \Rightarrow 2^2 + 4^2 + 3^2 - 8 - 5 - 6$

$\Rightarrow 4 + 16 + 9 - 19$

$\Rightarrow 29 - 19$

$\Rightarrow 10$