SECTION-1
PART-A
MATHEMATICS

1. The number of four-letter words that can be formed with letters a.b.c such that all three letters occur is

(A) 30 (B) 36
(C) 81 (D) 256

Ans. (B)

Sol. $^{3}C_{1} \cdot \frac{4!}{2!} = 36$

2. Let

$$A = \left\{ \theta \in \mathbb{R} \left| \left(\frac{1}{3}\sin(\theta) + \frac{2}{3}\cos(\theta)\right)^2 = \frac{1}{3}\sin^2(\theta) + \frac{2}{3}\cos^2(\theta) \right. \right\}$$

Then

(A) $A \cap [0, \pi]$ is an empty set
(B) $A \cap [0, \pi]$ has exactly one point
(C) $A \cap [0, \pi]$ has exactly two points
(D) $A \cap [0, \pi]$ has more than two points

Ans. (B)

Sol. $\frac{\sin^2\theta}{9} + \frac{4}{9}\cos^2\theta + \frac{4}{9}\cos\theta\sin\theta = \frac{\sin^2\theta}{3} + \frac{2}{3}\cos^2\theta$

$\Rightarrow \sin 2\theta = 1$

$\Rightarrow \theta = \frac{\pi}{4}$ in $[0, \pi]$**Ans. (B)**

$\frac{\sin^2\theta}{9} + \frac{4}{9}\cos^2\theta + \frac{4}{9}\cos\theta\sin\theta = \frac{\sin^2\theta}{3} + \frac{2}{3}\cos^2\theta$

$\Rightarrow \sin 2\theta = 1$

$\Rightarrow \theta = \frac{\pi}{4}$ in $[0, \pi]$

3. The area of the region bounded by the lines $x = 1$, $x = 2$, and the curves $x(y - e^x) = \sin x$ and $2xy = 2\sin x + x^3$ is

(A) $e^2 - e - \frac{1}{6}$ (B) $e^2 - e - \frac{7}{6}$

(C) $e^2 - e + \frac{1}{6}$ (D) $e^2 - e + \frac{7}{6}$

Ans. (B)

Sol. Area $= \left| \int_1^2 \left(\frac{2\sin x + x^3}{2x} - \left(\frac{\sin x}{x} + e^x \right) \right) dx \right|$

$= \left| \int_1^2 \left(\frac{x^2}{2} - e^x \right) dx \right|$

$= e^2 - e - \frac{7}{6}$

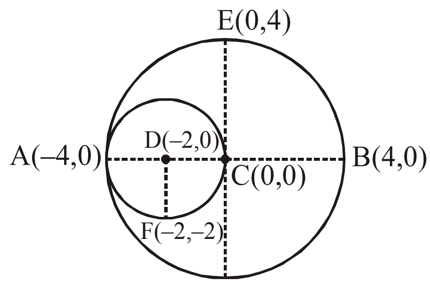
4. Let AB be a line segment with midpoint C, and D be the midpoint of AC. Let C_1 be the circle withdiameter AB. and C_2 be the circle with diameter AC. Let E be a point on C_1 such that EC isperpendicular to AB. Let F be a point on C_2 such that DF is perpendicular to AB, and E and F lie on opposite sides of AB. Then the value of $\sin \angle FEC$ is

(A) $\frac{1}{\sqrt{10}}$ (B) $\frac{2}{\sqrt{10}}$

(C) $\frac{1}{\sqrt{13}}$ (D) $\frac{2}{\sqrt{13}}$

Ans. (A)

Sol.



$\sin(\angle FEC) = \frac{2}{\sqrt{4+36}} = \frac{1}{\sqrt{10}}$

5. The number of integers x satisfying

$$-3x^4 + \det\begin{bmatrix} 1 & x & x^2 \\ 1 & x^2 & x^4 \\ 1 & x^3 & x^6 \end{bmatrix} = 0$$

is equal to

(A) 1 (B) 2
(C) 5 (D) 8

Ans. (B)

Sol. $3x^4 = \begin{vmatrix} 1 & x & x^2 \\ 1 & x^2 & x^4 \\ 1 & x^3 & x^6 \end{vmatrix}$

$3x^4 = (x - x^2)(x^2 - x^3)(x^3 - x)$

$\Rightarrow 3x^4 = x^4(x - 1)^3 (x + 1)$

$\Rightarrow x = 0 \ \& \ x = 2$

6. Let P be a non-zero polynomial such that $P(1 + x) = P(1 - x)$ for all real x, and $P(1) = 0$. Let m be the largest integer such that $(x - 1)^m$ divides $P(x)$ for all such $P(x)$. Then m equals

(A) 1 (B) 2

(C) 3 (D) 4

Ans. (B)

Sol. $P(1 + x) = P(1 - x)$

$\because$ P(x) is a polynomial

$\therefore P'(1 + x) = -P'(1 - x) \ \& \ P'(1) = 0$

$\Rightarrow P(x) = (x - 1)^2 Q(x)$

7. Let

$$f(x) = \begin{cases} x\sin\left(\dfrac{1}{x}\right) & \text{when } x \neq 0 \\ 1 & \text{when } x = 0 \end{cases}$$

and $A = \{x \in \mathbb{R} : f(x) = 1\}$. Then A has

(A) exactly one element

(B) exactly two elements

(C) exactly three elements

(D) infinitely many elements

Ans. (A)

Sol. $f(x) = 1$

$\Rightarrow 1 = 1$ when $x = 0$ & $x\sin\dfrac{1}{x} = 1$ when $x \neq 0$

$\because \sin\dfrac{1}{x} = \dfrac{1}{x}$ has no solution

$\therefore x = 0$ is only solution

8. Let S be a subset of the plane defined by:

$S = \{(x,y): |x| + 2|y| = 1\}$.

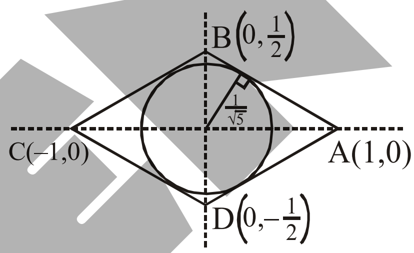
Then the radius of the smallest circle with centre at the origin and having non-empty intersectionwith S is

(A) $\dfrac{1}{5}$ (B) $\dfrac{1}{\sqrt{5}}$

(C) $\dfrac{1}{2}$ (D) $\dfrac{2}{\sqrt{5}}$

Ans. (B)

Sol.



minimum radius = $\dfrac{1}{\sqrt{5}}$

9. The number of solutions of the equation

$\sin(9x) + \sin(3x) = 0$

in the closed interval $[0, 2\pi]$ is

(A) 7 (B) 13

(C) 19 (D) 25

Ans. (B)

Sol. $\sin 9x + \sin 3x = 0$

$\Rightarrow \sin 6x \sin 3x = 0$

$\Rightarrow x = \dfrac{n\pi}{6}$

$\Rightarrow$ 13 solutions

10. Among all the parallelograms whose diagonals are 10 and 4, the one having maximum area has its perimeter lying in the interval

(A) (19, 20] (B) (20, 21]

(C) (21, 22] (D) (22, 23]

Ans. (C)

Sol. Area of parallelogram = $\frac{1}{2} d_1 d_2 \sin\phi$

max area = $\frac{1}{2} \cdot d_1 d_2$

$\Rightarrow$ It is a rhombus

$\Rightarrow$ side length = $\sqrt{\left(\frac{d_1}{2}\right)^2 + \left(\frac{d_2}{2}\right)^2}$

$= \sqrt{25+4} = \sqrt{29}$

$\Rightarrow$ perimeter = 4a = $4\sqrt{29} \in [21, 22)$

11. The number of ordered pairs (a, b) of positive integers such that

$$\frac{2a-1}{b} \text{ and } \frac{2b-1}{a}$$

are both integers is

(A) 1 (B) 2

(C) 3 (D) more than 3

Ans. (C)

Sol. $2a - 1 = \lambda b$ λ = odd

$2b - 1 = \mu a$ μ = odd

$\Rightarrow 4a - 2 = 2\lambda b = \lambda(\mu a + 1)$

$\Rightarrow (4 - \lambda\mu)a = \lambda + 2$

$1 \le \lambda\mu \le 3$

$\lambda = 1, \mu = 1 \Rightarrow a = 1, b = 1$

$\lambda = 1, \mu = 3 \Rightarrow a = 3, b = 5$

$\lambda = 3, \mu = 1 \Rightarrow a = 5, b = 3$

12. Let z = x + iy and w = u + iv be complex numbers on the unit circle such that $z^2 + w^2 = 1$. Then the number of ordered pairs (z, w) is

(A) 0 (B) 4

(C) 8 (D) infinite

Ans. (C)

Sol. Let $z = e^{i\alpha} = \cos\alpha + i\sin\alpha$

$w = e^{i\beta} = \cos\beta + i\sin\beta$

since $z^2 + w^2 = 1$

$\therefore \cos2\alpha + i\sin2\alpha + \cos2\beta + i\sin2\beta = 1$

$\therefore \cos2\alpha + \cos2\beta = 1$

and $\sin2\alpha + \sin2\beta = 0$

$2\cos(\alpha + \beta).\cos(\alpha - \beta) = 1$

and $\sin2\alpha = -\sin2\beta$

$\therefore \sin^2 2\alpha = \sin^2 2\beta$

$\cos2\alpha = \pm\cos2\beta$

$\cos2\alpha = \cos2\beta$

or $\cos2\alpha + \cos2\beta = 0$ (Cancelled)

If $\cos2\alpha = \cos2\beta$

$\therefore 2\cos2\alpha = 1$

$$\cos 2\alpha = \frac{1}{2}$$

$$\therefore 2\alpha = \frac{\pi}{3},\ 2\alpha = 2\pi - \frac{\pi}{3},$$

$$2\alpha = 2\pi + \frac{\pi}{3},\ 2\alpha = 4\pi - \frac{\pi}{3}$$

$$\alpha = \frac{\pi}{6},\ \alpha = \frac{5\pi}{6},\ \alpha = \frac{7\pi}{6},\ \alpha = \frac{11\pi}{6}$$

$$\therefore (\alpha, \beta) \equiv \left(\frac{\pi}{6}, \frac{5\pi}{6}\right), \left(\frac{\pi}{6}, \frac{11\pi}{6}\right), \left(\frac{5\pi}{6}, \frac{\pi}{6}\right),$$

$$\left(\frac{5\pi}{6}, \frac{7\pi}{6}\right), \left(\frac{7\pi}{6}, \frac{5\pi}{6}\right),$$

$$\left(\frac{7\pi}{6}, \frac{11\pi}{6}\right),$$

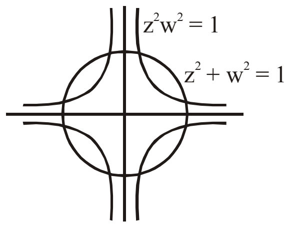
$$\left(\frac{11\pi}{6}, \frac{\pi}{6}\right), \left(\frac{11\pi}{6}, \frac{7\pi}{6}\right)$$

Alternate :

$z^2 + w^2 = 1$

$\bar{z}^2 + \bar{w}^2 = 1$

$\frac{1}{z^2} + \frac{1}{w^2} = 1$



$\Rightarrow z^2 + w^2 = z^2w^2 = 1$

13. Let E denote the set of letters of the English alphabet. V = {a, e, i, o, u}, and C be the complement of V in E. Then, the number of four-letter words (where repetitions of letters are allowed) having at least one letter from V and at least one letter from C is

(A) 261870 (B) 314160

(C) 425880 (D) 851760

Ans. (A)

Sol. 'V' denotes vowels and 'C' denotes consonants.

total 4 letter words = $(26)^4$

Number of 4 letter words which contains only vowels = $(5)^4$

No. of is letter words which contains only consonants = $(21)^4$

$\therefore$ No. of words which contains at least one vowel and at least one consonants :

$(26)^4 - (21)^4 - 5^4 = 261870$

14. Let σ_1, σ_2, σ_3 be planes passing through the origin. Assume that σ_1 is perpendicular to the vector (1,1,1), σ_2 is perpendicular to a vector (a, b, c), and σ_3 is perpendicular to the vector (a^2, b^2, c^2). What are all the positive values of a, b, and c so that $\sigma_1 \cap \sigma_2 \cap \sigma_3$ is a single point?

(A) Any positive value of a. b, and c other than 1

(B) Any positive values of a, b, and c where either $a \neq b, b \neq c$ or $a \neq c$

(C) Any three distinct positive values of a, b, and c

(D) There exist no such positive real numbers a. b, and c

Ans. (C)

Sol. σ_1 is perpendicular to $\left(\hat{i}+\hat{j}+\hat{k}\right)$

σ_2 is perpendicular to $\left(a\hat{i}+b\hat{j}+c\hat{k}\right)$

σ_3 is perpendicular to $\left(a^2\hat{i}+b^2\hat{j}+c^2\hat{k}\right)$

$\therefore$ planes are

$\sigma_1 \rightarrow x + y + z = 0$

$\sigma_2 \rightarrow ax + by + cz = 0$

$\sigma_3 \rightarrow a^2x + b^2y + c^2z = 0$

$$\therefore \begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^2 & b^2 & c^2 \end{vmatrix} \neq 0$$

$(a - b)(b - c)(c - a) \neq 0$

a, b, c must be distinct and positive

15. Ravi and Rashmi are each holding 2 red cards and 2 black cards (all four red and all four black cards are identical). Ravi picks a card at random from Rashmi, and then Rashmi picks a card at random from Ravi. This process is repeated a second time. Let p be the probability that both have all 4 cards of the same colour. Then p satisfies

(A) $p \leq 5\%$ (B) $5\% < p \leq 10\%$

(C) $10\% < p \leq 15\%$ (D) $15\% < p$

Ans. (A)

Sol. Ravi $\begin{cases} \nearrow 2R \\ \searrow 2B \end{cases}$, Rashmi $\begin{cases} \nearrow 2R \\ \searrow 2B \end{cases}$

If Ravi withdraw Red cards from Rashmi, then Rashmi withdraw Black card from Ravi and this process repeat again, Vice-versa. If Ravi withdraw Black card from Rashmi

$$P = 2\left(\frac{2}{4}\times\frac{2}{5}\times\frac{1}{4}\times\frac{1}{5}\right)$$

$$P = \frac{1}{50} = 2\%$$

16. Let A_1, A_2, and A_3 be the regions on $\mathbb{R}^2$ defined by

$A_1 = \{(x,y): x \geq 0, y \geq 0,$
$\qquad 2x + 2y - x^2 - y^2 > 1 > x + y\},$

$A_2 = \{(x, y): x \geq 0, y \geq 0, x + y > 1 > x^2 + y^2\},$

$A_3 = \{(x,y): x \geq 0, y > 0, x + y > 1 > x^3 + y^3\}.$

Denote by $|A_1|$, $|A_2|$, and $|A_3|$ the areas of the regions A_1, A_2, and A_3 respectively. Then

(A) $|A_1| > |A_2| > |A_3|$

(B) $|A_1| > |A_3| > |A_2|$

(C) $|A_1| = |A_2| < |A_3|$

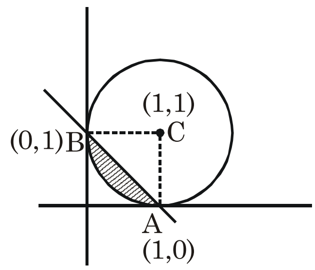
(D) $|A_1| = |A_3| > |A_2|$

Ans. (C)

Sol. $A_1 \equiv 2x + 2y - x^2 - y^2 > 1 > x + y$

$\therefore x^2 + y^2 - 2x - 2y + 1 < 0$ and $x + y < 1$

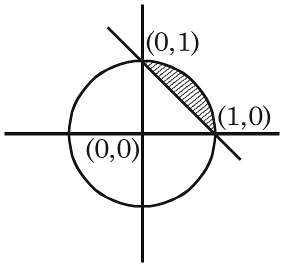
$(x - 1)^2 + (y - 1)^2 < 1$ and $x + y < 1$



$$A_1 = \frac{\pi(1)^2}{4} - \frac{1}{2}(1)(1) = \frac{\pi}{4} - \frac{1}{2}$$

$A_2 = x + y > 1 > x^2 + y^2$

$$A_2 = \frac{\pi}{4} - \frac{1}{2}$$



$A_3 = x + y > 1 > x^3 + y^3$

clearly $A_3 > A_2 = A_1$

17. Let f : $\mathbb{R} \to \mathbb{R}$ be a continuous function such that $f(x^2) = f(x^3)$ for all $x \in \mathbb{R}$. Consider the following statements.

I. f is an odd function.

II. f is an even function.

III. f is differentiable everywhere.

Then

(A) I is true and III is false

(B) II is true and III is false

(C) both I and III are true

(D) both II and III are true

Ans. (D)

Sol. If R $\to$ R, continuous function

$f(x^2) = f(x^3) \quad \forall\ x \in R$

Let $t = x^3 \Rightarrow x = t^{1/3}$

$f(t^{2/3}) = f(t)$ ...(1)

Replace $t \to -t$

$f((-t)^{2/3}) = f(-t)$

$f(t^{2/3}) = f(-t)$ ...(2)

$\therefore\ f(t) = f(-t) \Rightarrow f(x)$ is even function

$f(x^{3/2}) = f(x) = f(x^{2/3}) = f(x^{4/9})$ ..... $= f\left(x^{(2/3)^n}\right)$

As $n \to \infty$, $f\left(x^{(2/3)^n}\right) = f(x^0) = f(1)$ = constant function it is always differentiable

18. Suppose a continuous function $f : [0, \infty) \to \mathbb{R}$ satisfies

$$f(x) = 2\int_0^x tf(t)dt + 1 \text{ for all } x \geq 0.$$

Then f(1) equals

(A) e (B) e^2

(C) e^4 (D) e^6

Ans. (A)

Sol. $f : [0, \infty) \to R$

$$f(x) = 2\int_0^x t f(t)\,dt + 1, \quad \forall\ x > 0$$

$\therefore\ f'(x) = 2xf(x)$

$$\frac{f'(x)}{f(x)} = 2x$$

$$\int \frac{f'(x)}{f(x)} dx = \int 2x\,dx$$

$\ell n f(x) = x^2 + c$

$$f(x) = e^{x^2} + c$$

Since $f(0) = 1 \Rightarrow f(0) = e^c = 1 \Rightarrow c = 0$

$\therefore\ f(x) = e^{x^2}$

$f(1) = e$

19. Let $a > 0$, $a \neq 1$. Then the set S of all positive real numbers b satisfying

$(1 + a^2)(1 + b^2) = 4ab$

is

(A) an empty set

(B) a singleton set

(C) a finite set containing more than one element

(D) $(0, \infty)$

Ans. (A)

Sol. $a > 0$, $a \neq 1$

$(1 + a^2)(1 + b^2) = 4ab$

$$\Rightarrow \underbrace{\left(\frac{1}{a} + a\right)}_{\geq 2}\underbrace{\left(\frac{1}{b} + b\right)}_{\geq 2} = 4$$

$$\therefore \left(\frac{1}{a} + a\right)\left(\frac{1}{b} + b\right) \geq 4$$

equality holds true when a = b = 1

but it is given in the question that $a \neq 1$ hence there is no values of b

20. Let $f:\mathbb{R}\to\mathbb{R}$ be a function defined by

$$f(x)=\begin{cases}\dfrac{\sin(x^2)}{x} & \text{if } x\neq 0,\\ 0 & \text{if } x=0\end{cases}$$

Then, at x = 0, f is

(A) not continuous

(B) continuous but not differentiable

(C) differentiable and the derivative is not continuous

(D) differentiable and the derivative is continuous

Ans. (D)

Sol. $f:R\to R,\ f(x)=\begin{cases}\dfrac{\sin x^2}{x} & , & x\neq 0\\ 0 & , & x=0\end{cases}$

Continuity :

$$\lim_{x\to 0} f(x)=\lim_{x\to 0}\frac{\sin x^2}{x}=0=f(0)$$

hence $f(x)$ is continuous at x = 0

differentiability :

$$\text{R.H.D.}=\lim_{h\to 0}\frac{f(0+h)-f(0)}{h}=\lim_{h\to 0}\frac{\dfrac{\sin h^2}{h}-0}{h}=1$$

$$\text{L.H.D.}=\lim_{h\to 0}\frac{f(0-h)-f(0)}{-h}=\lim_{h\to 0}\frac{\dfrac{\sin h^2}{-h}-0}{-h}=1$$

R.H.D. = L.H.D.

$f(x)$ is differentiable at x = 0

$$f'(x)=\begin{cases}2\cos x^2-\dfrac{\sin x^2}{x} & , & x\neq 0\\ 1 & , & x=0\end{cases}$$

$\lim\limits_{h\to 0} f'(x) = 2 - 1 = 1$, derivative is continuous at x $=0$

SECTION-2

PART-A

PHYSICS

21. In a muonic atom a muon of mass of 200 times of that of electron and same charge is bound to the proton. The wavelengths of its Balmer series are in the range of

(A) X-rays. (B) infrared.

(C) γ rays. (D) microwave.

Ans. (A)

Sol. Energy of an orbit

$$E=\frac{-me^4}{8\varepsilon_0^2h^2}\frac{z^2}{n^2}$$

Since energy increases by 200 times wavelength will decrease by 200 times and will be in the range of X-ray.

22. We consider the Thomson model of the hydrogen atom in which the proton charge is distributed uniformly over a spherical volume of radius 0.25 angstrom. Applying the Bohr condition in this model the ground state energy (in eV) of the electron will be close to

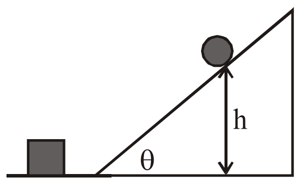
(A) –13.6/4 (B) –13.6

(C) $-\dfrac{13.6}{2}$ (D) – 2 × 13.6

Ans. (B)

Sol. If electron is orbiting a proton or a positive sphere having charge equal to proton then its K.E. & P.E. will be same. Hence, its total energy will be –13.6 eV.

23. A spherical rigid ball is released from rest and starts rolling down an inclined plane from height h = 7 m, as shown in the figure. It hits a block at rest on the horizontal plane (assume elastic collision). If the mass of both the ball and the block is m and the ball is rolling without sliding, then the speed of the block after collision is close to



(A) 6 m/s (B) 8 m/s

(C) 10 m/s (D) 12 m/s

Ans. **(C)**

Sol. $\frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 = mgh$

$$\frac{1}{2}mv^2 + \frac{1}{2}\left(\frac{2}{5}mR^2\right)\frac{V^2}{R^2} = mgh$$

$$V = \sqrt{\frac{10gh}{7}}$$

On collision it will transfer whole velocity so

velocity of the block = $\sqrt{\frac{10 \times g \times 7}{7}} = 10m/s$

24. A girl drops an apple from the window of a train which is moving on a straight track with speed increasing with a constant rate. The trajectory of the falling apple as seen by the girl is

(A) parabolic and in the direction of the moving train.

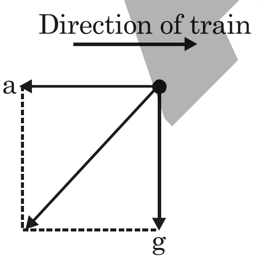
(B) parabolic and opposite to the direction of the moving train.

(C) an inclined straight line pointing in the direction of the moving train.

(D) an inclined straight line pointing opposite to the direction of the moving train.

Ans. **(D)**

Sol. Let's assume acceleration of train is 'a' then in frame of the train ball will have net acceleration at an incline pointing away from direction of the train.

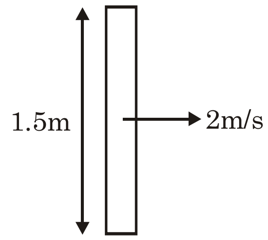


25. A train is moving slowly at 2 m/s next to a railway platform A man, 1.5 m tall, alights from the train such that his feet are fixed on the ground. Taking him to be a rigid body, the instantaneous angular velocity (in rad/sec) is

(A) 1.5 (B) 2.0 (C) 2.5 (D) 3.0

Ans. **(B)**

Sol. Man is considered a rigid body. Applying conservation of angular momentum about feet just before & after landing



$$mv\frac{\ell}{2} = I\omega$$

$$m \times v \times \frac{\ell}{2} = \frac{ML^2}{3} \times \omega$$

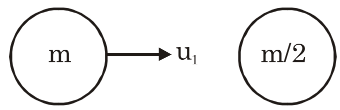
$$\Rightarrow \omega = \frac{3v}{2\ell} = \frac{3 \times 2}{2 \times \left(\frac{3}{2}\right)} = 2rad/sec$$

26. A point mass M moving with a certain velocity collides with a stationary- point mass M/2. The collision is elastic and in one dimension. Let the ratio of the final velocities of M and M/2 be x. The value of x is

(A) 2 (B) 3 (C) 1/2 (D) l/4

Ans. **(D)**

Sol.



$$v_1 = \left(\frac{m - \frac{m}{2}}{m + \frac{m}{2}}\right)u_1 = \frac{u_1}{3}$$

$$v_2 = \frac{2m}{m + \frac{m}{2}}u_1 = \frac{4u_1}{3}$$

$$\frac{v_1}{v_2} = \frac{1}{4}$$

27. A particle of mass 2/3 kg with velocity $v = -15$ m/s at $t = -2$s is acted upon by a force $f = k - \beta t^2$. Here $k = 8$ N and $\beta = 2$ N/s^2. The motion is one dimensional. Then the speed at which the particle acceleration is zero again, is
(A) 1 m/s (B) 16 m/s (C) 17 m/s (D) 32 m/s

Ans. (C)

Sol. $m\frac{dv}{dt} = 8 - 2t^2$

$mv = 8t - \frac{2t^3}{3} + C$

$\frac{2}{3}(-15) = 8(-2) - \frac{2}{3}(-2)^3 + C$

$-10 = -16 + \frac{16}{3} + C$

$C = \frac{2\times16}{3} - 10 = \frac{2}{3}$

F is zero again at t = 2 sec.

$\frac{2}{3}\times v' = 8\times 2 - \frac{2}{3}\times(2)^3 + \frac{2}{3}$

$\frac{2}{3}v' = 16 - \frac{16}{3} + \frac{2}{3}$

$v' = 17$ m/s

28. A certain stellar body has radius 50 R_s and temperature 2 T_s and is at a distance of 2×10^{10} A.U. from the earth. Here A.U. refers to the earth sun distance and R_s and T_s refer to the sun's radius and temperature respectively. Take both star and sun to be ideal black bodies. The ratio of the power received on earth from the stellar body as compared to that received from the sun is close to
(A) 4×10^{-20} (B) 2×10^{-6} (C) 10^{-8} (D) 10^{-16}

Ans. (D)

Sol. $P_{body} = \sigma\left(4\pi(50R_S)^2\right)(2T_S)^4$

$P_{body} = 50^2 \times 2^4 \times P_{sun}$

Intensity at earth due to body

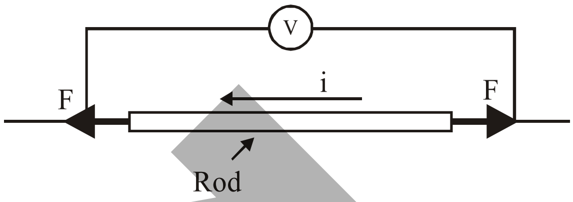
$= \frac{P_{body}}{4\pi\left(2\times10^{10}\,AU\right)^2}$

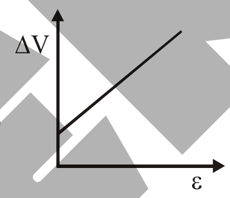
$= \frac{50^2\times2^4\times P_{sun}}{4\times10^{20}\times(4\pi AU)^2}$

$= \frac{50^2\times2^4}{4\times10^{20}}\times I_{Sun}$

$I_{body} = 10^{-16}\times I_{sun}$

29. As shown in the schematic below, a rod of uniform cross-sectional area A and length l is carrying a constant current i through it and voltase across the rod is measured using an ideal voltmeter. The rod is stretched by the application of a force F. Which of the following graphs would show the variation in the voltage across the rod as function of the strain, ε, when the strain is small. Neglect Joule heating.



(A)


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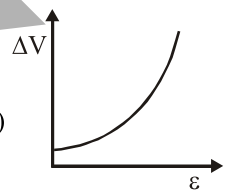
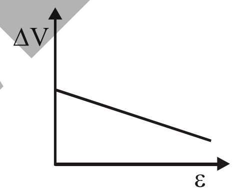
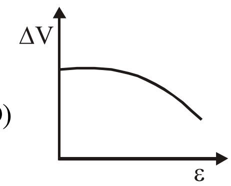


(C)


(D)


Ans. (A)

Sol. $v = iR$

i = constant

$v = i(R_0 + \Delta R)$

$R = \frac{\rho\ell}{A}$

$\frac{\Delta R}{R} = \rho\left(\frac{\Delta\ell}{\ell} - \frac{\Delta A}{A}\right)$

$\frac{\Delta A}{A} = -\frac{\Delta\ell}{\ell}$ if ρ = constant.

$\Delta R = \rho R\left(2\frac{\Delta\ell}{\ell}\right) = 2\rho R\varepsilon$

$V = i\,(R_0 + 2\rho R\varepsilon)$

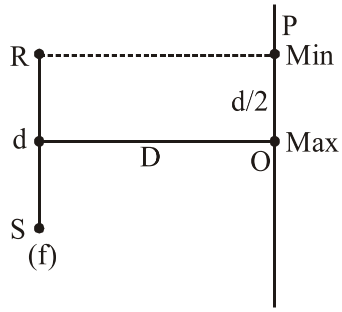
V will increase linearly with strain from an initial value.

30. Two identical coherent sound sources R and S with frequency f are 5 m apart. An observer standing equidistant from the sources and at a perpendicular distance of 12 m from the line RS hears maximum sound intensity. When he moves parallel to RS the sound intensity varies and is a minimum when he comes directly in front of one of the two sources. Then a possible value of f is close to (the speed of sound is 330 m/s)

(A) 495 Hz (B) 275 Hz
(C) 660 Hz (D) 330 Hz

Ans. (A)

Sol.



d = 5m
D = 12 m

$$\Delta r_p = d \cdot \frac{d/2}{D}$$

$$\Delta r_p = \frac{d^2}{2D} \quad \text{... (i)}$$

$$\Rightarrow \Delta\phi_p = \Delta r_p \cdot \frac{2\pi}{\lambda} = \frac{d^2}{2D} \cdot \frac{2\pi}{\lambda} \quad \text{.... (ii)}$$

$\Rightarrow$ At p there is minima

$\Rightarrow \Delta\phi_p = (2n + 1)\pi$ ... (iii)

$$\Rightarrow \frac{d^2}{2D} \cdot \frac{2}{\lambda} = (2n+1)$$

$$\Rightarrow \frac{d^2}{2D} = (2n+1)\frac{\lambda}{2} = (2n+1) \cdot \frac{v}{2f}$$

$$\Rightarrow f = \frac{(2n+1)v \cdot D}{d^2}$$

$$\Rightarrow f = (2n+1) \cdot 330 \times \frac{12}{25} = (2n+1)158 \cdot 4$$

For n = 1, f = 475.2 Hz

31. A photon falls through a height of 1 km through the earth's gravitational field. To calculate the change in its frequency, take its mass to be $h\nu/c^2$. The fractional change in frequency ν is close to

(A) 10^{-20} (B) 10^{-17}
(C) 10^{-13} (D) 10^{-10}

Ans. (C)

Sol. mgH = extra energy

$\Rightarrow$ final energy of photon

$$\Rightarrow h\nu' = h\nu + mgH \quad \text{... (i)}$$

$$\Rightarrow \frac{\nu' - \nu}{\nu} = \text{Ans} \quad \text{... (ii)}$$

$$\Rightarrow h\nu' = h\nu + \frac{h\nu}{c^2} \cdot gH$$

$$\Rightarrow \frac{\nu'}{\nu} = 1 + \frac{gH}{c^2}$$

$$\Rightarrow \frac{\nu'}{\nu} - 1 = \frac{gH}{c^2}$$

$$\Rightarrow \frac{\nu' - \nu}{\nu} = \frac{gH}{c^2}$$

$$= \frac{10 \times 1000}{\left[3 \times 10^8\right]^2} \simeq 1.12 \times 10^{-13}$$

32. 0.02 moles of an ideal diatomic gas with initial temperature 20°C is compressed from 1500 cm^3 to 500 cm^3. The thermodynamic process is such that $PV^2 = \beta$ where β is a constant. Then the value of β is close to:
(The gas constant, R = 8.31 J/K/mol)

(A) 7.5×10^{-2} Pa.m^6 (B) 1.5×10^2 Pa.m^6
(C) 3×10^{-2} Pa.m^6 (D) 2.2×10^1 Pa.m^6

Ans. (A)

Sol. n = 0.02, f = 5, T_1 = 20°C

$V_1 = 1500 \times 10^{-6}$, $V_2 = 500 \times 10^{-6}$

$$PV^2 = \beta, \quad \frac{nRT}{V} \cdot V^2 = \beta$$

$\Rightarrow$ nRTV = β

$\Rightarrow 0.02 \times 8.31 \times (273 + 20) \times 1500 \times 10^{-6} = \beta$

$\Rightarrow \beta = 0.073$

33. A heater supplying constant power P watts is switched on at time t = 0 minutes to raise the temperature of a liquid kept in a calorimeter of negligible heat capacity. A student records the temperature of the liquid T(*t*) at equal time intervals. A graph is plotted with T(*t*) on the y-axis versus *t* on the x-axis. Assume that there is no heat loss to the surroundings during heating. Then.

(A) the graph is a straight line parallel to the time axis.

(B) the heat capacity of the liquid is inversely proportional to the slope of the graph.

(C) if some heat were lost at a constant rate to the surroundings during heating, the graph would be a straight line but with a larger slope.

(D) the internal energy of the liquid increases quadratically with time.

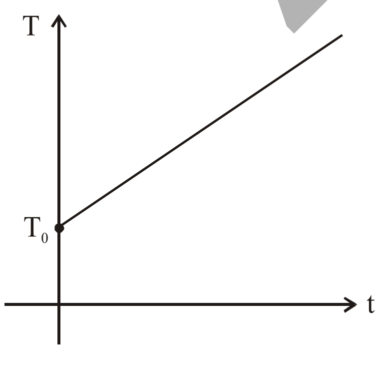
Ans. (B)

Sol. $P = ms\dfrac{dT}{dt}$

$dT = \dfrac{P}{ms}dt$

$\Rightarrow T = \dfrac{P}{ms}t + T_0$

T_0 = Temperature at t = 0



34. Unpolanzed red light is incident on the surface of a lake at incident angle θ_R. An observer seeing the light reflected from the water surface through a polarizer notices that on rotating the polarizer, the intensity of light drops to zero at a certain orientation. The red light is replaced by unpolarized blue light. The observer sees the same effect with reflected blue light at incident angle θ_B. Then,

(A) $\theta_B < \theta_R < 45°$ (B) $\theta_B = \theta_R$

(C) $\theta_B > \theta_R > 45°$ (D) $\theta_R > \theta_B > 45°$

Ans. (C)

Sol. By Cauchy's theorem we know that

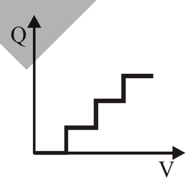
$\mu_{Red} < \mu_{Blue}$

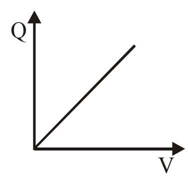
$\Rightarrow$ reflected light is polarized, so incidence angle must be equal to Brewster's angle.

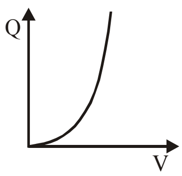
$\Rightarrow$ We know, $i_B = \tan^{-1}(\mu)$

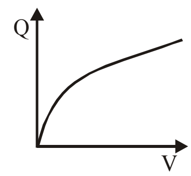
$\Rightarrow$ So, $\theta_R < \theta_B$

35. A neutral spherical copper particle has a radius of 10 nm (1 nm = 10^{-9} m). It gets charged by applying the voltage slowly adding one electron at a time. Then the graph of the total charge on the particle vs the applied voltage would look like:

(A) 

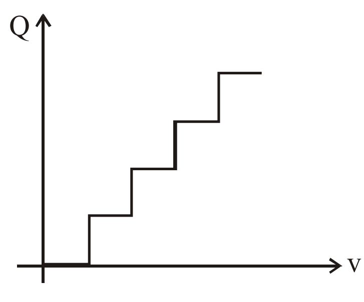
(B) 

(C) 

(D) 

Ans. (A)

Sol. $V = \dfrac{kQ}{r}$, since charge is increasing in discrete manner, so till the time charge is same on sphere, the potential will remain same so the graph will be



36. A charge +q is distributed over a thin ring of radius r with line charge density $\lambda = q\sin^2\theta/(\pi r)$. Note that the ring is in the x-y plane and θ is the angle made by $\vec{r}$ with the x-axis. The work done by the electric force in displacing a point charge +Q from the center of the ring to infinity is

(A) equal to $qQ/2\pi\varepsilon_0 r$.

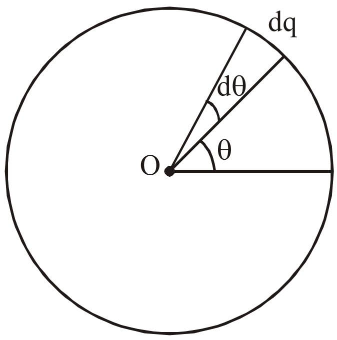
(B) equal to $qQ/4\pi\varepsilon_0 r$.

(C) equal to zero only if the path is a straight line perpendicular to the plane of the ring.

(D) equal to $qQ/8\pi\varepsilon_0 r$.

Ans. (B)

Sol.



Potential at O

$$\Rightarrow \int dV_0 = \int \frac{kdq}{r}$$

$$\Rightarrow V_0 = \int \frac{k}{r} \cdot \lambda \cdot Rd\theta$$

$$\Rightarrow V_0 = \int \frac{k}{r} \cdot \frac{q\sin^2\theta}{\pi r} \cdot rd\theta$$

$$\Rightarrow V_0 = \frac{kq}{r\pi}\int_0^{2\pi} \sin^2\theta d\theta \quad [\cos 2\theta = 1-2\sin^2\theta]$$

$$\Rightarrow V_0 = \frac{kq}{r\pi}\int_0^{2\pi}\left(\frac{1-\cos 2\theta}{2}\right)d\theta$$

$$\Rightarrow V_0 = \frac{kq}{r\pi}\left[\frac{1}{2}\cdot 2\pi - \frac{1}{2}\cdot\frac{1}{2}\left[\sin 2\theta\right]_0^{2\pi}\right]$$

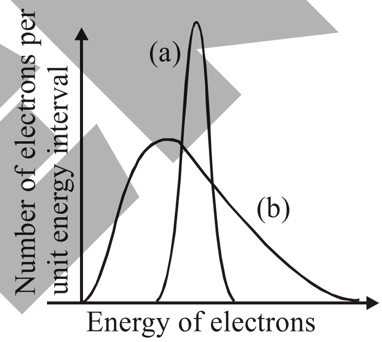
$$\Rightarrow V_0 = \frac{kq}{r\pi} \times \pi = \frac{kq}{r}$$

This can be directly stated also without any calculation

So, work done by external agent

$$\Rightarrow \Delta U = \frac{kqQ}{r} = \frac{qQ}{4\pi\varepsilon_0 r}$$

37. Originally the radioactive beta decay was thought as a decay of a nucleus with the emission of electrons only (Case I). However, in addition to the electron, another (nearly) massless and electrically neutral particle is also emitted (Case II). Based on the figure below, which of the following is correct:



(A) (a) in both cases I and II

(B) (a) in case I and (b) in case II.

(C) (a) in case II and (b) in case I.

(D) (b) in both cases I and II.

Ans. (B)

Sol. In case-I, no neutrino, or antineutrino is coming out so energy of β-particle will be same for all the decays.

In case-II, since neutrino or anti-neutrino is also coming out so energy of β-particle will become variable.

38. One gram-mole of an ideal gas A with the ratio of constant pressure and constant volume specific heats, $\gamma_A = 5/3$ is mixed with n gram-moles of another ideal gas B with $\gamma_B = 7/5$. If the γ for the mixture is 19/13 what will be the value of n?

(A) 0.75 (B) 2

(C) 1 (D) 3

Ans. (B)

Sol. Internal energy will remains conserved in the mixing process.

$$\Rightarrow \frac{f_1}{2}n_1RT + \frac{f_2}{2}n_2RT = \frac{f_{mix}}{2}(n_1+n_2)RT$$

$$\Rightarrow f_{mix} = \frac{f_1n_1+f_2n_2}{n_1+n_2} \qquad \gamma = 1+\frac{2}{f}$$

$$\Rightarrow f = \frac{2}{\gamma-1}$$

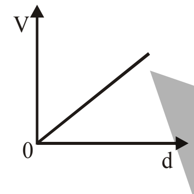
$$\Rightarrow \frac{2}{\gamma_{mix}-1} = \left(\frac{n_1}{n_1+n_2}\right)\cdot\left(\frac{2}{\gamma_1-1}\right)$$

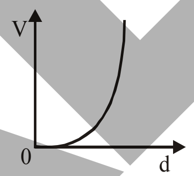
$$+\left(\frac{n_2}{n_1+n_2}\right)\left(\frac{2}{\gamma_2-1}\right)$$

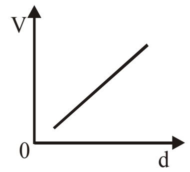
$$\Rightarrow \gamma_{mix} = \frac{19}{13}, \gamma_1 = \frac{5}{3}, \gamma_2 = \frac{7}{3}$$

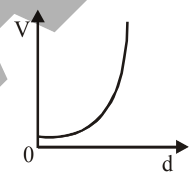
So, $n_2 = 2$

39. How will the voltage (V) between the two plates of a parallel plate capacitor depend on the distance (d) between the plates, if the charge on the capacitor remains the same ?

(A) 

(B) 

(C) 

(D) 

Ans. (C)

Sol. $Q = \frac{\varepsilon_0 A}{d}\cdot V$

$$\Rightarrow V = \left(\frac{Q}{\varepsilon_0 A}\right)d$$

The confusion comes between (A) and (C) options. Option (A) will be negated because value of d cannot be zero.

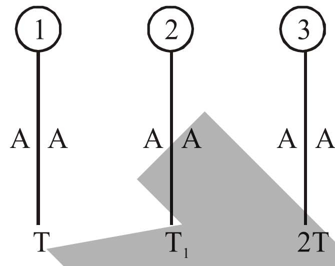
So answer is (C)

40. Three large identical plates are kept close and parallel to each other. The outer two plates are maintained at temperatures T and 2T, respectively. The temperature of the middle plate in steady state will be close to

(A) 1.1 T (B) 1.3T

(C) 1.7T (D) 1.9T

Ans. (C)

Sol. 

Assuming that all the plates are perfectly black bodies.

So in steady state,

Heat lost by (2) = Heat gained by (2)

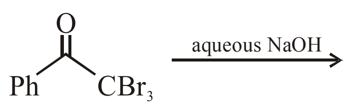
$$\Rightarrow \sigma(2A)T_1^4 = \sigma AT^4 + \sigma A(2T)^4$$

$$\Rightarrow 2T_1^4 = T^4 + 16T^4 = 17T^4$$

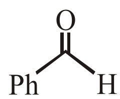
$$\Rightarrow T_1 = \left(\frac{17}{2}\right)^{1/4}\cdot T = 1.7T$$

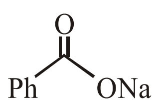
SECTION-3
PART-A
CHEMISTRY

41. The major products of the following reaction

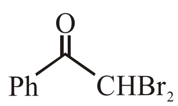


are

(A) Br_3C-OH and 

(B) 

and $CHBr_3$

(C) 

and NaBr

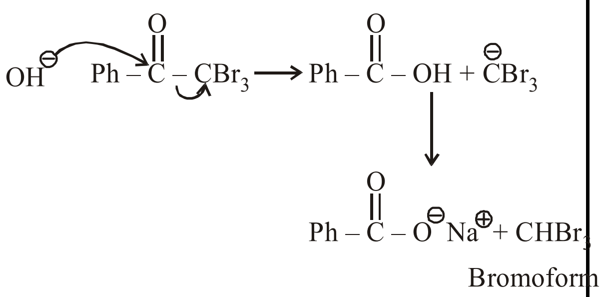
(D) PhH and CBr_3CO_2Na

Ans.(B)

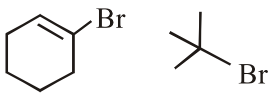
Sol. $Ph-\overset{\overset{O}{\|}}{C}-CBr_3 \xrightarrow{aq.NaOH}$

$\therefore$ $-CBr_3$ group is a leaving group

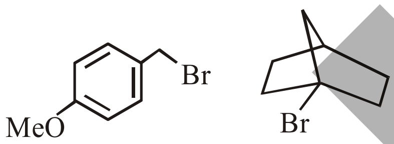
$\therefore$



42. Among the following ,



I **II**



III **IV**

the compounds which can undergo an S_N1 reaction in an aqueous solution, are

(A) I and IV only

(B) II and IV only

(C) II and III only

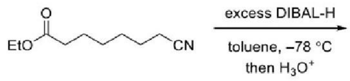
(D) II. III and IV only

Ans.(C)

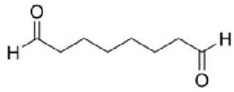
Sol. $\therefore$ S_N1 reaction is governed by stable carbocation.

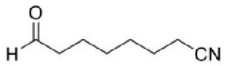
$\therefore$ II and III carbocation is stablised by hyper conjugation and resonance.

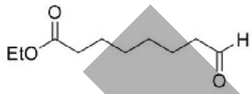
43. The major product of the following reaction

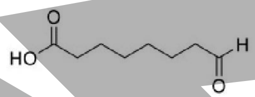


is

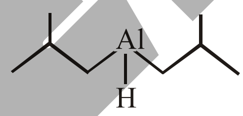
(A) 

(B) 

(C) 

(D) 

Ans.(A)

Sol. $\therefore$  Partial reduction takes

(DIBAL-H)

place

(Di isobutyl aluminium hydride)

44. Permanent hardness of water can be removed by

(A) heating

(B) treating with sodium acetate (CH_3CO_2Na)

(C) treating with $Ca(HCO_3)_2$

(D) treatment with sodium hexametaphosphate ($Na_6P_6O_{18}$)

Ans.(D)

Sol. Permanent hardness of water is removed by treatment with $Na_6P_6O_{18}$ (Calgon)

$Na_2[Na_4(PO_3)_6] + 2Ca^{+2}/Mg^{2+} \longrightarrow$

$$\underset{\text{soluble complex}}{Na_2[Ca_2/Mg_2(PO_3)_6]} + 4Na^+$$

45. Alkali metals (M) dissolve in liquid NH_3 to give

(A) MNH_2

(B) MH

(C) $[M(NH_3)_x]^+ + [e(NH_3)_y]^-$

(D) M_3N

Ans.(C)

Sol. Alkali metals dissolve in liquid ammonia to give ammoniated cation and ammoniated electron

$M_{(s)} + (x+y)NH_{3(\ell)} \longrightarrow [M(NH_3)_x]^+ + [e(NH_3)_y]^-$

46. The absolute configurations of the following compounds

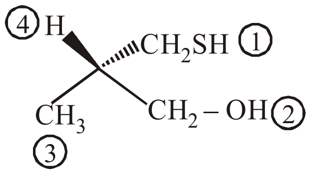
respectively, are

(A) R and R (B) S and S

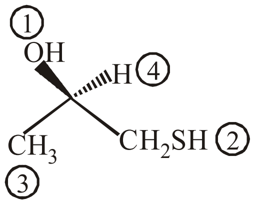
(C) R and S (D) S and R

Ans.(D)

Sol. $\therefore$ Priority decided by CIP rules



S form



R form

47. The diamagnctic species among the following is

(A) O_2^+ (B) O_2^-

(C) O_2 (D) O_2^{2-}

Ans.(D)

Sol. O_2^{2-} is diamagnetic in nature due to absence of unpaired electron in M.O. configuration.

Total electrons in O_2^{2-} = 18

$\sigma 1s^2\ \overset{*}{\sigma}1s^2,\ \sigma 2s^2\ \overset{*}{\sigma}2s^2, \sigma 2p_z^2\ \ \pi 2p_x^2 = \pi 2p_y^2$

$\overset{*}{\pi}2p_x^2 = \overset{*}{\pi}2p_y^2$

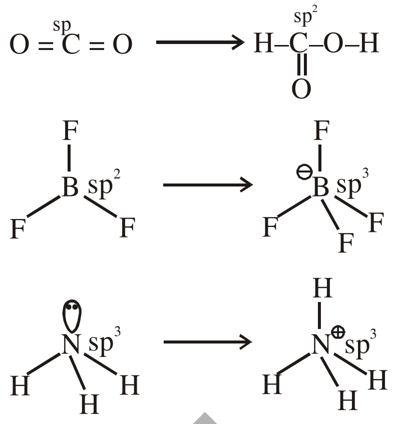
48. Among the following transformations, the hybridization of the central atom remains unchanged in

(A) $CO_2 \longrightarrow HCOOH$ (B) $BF_3 \longrightarrow BF_4^-$

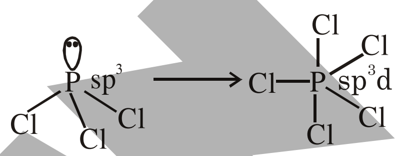
(C) $NH_3 \longrightarrow NH_4^+$ (D) $PCl_3 \longrightarrow PCl_5$

Ans.(C)

Sol.



do not change



49. For an octahedral complex MX_4Y_2 (M = a transition metal, X and Y are monodentate achiral ligands), the correct statement, among the following, is

(A) MX_4Y_2 has 2 geometrical isomers one of which is chiral

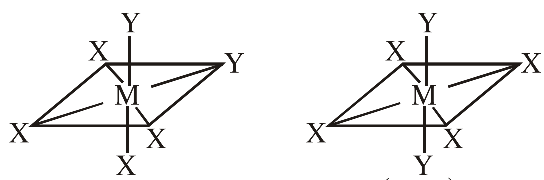
(B) MX_4Y_2 has 2 geometrical isomers both of which are achiral

(C) MX_4Y_2 has 4 geometrical isomers all of which are achiral

(D) MX_4Y_2 has 4 geometrical isomers two of which are chiral

Ans.(B)

Sol. $[MX_4Y_2]$ type octahedral complex contain two G.I. and both are achiral.



(cis) (trans)

50. The values of the Henry's law constant of Ar, CO_2, CH_4 and O_2 in water at 25°C are 40.30, 1.67, 0.41 and 34.86 kbar. respectively. The order of their solubility in water at the same temperature and pressure is

(A) $Ar > O_2 > CO_2 > CH_4$

(B) $CH_4 > CO_2 > Ar > O_2$

(C) $CH_4 > CO_2 > O_2 > Ar$

(D) $Ar > CH_4 > O_2 > CO_2$

Ans.(C)

Sol. $P_{gas} = K_H.X_{gas}$

X_{gas} (Mole fraction) $\propto$ solubility of gas $\propto \dfrac{1}{K_H}$

K_H value for Ar, CO_2, CH_4, O_2 are 40.30, 1.67, 0.41, 34.86 so solubility order is

$CH_4 > CO_2 > O_2 > Ar$

51. Thermal decomposition of N_2O_5 occurs as per the equation below

$2\ N_2O_5 \longrightarrow 4\ NO_2 + O_2$

The correct statement is

(A) O_2 production rate is four times the NO_2 production rate

(B) O_2 production rate is the same as the rate of disappearance of N_2O_5

(C) rate of disappearance of N_2O_5 is one-fourth of NO_2 production rate

(D) rate of disappearance of N_2O_5 is twice the O_2 production rate

Ans.(D)

Sol. $2N_2O_5 \longrightarrow 4NO_2 + O_2$

$$\text{rate} = \frac{1}{2}\left(\frac{-d[N_2O_5]}{dt}\right) = \frac{1}{4}\left(\frac{d[NO_2]}{dt}\right) = \frac{d[O_2]}{dt}$$

52. For a I[st] order chemical reaction.

(A) the product formation rate is independent of reactant concentration

(B) the time taken for the completion of half of the reaction ($t_{1/2}$) is 69.3% of the rate constant (k)

(C) the dimension of Arrhenius pre-exponential factor is reciprocal of time

(D) the concentration vs time plot for the reactanl should be linear with a negative slope

Ans.(C)

Sol. For I[st] order reaction

(A) Product formation rate depends upon reactant concentration.

(B) $t_{1/2} = \dfrac{0.693}{k} = 69.3\%$ of $\left(\dfrac{1}{k}\right)$

(C) $k = Ae^{-\frac{E_a}{RT}}$

Dimension of A is same as k (time^{-1})

(D) $A_t = A_0e^{-kt}$ (exponential)

53. The boiling point of 0.001 M aqueous solutions of NaCl, Na_2SO_4, K_3PO_4, and CH_3COOH should follows the order

(A) $CH_3COOH < NaCl < Na_2SO_4 < K_3PO_4$

(B) $NaCl < Na_2SO_4 < K_3PO_4 < CH_3COOH$

(C) $CH_3COOH < K_3PO_4 < Na_2SO_4 < NaCl$

(D) $CH_3COOH < K_3PO_4 < NaCl < Na_2SO_4$

Ans.(A)

Sol. $\Delta T_b \propto i.m$

$T_b' - T_b^0 \propto i.m \Rightarrow T_b' \propto i.m$

B.P. solution $\propto$ van't Hoff factor $\times$ conc.

T_b' order is.

$CH_3COOH < NaCl < Na_2SO_4 < K_3PO_4$

54. An allotrope of carbon which exhibits only two types of C–C bond distance of 143.5 pm and 138.3 pm, is

(A) charcoal (B) graphite

(C) diamond (D) fullerene

Ans.(D)

Sol. Fullerene exhibits two type of bond distances due to presence of two types of C–C single and C=C double bonds.

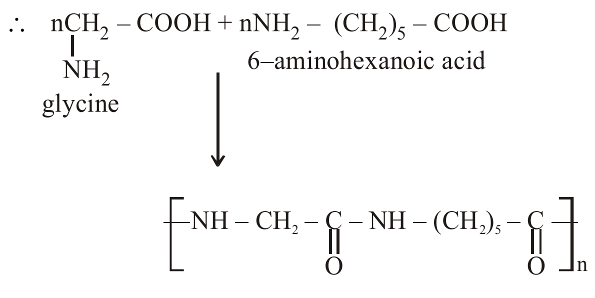
55. Nylon-2-nylon-6 is a co-polymer of 6-aminohexanoic acid and

(A) glycine (B) valine

(C) alanine (D) leucine

Ans.(A)

Sol.



Nylon-2-nylon-6

56. A solid is hard and brink. It is an insulator in solid state but conducts electricity in molten state. The solid is a

(A) molecular solid (B) ionic solid

(C) metallic solid (D) covalent solid

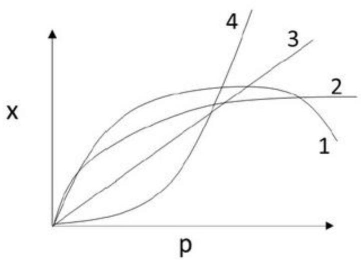
Ans.(B)

Sol. Ionic solids are hard and brittle.

Ionic solids conduct electricity in molten state due to presence of ions

eg. $NaCl_{(molten)} \longrightarrow Na^+ + Cl^-$

57. The curve that best describes the adsorption of a gas (X g) on 1.0 g of a solid substrate as a function of pressure (p) at a fixed temperature

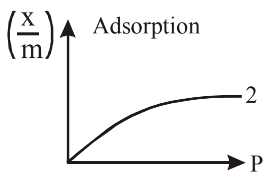


(A) 1 (B) 2

(C) 3 (D) 4

Ans.(B)

Sol.



58. The octahedral complex $CoSO_4Cl\cdot5\ NH_3$ exists in two isomeric fonns **X** and **Y**. Isomer **X** reacts with $AgNO_3$ to give a white precipitate, but does not react with $BaCl_2$. Isomer **Y** gives white precipitate with $BaCl_2$ but does not react with $AgNO_3$. Isomers **X** and **Y** are

(A) ionization isomers

(B) linkage isomers

(C) coordination isomers

(D) solvate isomers

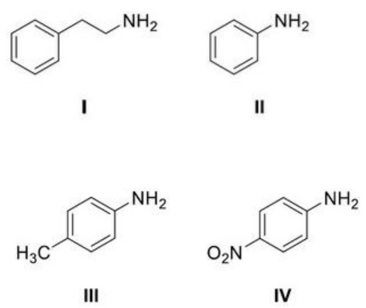
Ans.(A)

Sol. $\underset{(X)}{[Co(NH_3)_5SO_4]Cl} \xrightarrow{AgNO_3} \underset{\text{white ppt.}}{AgCl\downarrow}$

$\underset{(Y)}{[Co(NH_3)_5Cl]SO_4} \xrightarrow{BaCl_2} \underset{\text{white ppt.}}{BaSO_4\downarrow}$

[X] and [Y] are ionisation isomers.

59. The correct order of basicity of the following amines



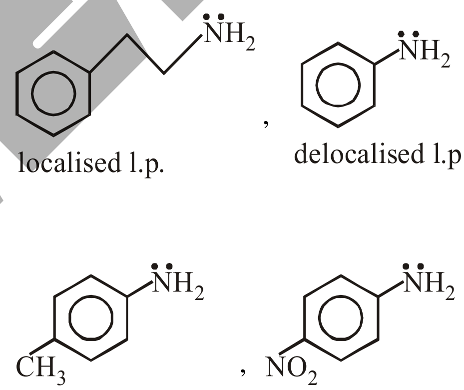
(A) I > II > III > IV (B) I > III > II > IV

(C) III > II > I > IV (D) IV > III > II > I

Ans.(B)

Sol. $\therefore$ Basic strength $\propto$ +M, +H, +I $\propto \dfrac{1}{-M,-I}$

$\therefore$



localised l.p., delocalised l.p.

delocalised l.p., delocalised l.p.

I > III > II > IV

60. Electrolysis of a concentrated aqueous solution of NaCl results in

(A) increase in pH of the solution

(B) decrease in pH of the solution

(C) O_2 liberation at the cathode

(D) H_2 liberation at the anode

Ans.(A)

Sol. $NaCl(conc.) \xrightleftharpoons{\text{Electrolysis}} H_2(g)\uparrow + Cl_2(g)\uparrow$

(Na^+, OH^-) NaOH increase in solution

so, solution becomes more basic

$[OH^-]\uparrow$; $[H+]\downarrow$; pH increase

SECTION-4

PART-A

BIOLOGY

61. Ethanol is used to treat methanol toxicity because ethanol

(A) is a competitive inhibitor of alcohol dehydrogenase

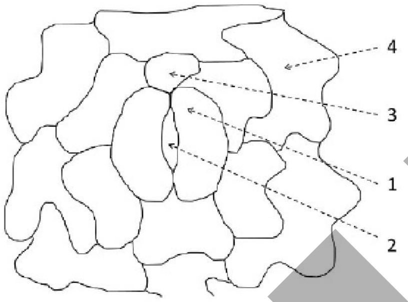
(B) is a non-competitive inhibitor of alcohol dehydrogenase

(C) activates enzymes involved in methanol metabolism

(D) inhibits methanol uptake by cells

Ans. (A)

62. Given below is a diagram of the stomatal apparatus. Match the labels with the corresponding of the components.



Choose the CORRECT combination.

(A) 1 – Stomatal pore; 2 –Guard cell; 3 – Epidermal cell; 4 – Subsidiary cell

(B) 1 – Guard cell; 2 – Stomatal pore: 3 – Subsidiary cell; 4 – Epidermal cell

(C) 1 – Subsidiary cell; 2 – Guard cell; 3 – Stomatal pore; 4 – Epidermal cell

(D) 1 – Guard cell; 2 – Stomatal pore; 3 – Epidermal cell; 4 – Subsidiary cell

Ans. (B)

63. Which one of the following pairs was excluded from Whittaker s five kingdom classification?

(A) Viruses and lichens

(B) Algae and euglena

(C) Lichens and algae

(D) Euglena and viruses

Ans. (A)

64. A plant species when grown in shade tends to produce thinner leaves with more surface area, and when grown under abundant sunlight starts producing thicker leaves with reduced surface area. This phenomenon is an example of

(A) character displacement

(B) phenotypic plasticity

(C) natural selection

(D) genotypic variation

Ans. (B)

65. Sacred groves found in several regions in India are an example

(A) in situ conservation

(B) ex situ conservation

(C) reintroduction

(D) restoration

Ans. (A)

66. Which one of the following immune processes is most effectively controlled by anti-histamines?

(A) Cell-mediated autoimmunity

(B) IgE-mediated exaggerated immune response

(C) IgG-mediated humoral immune response

(D) IgM-mediated humoral immune response

Ans. (B)

67. Which one of the following is explained by the endosymbiotic theory?

(A) The interaction between bacteria and viruses

(B) The symbiosis between plants and animals

(C) The origin of mitochondria and chloroplast

(D) The evolution of multicellular organisms from unicellular ones

Ans. (C)

68. According to the logistic population growth model, the growth rate is independent of

(A) per capita birth rate

(B) per capita death rate

(C) resource availability

(D) environmental fluctuations

Ans. (D)

69. A violent volcanic eruption wiped out most of the life forms in an island. Over time, different forms of simple organisms colonised this region, followed by the emergence of other organisms such as shrubs, woody plants, invertebrates and mammals. This ecological process is referred to as

(A) generation (B) replacement

(C) succession (D) turnover

Ans. (C)

70. Which one of the following microbial product is called "clot buster"?

(A) Cyclosporin A (B) Paracetamol

(C) Statins (D) Streptokinase

Ans. (D)

71. Which one of the following elements is NOT directly involved in transcription?

(A) Promoter (B) Terminator

(C) Enhancer (D) *OriC*

Ans. (D)

72. Which one of the following phyla is a pseudocoelomate?

(A) Cnidaria (B) Nematoda

(C) Mollusca (D) Chordate

Ans. (B)

73. Which one of the following glands does NOT secrete saliva?

(A) Submaxillary gland (B) Lacrimal gland

(C) Parotid gland (D) Sublingual gland

Ans. (B)

74. Which one of the following options correctly represents the tissue arrangement in roots?

(A) Cortex, pericycle. casparian strip, vascular bundle

(B) Pericycle. coitex. casparian strip, vascular bundle

(C) Cortex, casparian strip, pericycle. vascular bundle

(D) Casparian strip, pericycle. coitex. vascular bundle

Ans. (C)

75. During fermentation of glucose to ethanol glucose is

(A) first reduced and then oxidised

(B) only oxidised

(C) neither oxidised nor reduced

(D) only reduced

Ans. (C)

76. Which of the following is are the product(s) of cyclic photophosphorylation?

(A) Both NADPH and H^+

(B) NADPH

(C) ATP

(D) Both ATP and NADPH

Ans. (C)

77. Which one of the following amino acids is least likely to be in the core of a protein?

(A) Phenylalanine (B) Valine

(C) Isoleucine (D) Arginine

Ans. (D)

78. Which one of the following statements is a general feature of global species diversity?

(A) It increases from high to low latitudes

(B) It increases from low to high latitudes

(C) It changes over time but not spatially

(D) It changes randomly across space and time

Ans. (A)

79. Which one of the followmg conditions is NOT responsible for the presence of deoxygenated blood in the arteries of a newborn?

(A) Pneumonia

(B) Atrial septal defect

(C) Shunt between pulmonary artery and aorta

(D) Phenylketonuria

Ans. (D)

80. *Rhizobium* forms symbiotic association with roots in legumes and fixes atmospheric nitrogen. Which one of the following statement is CORRECT about this process?

(A) Activity of nitrogenase is sensitive to oxygen

(B) Activity of nitrogenase is insensitive to oxygen

(C) Anaerobic conditions allow ATP independent conversion of nitrogen to ammonia

(D) Under aerobic conditions, atmospheric nitrogen can be converted to nitrates by *Rhizobium*

Ans. (A)

SECTION-5

PART-B

MATHEMATICS

81. The points C and D on a semicircle with AB as diameter are such that AC = 1, CD = 2, and DB = 3

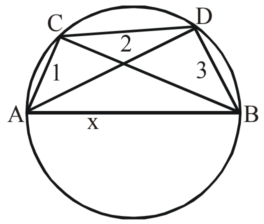
Then the length of AB lies in the interval

(A) [4,4.1) (B) [4.1,4.2)

(C) [4.2,4.3) (D) [4, 3, ∞)

Ans. (B)

Sol.



$BC=\sqrt{x^2-1}, AD=\sqrt{x^2-9}$

Apply ptolemy theoram we get

AB.CD + AC.BD = AD.BC

$2x+3=\sqrt{x^2-1}\sqrt{x^2-9}$

on squaring we get

$4x^2 + 12x + 9 = x^4 - 10x^2 + 9$

$x^4 - 14x^2 - 12x = 0 \Rightarrow x^3 - 14x - 12 = 0$

$f(x) = x^3 - 14x - 12$

$f(4.1) = -0.479$

$f(4.2) = 3.288$

$f(4.1).\ f(4.2) < 0$

Hence $x \in (4.1, 4.2)$

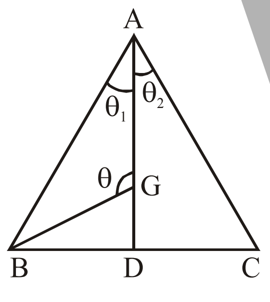
82. Let ABC be a triangle and let D be the midpoint of BC. Suppose cot($\angle$CAD): cot.($\angle$BAD) = 2:1. If G is the centroid of triangle ABC. then the measure of $\angle$BGA is

(A) 90° (B) 105°

(C) 120' (D) 135°

Ans. (A)

Sol.



$\cot\theta_2 = 2\cot\theta_1$

$$\frac{\cos\theta_2}{\sin\theta_2}=\frac{2\cos\theta_1}{\sin\theta_1}$$

$$\frac{AD^2+b^2-\frac{a^2}{4}}{2AD\,.\,b\sin\theta_2} = 2\left(\frac{AD^2+c^2-\frac{a^2}{4}}{2\,.\,AD\,.\,c\sin\theta_1}\right)$$

$$\frac{AD^2+b^2-\frac{a^2}{4}}{4\text{ area }(\Delta\text{ ADC})}=\frac{2\left(AD^2+c^2-\frac{a^2}{4}\right)}{4\text{ area }(\Delta\text{ ADC})}$$

$$AD^2 + b^2 - \frac{a^2}{4} = 2AD^2 + 2c^2 - \frac{a^2}{2}$$

$$\left(\text{use } AD=\frac{1}{2}\sqrt{2b^2+2c^2-a^2}\right)$$

$$\frac{a^2}{4}+b^2=2c^2+\frac{1}{4}(2b^2+2c^2-a^2)$$

$$\frac{a^2}{2}+\frac{b^2}{2}=\frac{5c^2}{2}\Rightarrow a^2+b^2=5c^2$$

$$\cos\theta = \frac{AG^2+BG^2-AB^2}{2AG\,.\,BG}$$

$$\cos\theta = \frac{\frac{1}{9}(2b^2+2c^2-a^2)+\frac{1}{9}(2a^2+2c^2-b^2)-c^2}{2AG\,.\,BG}$$

$$\cos\theta = \frac{\frac{1}{9}(a^2+b^2+4c^2)-c^2}{2AG\,.\,BG} = 0 \Rightarrow \theta = \frac{\pi}{2}$$

83. Let $f(x) = x^6 - 2x^5 + x^3 + x^2 - x - 1$ and $g(x) = x^4 - x^3 - x^2 - 1$ be two polynomials. Let a, b, c and d the roots of g(x) = 0. Then the value of f(a) + f(b) + f(c) + f(d) is :

(A) – 5 (B) 0

(C) 4 (D) 5

Ans. (B)

Sol. $g(x)=x^4-x^3-x^2-1=0$ (roots a, b, c, d)

$\sum a = 1$

$\sum ab = -1$

$(\sum a)^2=\sum a^2+2\sum ab \Rightarrow \sum a^2 = 3$

$f(x) = x^6 - 2x^5 + x^3 + x^2 - x - 1$

$x^2(x^4 - x^3 - x^2 - 1) - x(x^4 - x^3 - x^2 - 1) + 2x^2 - 2x - 1$

$f(a) = 2a^2 - 2a - 1$

$\sum f(a) = 2\sum a^2 - 2\sum a - 4 = 2(3) - 2(1) - 4 = 0$

84. Let $\vec{a}=\hat{i}+\hat{j}+\hat{k}$, $\vec{b}=2\hat{i}+2\hat{j}+\hat{k}$ and $\vec{c}=5\hat{i}+\hat{j}-\hat{k}$ be three vectors. The area of the region formed by the set of points whose position vector $\vec{r}$ satisfy the equations $\vec{r}.\vec{a}=5$ and $|\vec{r}-\vec{b}|+|\vec{r}-\vec{c}|=4$ is closest to the integer.

(A) 4 (B) 9

(C) 14 (D) 19

Ans. (A)

Sol. $\vec{r}.\vec{a}=5 \Rightarrow x+y+z=5$

$\left|\vec{r}-\vec{b}\right|+\left|\vec{r}-\vec{c}\right|=4$ ellipsoid

$\left|\vec{b}-\vec{c}\right|=\sqrt{14}$

with $\vec{b}$ and $\vec{c}$ lie on the plane x + y + z = 5

Now, $2ae=\sqrt{14}$ and 2a = 4

$$e=\frac{\sqrt{14}}{4}$$

$$e^2=1-\frac{b^2}{a^2}=\frac{14}{16}\Rightarrow b^2=\frac{1}{2}$$

Area of ellipse = $\pi ab = \pi \times 2 \times \frac{1}{\sqrt{2}}=\sqrt{2}\pi$

85. The number of solutions to $\sin(\pi \sin^2(\theta)) + \sin(\pi \cos^2(\theta))$

$$= 2\cos(\frac{\pi}{2}\cos(\theta))$$

Satisfying $0 \le \theta \le 2\pi$ is :

(A) 1 (B) 2

(C) 4 (D) 7

Ans. (D)

Sol. $\sin(\pi\sin^2\theta) + \sin(\pi\cos^2\theta) = 2\cos\left(\frac{\pi}{2}\cos\theta\right)$

$$2\sin\left(\frac{\pi}{2}\right)\cos\left(\frac{\pi\left(\cos^2\theta-\sin^2\theta\right)}{2}\right)$$

$$= 2\cos\left(\frac{\pi}{2}\cos\theta\right)$$

$$\cos\left(\frac{\pi}{2}\cos 2\theta\right) = \cos\left(\frac{\pi}{2}\cos\theta\right)$$

$$\frac{\pi}{2}\cos 2\theta = 2n\pi \pm \frac{\pi}{2}\cos\theta$$

$\cos 2\theta = 4n \pm \cos\theta$

C-1 $\cos 2\theta - \cos\theta = 4n$

only possiblity is $\cos 2\theta = \cos\theta$

$2\cos^2\theta - \cos\theta - 1 = 0$

$\cos\theta = 1, -\frac{1}{2}$

$\theta = 2n\pi, 2n\pi \pm \frac{2\pi}{3}$ $\quad \theta \in \left\{0, 2\pi, \frac{2\pi}{3}, \frac{4\pi}{3}\right\}$

C-2 $\cos 2\theta + \cos\theta = 4n$

only possiblity is $\cos 2\theta + \cos\theta = 0$

$2\cos^2\theta + \cos\theta - 1 = 0$

$\cos\theta = -1, \frac{1}{2}$

$\theta = (2n + 1)\pi, 2n\pi \pm \frac{\pi}{3}$

$\theta \in \left\{\pi, \frac{\pi}{3}, \frac{5\pi}{3}\right\}$

Total = 7 solutions

86. Let $J=\int_0^1 \frac{x}{1+x^8}dx$

Consider the following assertions :

(I) $J > \frac{1}{4}$ (II) $J < \frac{\pi}{8}$

Then

(A) Only I is true

(B) Only II is true

(C) Both I and II are true

(D) neither I nor II is true

Ans. (A)

Sol. $J = \int_0^1 \frac{x}{1+x^8}dx$

Now $1 + x^8 < 2$

$$\frac{1}{1+x^8} > \frac{1}{2} \Rightarrow \frac{x}{1+x^8} > \frac{x}{2}$$

$$\int_0^1 \frac{x}{1+x^8}dx > \int_0^1 \frac{x}{2}dx$$

$J > \frac{1}{4} \Rightarrow$ Statement I is true

$1 + x^4 > 1 + x^8$

$\frac{1}{1+x^8} > \frac{1}{1+x^4} \Rightarrow \frac{x}{1+x^8} > \frac{x}{1+x^4}$

$J > \int_0^1 \frac{x}{1+x^4}dx$ Put $x^2 = t$

$J > \frac{1}{2}\int_0^1 \frac{dt}{1+t^2} \Rightarrow J > \frac{\pi}{8} \Rightarrow$ II statement is not true

87. Let f: (–1, 1) → $\mathbb{R}$ be a differentiable function satisfying

$(f'(x))^4 = 16(f(x))^2$ for all $x \in (-1, 1)$,

$f(0) = 0$

The number of such functions is :-

(A) 2 (B) 3

(C) 4 (D) More than 4

Ans. (D)

Sol. $(f'(x))^4 = 16(f(x))^2$

$(f'(x))^2 = \pm 4 f(x)$

Case-I

$f(x) \geq 0$

$(f'(x)^2 = 4f(x)$

$f'(x) = \pm 2\sqrt{f(x)}$

$f'(x) = 2\sqrt{f(x)}$ or $f'(x) = -2\sqrt{f(x)}$

$f(x) = x^2, 1 > x \geq 0$ or $f(x) = x^2, -1 < x < 0$

Case-II

$f(x) < 0$

$(f'(x))^2 = -4f(x)$

$f'(x) = \pm 2\sqrt{-f(x)}$

$f'(x) = 2\sqrt{-f(x)}$ or $f'(x) = -2\sqrt{-f(x)}$

$f(x) = -x^2, -1 < x < 0$ or $f(x) = -(x)^2, 1 > x > 0$

Hence functions can be

$f(x) = x^2, -1 < x < 1$

$f(x) = -x^2, -1 < x < 1$

$f(x) = \begin{cases} x^2 & -1 < x < 0 \\ -x^2 & 0 \leq x < 1 \end{cases}$

$f(x) = \begin{cases} -x^2 & -1 < x < 0 \\ x^2 & 0 \leq x < 1 \end{cases}$

$f(x) = 0 \ \forall x \in (-1, 1)$

$f(x) = \begin{cases} x^2 & -1 < x < 0 \\ 0 & 0 \leq x < 1 \end{cases}$, more functions are also possible.

Hence number of such functions are more than 4.

88. For $x \in \mathbb{R}$, let $f(x) = |\sin x|$ and $g(x) = \int_0^x f(t)dt$. Let $p(x) = g(x) - \frac{2}{\pi}x$. Then

(A) $p(x + \pi) = p(x)$ for all x

(B) $p(x + \pi) \neq p(x)$ for at least one but finitely many x

(C) $p(x + \pi) \neq p(x)$ for infinitely many x

(D) p is a one–one function.

Ans. (A)

Sol. $f(x) = |\sin x|$

$g(x) = \int_0^x f(t)\,dt$ and $p(x) = g(x) - \frac{2x}{\pi}$

$g(x + \pi) = \int_0^{x+\pi} f(t)\,dt = \int_0^x f(t)\,dt + \int_x^{x+\pi} f(t)\,dt$

$g(x + \pi) = g(x) + \int_0^{\pi} f(t)\,dt$

$g(x + \pi) = g(x) + \int_0^{\pi} \sin t\,dt$

$g(x + \pi) = g(x) + 2 \Rightarrow g(x + \pi) - g(x) = 2$

$p(x + \pi) = g(x + \pi) - \frac{2}{\pi}(x + \pi)$ .... (1)

$p(x) = g(x) - \frac{2x}{\pi}$ .... (2)

subtract

$p(x + \pi) - p(x) = g(x + \pi) - g(x) - 2 = 0$

$p(x + \pi) = p(x)$ for all x

89. Let A be the set of vectors $\vec{a}=(a_1,a_2,a_3)$ satisfying

$$\left(\sum_{i=1}^{3}\frac{a_i}{2^i}\right)^2=\sum_{i=1}^{3}\frac{a_i^2}{2^i}$$

Then

(A) A is empty

(B) A contains exactly one element

(C) A has 6 elements

(D) A has infinitely meny elements.

Ans. (B)

Sol. $$\left(\sum_{i=1}^{3}\frac{a_i}{2^i}\right)^2=\sum_{i=1}^{3}\frac{a_i^2}{2^i}$$

$$\left(\frac{a_1}{2}+\frac{a_2}{2^2}+\frac{a_3}{2^3}\right)^2=\frac{a_1^2}{2}+\frac{a_2^2}{2^2}+\frac{a_3^2}{2^3}$$

$$\frac{a_1^2}{2^2}+\frac{3a_2^2}{2^4}+\frac{7a_3^2}{2^6}=\frac{a_1a_2}{2^2}+\frac{a_2a_3}{2^4}+\frac{a_1a_3}{2^3}$$

$16a_1^2 + 12a_2^2 + 7a_3^2 - 16a_1a_2 - 4a_2a_3 - 8a_1a_3 = 0$

$$\left(2\sqrt{2}a_1-2\sqrt{2}a_2\right)^2+\left(2\sqrt{2}a_1-\sqrt{2}a_3\right)^2+\left(2a_2-a_3\right)^2+4a_3^2=0$$

only possible when $a_1 = a_2 = a_3 = 0$

so only one element in the set.

90. Let f: [0, 1] → [0, 1] be a continuous function such that

$x^2 + (f(x))^2 \le 1$ for all $x \in [0, 1]$ and

$\int_0^1 f(x)dx=\frac{\pi}{4}$.

Then

$$\int_{\frac{1}{2}}^{\frac{1}{\sqrt{2}}}\frac{f(x)}{1-x^2}dx$$

equals

(A) $\frac{\pi}{12}$ (B) $\frac{\pi}{15}$

(C) $\frac{\sqrt{2}-1}{2}\pi$ (D) $\frac{\pi}{10}$

Ans. (A)

Sol. $y = f(x)$

$x^2 + y^2 \le 1 \quad \forall\, x \in [0, 1]$

$$y=\lambda\sqrt{1-x^2}$$

$$\lambda\int_0^1\sqrt{1-x^2}\,dx=\frac{\pi}{4}$$

$$\lambda\left(\frac{\pi}{4}\right)=\frac{\pi}{4}\Rightarrow\lambda=1$$

$$y=\sqrt{1-x^2}$$

$$\int_{\frac{1}{2}}^{\frac{1}{\sqrt{2}}}\frac{f(x)}{1-x^2}dx\Rightarrow\int_{\frac{1}{2}}^{\frac{1}{\sqrt{2}}}\frac{\sqrt{1-x^2}}{1-x^2}dx\Rightarrow\int_{\frac{1}{2}}^{\frac{1}{\sqrt{2}}}\frac{dx}{\sqrt{1-x^2}}$$

$$\left(\sin^{-1}x\right)_{\frac{1}{2}}^{\frac{1}{\sqrt{2}}}=\sin^{-1}\frac{1}{\sqrt{2}}-\sin^{-1}\frac{1}{2}=\frac{\pi}{4}-\frac{\pi}{6}=\frac{\pi}{12}$$

SECTION-6
PART-B
PHYSICS

91. A metal rod of cross-sectional area 10^{-4} m^2 is hanging in a chamber kept at 20°C with a weight attached to its free end. The coefficient of thermal expansion of the rod is 2.5×10^6 K^{-1} and its Young's modulus is 4×10^{12} N/m^2. When the temperature of the chamber is lowered to T then a weight of 5000 N needs to be attached to the rod so that its length is unchanged. Then T is

(A) 15°C (B) 12°C

(C) 5°C (D) 0°C

Ans. (A)

Sol. $$\frac{\Delta\ell}{\ell}=\alpha\Delta\theta$$

$$Y=\frac{\frac{F}{A}}{\frac{\Delta\ell}{\ell}}\Rightarrow\Delta\theta=\frac{F}{A\times\alpha\times Y}$$

$$\Rightarrow\ \Delta\theta=\frac{5000}{10^{-4}\times2.5\times10^{-6}\times4\times10^{12}}=5^0C$$

$\Rightarrow \Delta\theta = 20 - T = 5 \Rightarrow T = 15°C$

92. A short solenoid (length ℓ and radius r, with n turns per unit length) lies well inside and on the axis of a very long, coaxial solenoid (length L, radius R and N turns per unit length, with R > r). Current I flows in the short solenoid. Choose the correct statement.

(A) There is uniform magnetic field $\mu_0 nI$ in the long solenoid.

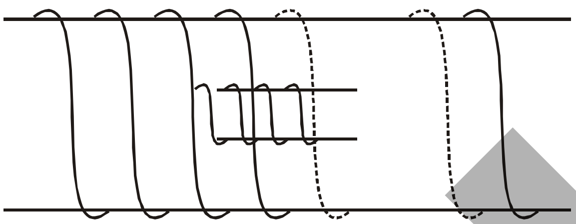
(B) Mutual inductance of the solenoids is $\pi\mu_0 r^2 nNI$.

(C) Flux through outer solenoid due to current I in the inner solenoid is proportional to the ratio R/r.

(D) Mutual inductance of the solenoids is $\pi\mu_0 rRnNIL/(rR)^{1/2}$.

Ans. (B)

Sol.



Mutual inductance

$\Rightarrow \mu_0 N \times n \times \ell\pi r^2$.

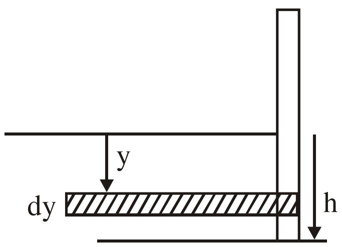
93. Consider the wall of a dam to be straight with height H and length L. It holds a lake of water of height h (h < H) on one side. Let the density of water be ρ_w Denote the torque about the axis along the bottom length of the wall by τ_1. Denote also a similar torque due to the water up to height h/2 and wall length L/2 by τ_2. Then τ_1/τ_2 (ignore atmospheric pressure) is

(A) 2 (B) 4

(C) 8 (D) 16

Ans. (D)

Sol.



$d\tau = (\rho g y)\, L\, dy \times (h - y)$

$$\tau = \rho g L\left(\frac{y^2}{2}\times h - \frac{y^3}{3}\right)_0^h$$

$$\tau = \rho g L\left(\frac{h^3}{2} - \frac{h^3}{3}\right)$$

$$\tau_1 = \rho g L\frac{h^3}{6}$$

$$\& \ \tau_2 = \frac{\rho g L_2 h_2^3}{6} = \frac{\rho g\left(\frac{L}{2}\right)\times\left(\frac{h}{2}\right)^3}{6}$$

$$\Rightarrow \frac{\tau_1}{\tau_2} = 16$$

94. Two containers CI and C2 of volumes V and 4V respectively hold the same ideal gas and are connected by a thin horizontal tube of negligible volume with a valve which is initially closed. The initial pressures of the gas in CI and C2 are P and 5P, respectively. Heat baths are employed to maintain the temperatures in the containers at 300 K and 400 K respectively. The valve is now opened. Select the correct statement:

(A) The gas will flow from the hot container to the cold one and the process is reversible.

(B) The gas will flow from one container to the other till the number of moles in two containers are equal.

(C) A long time after the valve is opened, the pressure in both the containers will be 3P.

(D) A long time after the valve is opened, number of moles of gas in the hot container will be thrice that of the cold one.

Ans. (D)

Sol. Finally $P_1 = P_2$

$$\frac{n_1RT_1}{V_1} = \frac{n_2RT_2}{V_2}$$

$$\frac{n_1 300}{V} = \frac{n_2\times 400}{4V} \Rightarrow \frac{n_1}{n_2} = \frac{1}{3}$$

95. Four electrons, each of mass m_e are in a one dimensional box of size L. Assume that the electrons are non-interacting, obey the Pauli exclusion principle and are described by standing de Broglie waves confined within the box. Define $\alpha = h^2/8m_eL^2$ and U_0 to be the ground state energy. Then

(A) The energy of the highest occupied state is 16α.

(B) $U_0 = 30\alpha$

(C) the total energy of the first excited state is $U_0 + 9\alpha$.

(D) The total energy of the second excited state is $U_0 + 8\alpha$.

Ans. (D)

Sol. $n\dfrac{\lambda}{2} = L$

$$P = \frac{h}{2L} \times n$$

$$E = \frac{P^2}{2m_e} = \frac{h^2n^2}{8L^2m_e} = n^2\alpha$$

$E_1 = \alpha = U_0$

$E_2 = 4\alpha$

$E_3 = 9\alpha \quad \Rightarrow \quad E_3 = U_0 + 8\alpha$

96. A rope of length L and uniform linear density is hanging from the ceiling. A transverse wave pulse, generated close to the free end of the rope, travels upwards through the rope. Select the correct option:

(A) The speed of the pulse decreases as it moves up.

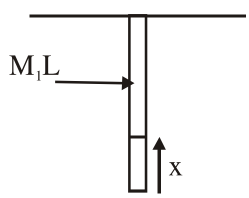
(B) The time taken by the pulse to travel the length of the rope is proportional to $\sqrt{L}$.

(C) The tension will be constant along the length of the rope.

(D) The speed of the pulse will be constant along the length of the rope.

Ans. (B)

Sol.



$$T = \frac{Mg}{L}x = \mu g x$$

$$\mu = \frac{M}{L}$$

$$\& \quad v = \sqrt{\frac{T}{\mu}} = \sqrt{gx} = \frac{dx}{dt}$$

$$\Rightarrow \int_0^t dt = \int_0^L \frac{dx}{\sqrt{gx}}$$

$$t = \frac{2}{\sqrt{g}}[x^{1/2}]_0^L = \frac{2}{\sqrt{g}}\left(\sqrt{L} - 0\right)$$

97. A circuit consists of a coil with inductance L and an uncharged capacitor of capacitance C. The coil is in a constant uniform magnetic field such that the flux through the coil is Φ. At time t = 0, the magnetic field is abruptly switched off. Let $w_0 = 1/\sqrt{LC}$ and ignore the resistance of the circuit. Then

(A) current in the circuit is $I(t) = (\Phi/L)\cos \omega_0 t$.

(B) magnitude of the charge on the capacitor is $|Q(t)| = 2C\omega_0|\sin \omega_0 t|$.

(C) initial current in the circuit is infinite.

(D) initial charge on the capacitor is $C\omega_0\Phi$.

Ans. (A)

Sol. $-\dfrac{q}{C} = L\dfrac{di}{dt}$

$$Li_0 - 0 = \phi \Rightarrow i_0 = \frac{\phi}{L}$$

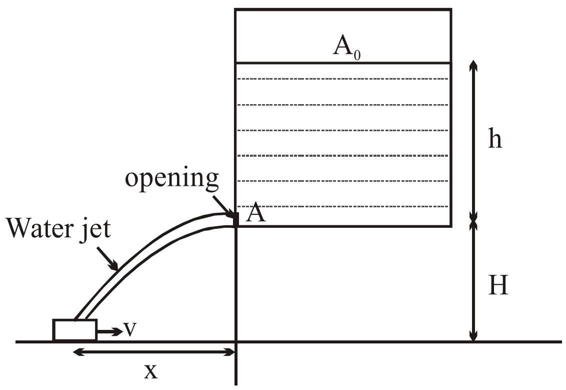
$$\Rightarrow \frac{d^2q}{dt^2} = -\frac{q}{LC}$$

$q = q_0 \sin\omega_0 t$

$\& \ i = q_0\omega \cos\omega_0 t$

$$i = \frac{\phi}{L}\cos\omega_0 t$$

98. Consider the configuration of a stationary water tank of cross section area A_0, and a small bucket as shown in figure below:



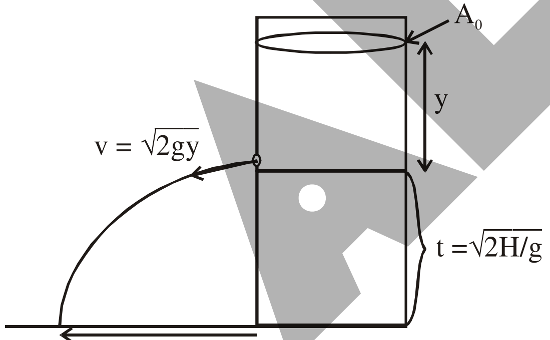
What should be the speed, v, of the bucket so that the water leaking out of a hole of cross-section area A (as shown) from the water tank does not fall outside the bucket? Take $h = 5m$, $H = 5m$, $g = 10\ m/s^2$, $A = 5\ cm^2$ and $A_0 = 500\ cm^2$.

(A) 1 m/s (B) 0.5 m/s

(C) 0.1 m/s (D) 0.05 m's

Ans. (C)

Sol.



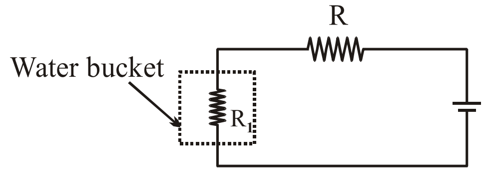
$x = v \times t \quad \Rightarrow \quad x = \sqrt{4yH}$

$$v = \frac{dx}{dt} = \sqrt{4\frac{H}{y}} \times \frac{1}{2} \times \frac{dy}{dt}$$

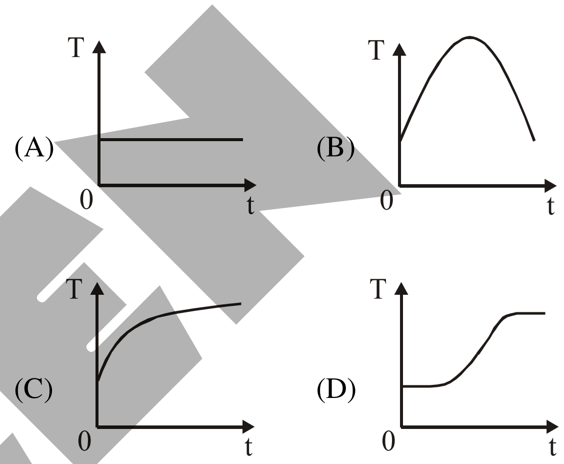
$$\frac{dy}{dt} = \sqrt{2gy} \times \frac{A}{A_0}$$

Now $v = \sqrt{\frac{H}{y}} \times \sqrt{2gy} \times \frac{A}{A_0}$ = 0.1 m/sec

99. The circuit below is used to heat water kept in a bucket.



Assuming heat loss only by Newton's law of cooling, the variation in the temperature of the water in the bucket as a function of time is depicted by:



Ans. (C)

Sol. $\frac{dQ}{dt} = ms\frac{dT}{dt} \propto (T - T_s)$

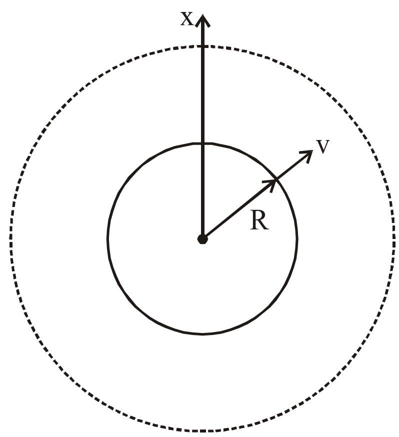
100. A bubble of radius R in water of density ρ is expanding uniformly at speed v. Give that water is incompressible, the kinetic energy of water being pushed is :

(A) Zero (B) $2\pi\rho R^3v^2$

(C) $2\pi\rho R^3v^2/3$ (D) $4\pi\rho R^3v^2/3$

Ans. (B)

Sol.



Velocity flux is same

$4\pi R^2 v = 4\pi x^2 v_x$

$$v_x = \frac{R^2 v}{x^2}$$

$$dK = \frac{1}{2} dm v_x^2$$

$$= \frac{1}{2}\rho \times 4\pi x^2 dx \times \frac{R^4 v^2}{x^4}$$

$$K = 2\pi\rho R^4 v^2 \int_R^\infty \frac{dx}{x^2}$$

$$= 2\pi\rho R^4 v^2 \left[-\frac{1}{R} \right]_R^\infty$$

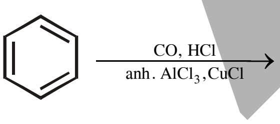
$= 2\pi\rho R^3 v^2$

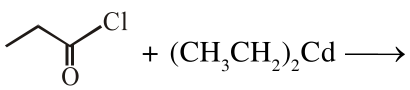
SECTION-7

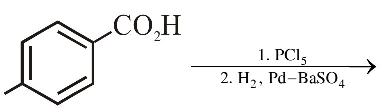
PART-B

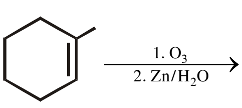
CHEMISTRY

101. The product of which of the following reactions forms a reddish brown precipitate when subjected to Fehling's test ?

(A) 

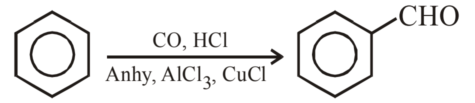
(B) 

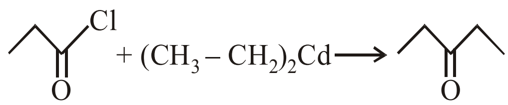
(C) 

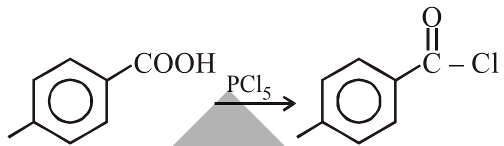
(D) 

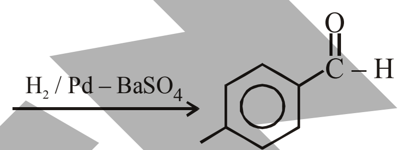
Ans.(D)

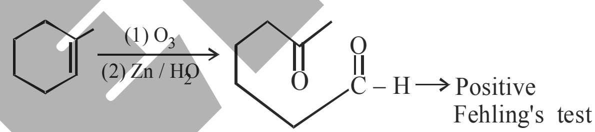
Sol. Ketones and aromatic aldehydes do not show positive Fehling's test.

(A) 

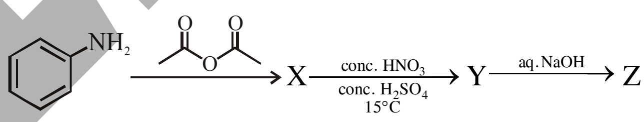
(B) 

(C) 

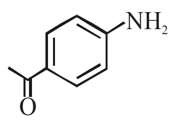
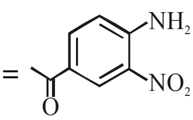


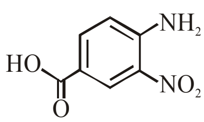
(D) 

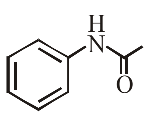
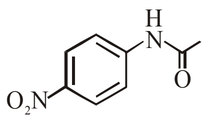
102. The major product **X, Y** and **Z** in the following sequence of transformations

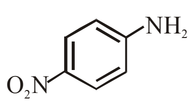


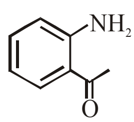
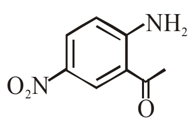
are :

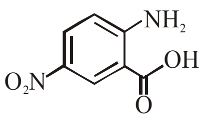
(A) X =  ; Y =  ;

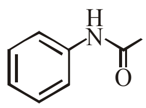
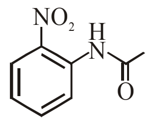
Z = 

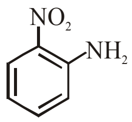
(B) X =  ; Y =  ;

Z = 

(C) X =  ; Y =  ;

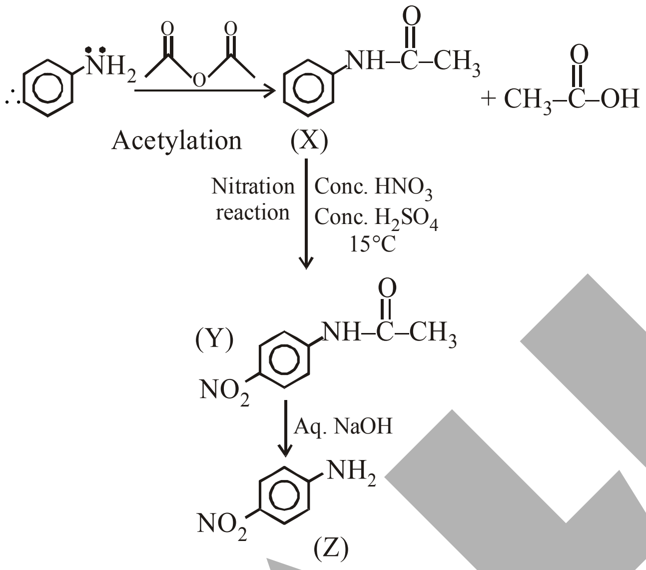
Z = 

(D) X =  ; Y =  ;

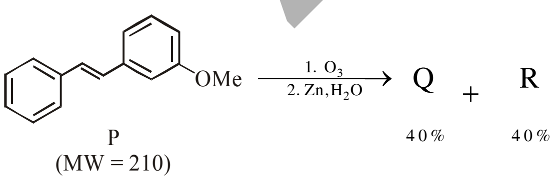
Z = 

Ans.(B)

Sol.



103. In the following reaction, **P** gives two products **Q** and **R**, each in 40% yield :



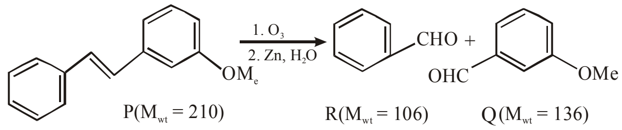
If the reaction is carried out with 420 mg of **P**, the reaction yields 108.8 mg of **Q**. The amount of R produced in the reaction is closed to :

(A) 97.6 mg (B) 108.8 mg

(C) 84.8 mg (D) 121.6 mg

Ans.(C)

Sol.



$n_p = \frac{420\text{ mg}}{210} = 2\text{ m moles}$ $\quad n_R = \frac{40}{100} \times 2\text{ m moles}$ $\quad n_Q = \frac{40}{100} \times 2\text{ m moles}$

mass of Q = $n_Q \times M_{wt}$

$108.8\text{ mg} = \frac{40}{100} \times 2\text{m. moles} \times M_{wt}$

$M_{wt} = 136 \Rightarrow$ this is Q

Mass of R = $n_R \times M_{wt}$

$= \frac{40}{100} \times 2\text{m. moles} \times 106$

= 84.8 mg

104. Solubility products of CuI and Ag_2CrO_4 have almost the same value ($\sim 4 \times 10^{-12}$). The ratio of solubilities of the two salts (CuI : Ag_2CrO_4) is closest to :

(A) 0.01 (B) 0.02

(C) 0.03 (D) 0.10

Ans.(B)

Sol. $K_{sp}(CuI) = S_1^2 = 4 \times 10^{-12} \Rightarrow S_1 = 2 \times 10^{-6}$

$K_{sp}(Ag_2CrO_4) = 4S_2^3 = 4 \times 10^{-12} \Rightarrow S_2 = 10^{-4}$

$$\frac{\text{Solubility(CuI)}}{\text{Solubility}(Ag_2CrO_4)} = \frac{S_1}{S_2} = \frac{2\times10^{-6}}{10^{-4}} = 0.02$$

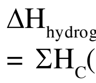
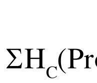
105. Given that the molar combustion enthalpy of benzene, cyclohexane and hydrogen are x, y and z respectively, the molar enthalpy of hydrogenation of benzene to cyclohexane is :

(A) x – y + z (B) x – y + 3z

(C) y – x + z (D) y – x + 3z

Ans.(B)

Sol. Hydrogenation of benzene

 $+ 3H_2 \longrightarrow$  ; $\Delta H = \Delta H_{hyd.}$

$\Delta H_{hydrogenation} = \Delta H$

$= \Sigma H_C(\text{Reactant}) - \Sigma H_C(\text{Product})$

$= H_C(\text{Benzene}) +$

$3 \times \Delta H_C(H_2) - \Delta H_C(\text{cyclohexane})$

$= x + 3z - y$

$= x - y + 3z$

106. Among the following, the pair of paramagnetic complexes is :

(A) $K_3[Fe(CN)_6]$ and $K_3[CoF_6]$
(B) $K_3[Fe(CN)_6]$ and $[Co(NH_3)_6]Cl_3$
(C) $K_4[Fe(CN)_6]$ and $K_3[CoF_6]$
(D) $K_4[Fe(CN)_6]$ and $[Co(NH_3)_6]Cl_3$

Ans.(A)

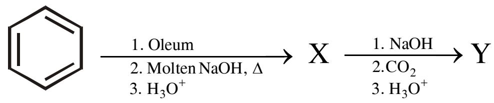
Sol. In $K_3[Fe(CN)_6]$, $Fe^{3+} \rightarrow [Ar]3d^5 4s^0$

d^2sp^3 hybridisation and paramagnetic due to presence of unpaired electron.

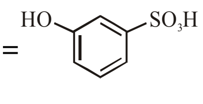
In $K_3[CoF_6]$, $Co^{+3} \rightarrow [Ar]3d^6 4s^0$

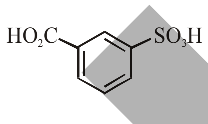
sp^3d^2 hybridisation, F^- is weak field ligand so no pairing of electrons take place therefore it is paramagnetic

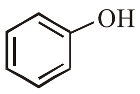
107. The major product **X** and **Y** in the following sequence of transformations :

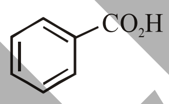


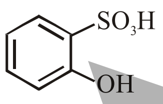
are :

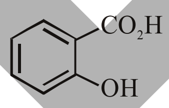
(A) X = 

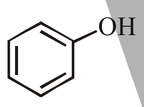
; Y = 

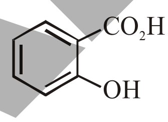
(B) X = 

; Y = 

(C) X = 

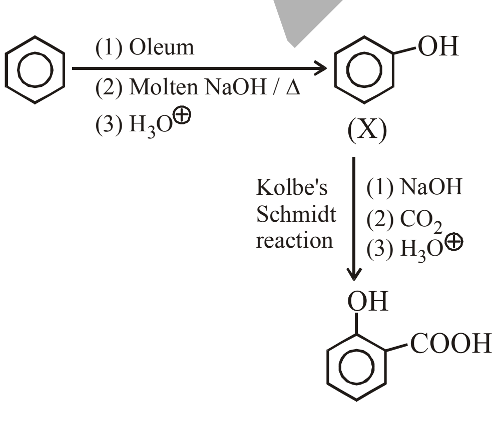
; Y = 

(D) X = 

; Y = 

Ans. (D)

Sol.



108. 3.0 g of oxalic acid $[(CO_2H)_2.2H_2O]$ is dissolved in a solvent to prepare a 250 mL solution. The density of the solution is 1.9 g/mL. The molality of normality of the solution, respectively, are closest to :

(A) 0.10 and 0.38
(B) 0.10 and 0.19
(C) 0.05 and 0.19
(D) 0.05 and 0.09

Ans.(C)

Sol. 3 gm of oxalic acid $[(CO_2H)_2.2H_2O]$

$n = \frac{3}{126}$ Mole

Volume of solution = 250 ml

Density of solution = 1.9 gm/ml

Mass of 250 ml solution = $1.9 \times 250 = 475$gm

Mass of solvent present in 250 ml solution

$= 475 - 3 = 472$ gm

472 gm solvent has $\frac{3}{126}$ Mole

$\therefore$ 1000 gm solvent has $\frac{3}{126} \times \frac{1000}{472} \simeq 0.05$

$\Rightarrow$ Molality = 0.05 m

$\frac{3}{126}$ mole oxalic acid

$= \frac{3}{63}$ gm eq of oxalic acid

$= \frac{1}{21}$ gm eq of oxalic acid

250 ml solution has = $\frac{1}{21}$ gm eq of oxalic acid.

1000 ml solution has $\frac{4}{21}$ gm eq. of oxalic acid.

$\Rightarrow$ Normality = $\frac{4}{21} N \simeq 0.19N$

109. In a titration experiment, 10 mL of an $FeCl_2$ solution consumed 25 mL of a standard $K_2Cr_2O_7$ solution to reach the equivalent point. The standard $K_2Cr_2O_7$ solution to prepared by dissolving 1.225 g of $K_2Cr_2O_7$ in 250 mL water. The concentration of the $FeCl_2$ solution is closest to :

[Given : molecular weight of $K_2Cr_2O_7$ = 294 g mol^{-1}]

(A) 0.25 N (B) 0.50 N

(C) 0.10 N (D) 0.04 N

Ans.(A)

Sol. With the action of $K_2Cr_2O_7$, the Fe^{2+} ions present in $FeCl_2$ solution is converted to Fe^{3+} ions.

$[Fe^{2+} \rightarrow Fe^{3+} + e^-] \times 6$

$[6e^- + K_2Cr_2O_7 \rightarrow 2Cr^{3+}] \times 1$

Molarity of standard $K_2Cr_2O_7$ solution

$$= \frac{1.225/294}{250/1000} = \frac{0.05}{3}$$

Normality of $K_2Cr_2O_7$ solution

$$= \frac{0.05}{3} \times 6 = 0.1N$$

$N_1V_1 = N_2V_2$

$(K_2Cr_2O_7)$ $(FeCl_2)$

$0.1 \times 25 = N_2 \times 10$

$\Rightarrow N = 0.25$ N

110. Atoms of an element Z form hexagonal closed pack (hcp) lattice and atoms of element X occupy all the tetrahedral voids. The formula of the compound is :

(A) XZ (B) XZ_2

(C) X_2Z (4) X_4Z_3

Ans.(C)

Sol. Z form hcp lattice

let no. of atom of Z is N

no. of THV = 2N

no. of OHV = N

X lies at all THV so

no. of atoms of X is 2 N

so, X : Z = 2N : N

= 2 : 1

so lattice is X_2Z

SECTION-8

PART-B

BIOLOGY

111. In a population, N_{AA} and N_{aa} are the number of homozygous individuals of allele 'A' and 'a', respectively, and N_{Aa} is the number of heterozygous individuals. Which one of the following options is the allele frequency of 'A' and 'a' in a population with N_{AA} = 90, N_{Aa} = 40 and N_{aa} = 70?

(A) A = 0.55 and a = 0.45

(B) A = 0.40 and a = 0.60

(C) A = 0.35 and a = 0.65

(D) A = 0.25 and a = 0.75

Ans. (A)

112. A newly discovered organism possesses a genetic material with a new base composition consisting of the sugar and phosphate backbone as found in existing natural DNA. The give novel bases in this genetic material - namely, P, Q, R, S, T - are heterocyclic structures with 1, 1, 2, 2 and 3 rings, respectively. Assuming the new DNA forms a double helix of uniform width, which of the following would be the most appropriate base pairing ?

(A) P with Q ; R with T ; S with T

(B) P with T ; R with S ; Q with T

(C) P with S ; Q with R ; S with T

(D) P with Q ; R with S ; S with T

Ans. (B)

113. Amino acid analysis of two globular protein samples yielded identical composition per mole. Which one of following characteristics is necessarily identical for the two proteins ?

(A) Disulphide bonds

(B) Primary structure

(C) Molecular mass

(D) Three-dimensional structure

Ans. (C)

114. Which of the following conversions in glycolysis is an example of substrate level phosphorylation?

(A) Glyceraldehyde-3-phosphate to 1,3-bisphosphoglycerate

(B) 1,3-bisphosphoglycerate to 3-phosphoglycerate

(C) Fructose-6-phosphate to fructose-1, 6-bisphosphate

(D) Glucose-6-phosphate to fructose-6-phosphate

Ans. (B)

115. A plant heterozygous for height and flower colour (TtRr) are selfed and 1600 of the resulting seeds are planted. If the distance between the loci controlling height and flower colour is 1 centimorgan, then how many offspring are expected to be short with white flower (ttrr) ?

(A) 1 (B) 10

(C) 100 (D) 400

Ans. (D)

Sol. Cis arrangement

Distance - 1cM

So recombination frequency is 1%.

TtRr

TR	Tr	tR	tr
49.5%	0.5%	0.5%	49.5%
P.C.			P.C.

		TR	Tr	tR	tr
	TR				
	Tr				
	tR				
49.50%	tr				ttrr (0.2450)

↑ 0.495×0.495

$0.245 \times 1600 = 392.04 \Rightarrow 400$

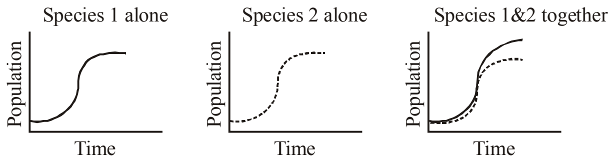
116. Which one of the following will be the ratio of heavy, intermediate and light bonds in meselson and Stahl's experiment after two generations if DNA replication were conservative ?

(A) 0 : 2 : 2 (B) 1 : 0 : 3

(C) 2 : 2 : 0 (D) 2 : 0 : 2

Ans. (B)

117. Given the graph below, the interaction between species 1 and 2 can be classified as :



(A) Amensalism (B) Commensalism

(C) Mutualism (D) Competition

Ans. (B)

118. The additional nuclear ploidy levels found in a diploid angiosperm species in full bloom compared to its vegetative stage are :

(A) 1N and 2N (B) 2N and 3N

(C) 3N and 4N (D) 1N and 3N

Ans. (D)

119. The bill sizes in a bird species of seedcrackers from West Africa shows a bimodal distribution. Their most abundant food sources are two types of marsh plants that produce hard and soft seeds, consumed preferentially by the large and small billed birds respectively. This bimodal distribution of bill sizes is a likely consequence of :

(A) Directional selection

(B) Stabilising selection

(C) Disruptive selection

(D) Sexual selection

Ans. (C)

120. The containers X and Y have 1 litre of pure water and 1 litre of 0.1 M sugar solution respectively. Which one of the following statements would be CORRECT regarding their water potential (ψ) and osmotic potential (ψ_S)

(A) Both ψ and ψ_S are zero in X

(B) Both ψ and ψ_S are zero in Y

(C) ψ in X is zero and ψ_S in Y is negative

(D) ψ in X is negative and ψ_S in Y is zero

Ans. (C,A)