KVPY QUESTION PAPER 2015
Class XII
Part-I
(One-Mark Questions)

[MATHEMATICS]

1. The number of ordered pairs (x, y) of real numbers that satisfy the simultaneous equations
$$x + y^2 = x^2 + y = 12 \text{ is}$$
(A) 0 (B) 1 (C) 2 (D) 4

Ans. (D)

Sol. $x^2 - y^2 = x - y$

$\Rightarrow$ $(x - y)(x + y) = x - y$

$\Rightarrow$ $x = y$ or, $x + y = 1$

$\Rightarrow$ $x + x^2 = 12$

$\Rightarrow$ $x = -4, 3$

The required ordered pairs are (–4, –4), (3, 3), & two irrational roots from $x^2 - x - 11 = 0$

Ans. (D)

2. If z is a comber satisfying $|z^3 + z^{-3}| \le 2$, then the maximum possible value of $|z + z^{-1}|$ is
(A) 2 (B) $\sqrt[3]{2}$ (C) $2\sqrt{2}$ (D) 1

Ans. (A)

Sol. $z^3 + z^{-3}$

Let, $|z + z^{-1}| = a$

$|(z + z^{-1})^3| = a^3$

$|z^3 + z^{-3} + 3(z + z^{-1})| = a^3$

$$a^3 \le \left|z^3 + \frac{1}{z^{-3}}\right| + 3a \le 3a + 2$$

$a^3 - 3a - 2 \le 0$

$a^3 - 8 - 3a + 6 \le 0$

$(a - 2)(a^2 + 2a + 4) - 3(a - 2) \le 0$

$(a - 2)(a + 1)^2 \le 0$

$$\Rightarrow \quad \left|z + \frac{1}{z}\right| = a \le 2$$

3. The largest perfect square that divides $2014^3 - 2013^3 + 2012^3 - 2011^3 + ... + 2^3 - 1^3$ is
(A) 1^2 (B) 2^2 (C) 1007^2 (D) 2014^2

Ans. (C)

Sol. $-(1^3 + 2^3 + 3^3 + ... + 2014)^3 + 2(2^3 + 4^3 + ... + 2014^3)$

$$= -\left(\frac{2014.2015}{2}\right)^2 + 2.2^3\left(\frac{1007.1008}{2}\right)^2$$

$= -(1007)^2 (2015)^2 + 4 . (1007)^2(1008)^2$

$= -(1007)^2 [(2016)^2 - (2015)^2]$

$= (1007)^2 . 4031$

4. Suppose OABC is a rectangle in the xy-plane where O is the origin and A, B lie on the parabola $y = x^2$. Then C must lie on the curve
(A) $y = x^2 + 2$ (B) $y = 2x^2 + 1$ (C) $y = -x^2 + 2$ (D) $y = -2x^2 + 1$

Ans. (A)

Sol. Point M $= t_1^2 + t_2^2 = 2k$ ... (1)

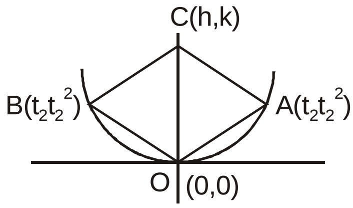
$M_{OA} \cdot m_{OB} = -1$

$t_1 \cdot t_2 = -1$

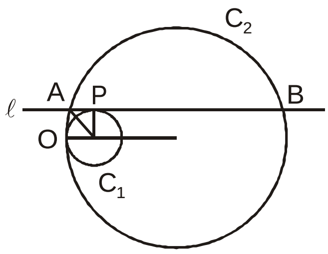
$k = (t_1 + t_2)^2 - 2t_1t_2$

$y = x^2 + 2$

Ans. (A)



5. Circles C_1 and C_2, of radii r and R respectively, touch each other as shown in the figure. The line ℓ, which is parallel to the line joining the centres of C_1 and C_2, is tangent to C_1 at P and intersects C_2 at A, B. If $R^2 = 2r^2$, then $\angle AOB$ equals



(A) $22\frac{1^o}{2}$ (B) 45º (C) 60º (D) $67\frac{1^o}{2}$

Ans. (B)

Sol. Equation of AB y = r :

$A\left(R - \sqrt{R^2 - r^2}, r\right)$ using $R^2 = 2r^2$

$B\left(R + \sqrt{R^2 - r^2}, r\right)$

point A(R – r, r) B(R + r, r)

slope OA $= \frac{r}{R-r} = m_1$

slope OB $= \frac{r}{R+r} = m_2$

$\tan\theta = \frac{m_1 - m_2}{1 + m_1.m_2} = 1$

$\theta = \frac{\pi}{4} = 45^o$

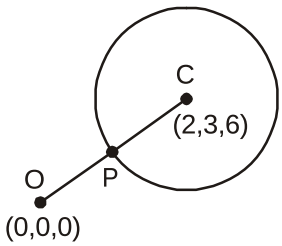
6. The shortest distance from the origin to a variable point on the sphere

$(x-2)^2 + (y-3)^2 + (z-6)^2 = 1$ is

(A) 5 (B) 6 (C) 7 (D) 8

Ans. (B)

Sol. Shortest distance $|OC - r| = \left|\sqrt{49} - 1\right| = 6$



7. The number of real numbers λ for which the equality

$$\frac{\sin(\lambda\alpha)}{\sin\alpha} - \frac{\cos(\lambda\alpha)}{\cos\alpha} = \lambda - 1,$$

holds for all real α which are not integral multiples of $\frac{\pi}{2}$ is

(A) 1 (B) 2 (C) 3 (D) Infinite

Ans. (C)

Sol. $\frac{\sin\lambda\alpha.\cos\alpha - \cos\lambda\alpha\sin\alpha}{\sin\alpha\cos\alpha} = \lambda - 1$

$\frac{\sin(\lambda\alpha - \alpha)}{\sin\alpha\cos\alpha} = \lambda - 1$

$\frac{\sin(\alpha)(\lambda-1)}{\sin\alpha\cos\alpha} = \lambda - 1$

$\Rightarrow \frac{2\sin\alpha(\lambda-1)}{\sin 2\alpha} = \lambda - 1$

$\lambda = 1$

$\Rightarrow 2\sin\alpha(\lambda - 1) = (\lambda - 1)\sin 2\alpha$

$\lambda - 1 = 2 \Rightarrow \lambda = 3$

$\lambda - 1 = -2 \Rightarrow \lambda = -1$

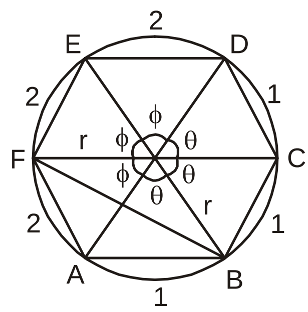
8. Suppose ABCDEF is a hexagon such that AB = BC = CD = 1 and DE = EF = FA = 2. If the vertices A, B, C, D, E, F are concyclic, the radius of the circle passing through them is

(A) $\sqrt{\frac{5}{2}}$ (B) $\sqrt{\frac{7}{3}}$ (C) $\sqrt{\frac{11}{5}}$ (D) $\sqrt{2}$

Ans. (B)

Sol. $\phi + \theta = 120^\circ$

$\angle A = 120^\circ$



$\cos A = \cos 120^\circ = \frac{1^2 + 2^2 - FB^2}{2(1.2)}$

$\Rightarrow FB = \sqrt{7}$

Cosine Rule :

$\cos(\theta + \phi) = \frac{r^2 + r^2 - 7}{2r^2}$

$-\frac{1}{2} = \frac{2r^2 - 7}{2r^2}$

$r = \sqrt{\frac{7}{3}}$

9. Let p(x) be a polynomial such that p(x) – p′(x) = x″, where n is a positive integer. Then p(0) equals

(A) n! (B) (n – 1)! (C) $\frac{1}{n!}$ (D) $\frac{1}{(n-1)!}$

Ans. (A)

Sol. $P(x) = \left(1 + \frac{x}{1!} + \frac{x^2}{2!} ..... \frac{x^n}{n!}\right)n!$

p(0) = n!

10. The value of the limit

$$\lim_{x\to 0}\left(\frac{x}{\sin x}\right)^{6/x^2} \text{ is}$$

(A) e (B) e^{-2} (C) $e^{-1/6}$ (D) e^{6}

Ans. (A)

Sol. $\lim_{x\to 0}\left(\frac{x}{\sin x}\right)^{6/x^2}$

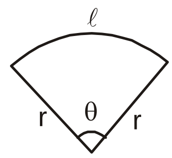
$$= e^{\lim_{x\to 0}\left(\frac{x}{\sin x}-1\right)\frac{6}{x^2}} = e^{\lim_{x\to 0}\left(\frac{x-\sin x}{\sin x}\right)\frac{6}{x^2}} = e^{\lim_{x\to 0}\left(\frac{6}{3!}\right)} = e$$

11. Among all sectors of a fixed perimeter, choose the one with maximum area. Then the angle at the center of this sector (i.e., the angle between the bounding radii) is

(A) $\frac{\pi}{3}$ (B) $\frac{3}{2}$ (C) $\sqrt{3}$ (D) 2

Ans. (D)

Sol. $\theta \,.\, r + 2r = 2(\cos t)$



Area of secter $= \left(\frac{r^2}{2}\right)\theta$

$$= \frac{r^2}{2}\left[\frac{k-2r}{r}\right]$$

$$A = \frac{1}{2}\left[kr - 2r^2\right]$$

$$\frac{dA}{dr} = \frac{1}{2}\left[k-4r\right] = 0$$

$$r = \frac{k}{4} \Rightarrow \theta = 2$$

12. Define a function $f:\mathbb{R}\to\mathbb{R}$ by $f(x) = \max\{|x|, |x-1|, \ldots, |x-2n|\}$,

where n is a fixed natural number. Then $\int_0^{2n} f(x)\,dx$ is

(A) n (B) n^2 (C) 2n (D) $3n^2$

Ans. (D)

Sol. $\int_0^{2n} f(x)\,dx$

$$= 2\int_0^{n}(2n-x)\,dx$$

$$= 2\left[2n^2 - \frac{n^2}{2}\right] = 2\left(\frac{3n^2}{2}\right)$$

$= 3n^2$

13. If p(x) is a cubic polynomial with p(1) = 3, p(0) = 2 and p(–1) = 4, then $\int_{-1}^{1} p(x)\,dx$ is

(A) 2 (B) 3 (C) 4 (D) 5

Ans. (D)

Sol. $P(x) = ax^3 + bx^2 + cx + d$

$$d = 2 \ \& \ b = \frac{3}{2}$$

$$\int_{-1}^{1} P(x)dx$$

$$= 2\int_{-1}^{1}\left(bx^2 + d\right)$$

$$= 2\left[\frac{bx^3}{3} + dx\right]_0^1$$

$$= 2\left[\frac{b}{3} + d\right]$$

$$= 5$$

14. Let $x > 0$ be a fixed real number. Then the integral $\int_0^{\infty} e^{-1}|x-t|dt$ is equal to

(A) $x + 2e^{-x} - 1$ (B) $x - 2e^{-x} + 1$ (C) $x + 2e^{-x} + 1$ (D) $-x - 2e^{-x} + 1$

Ans. (A)

Sol. $\int_0^{\infty} e^{-t}|x-t|dt$

$$= \int_0^{x} \underset{II}{e^{-t}}.\underset{I}{(-t+x)}dt + \int_x^{\infty} \underset{II}{e^{-t}}\underset{I}{(-x+t)}dt$$

apply Integration by parts

$$= \left[(x-t)\left(-e^{-t}\right) - (-1)e^{-t}\right]_0^x + \left[(x-t)e^{-t} - e^{-t}\right]_x^{\infty}$$

$$= x + 2e^{-x} - 1$$

15. An urn contains marbles of four colours : red, white, blue and green. When four marbles are drawn without replacement, the following events are equally likely :

(1) the selection of four red marbles;

(2) the selection of one white and three red marbles;

(3) the selection of one white, one blue and two red marbles;

(4) the selection of one marble of each colour.

The smallest total number of marbles satisfying the given condition is

(A) 19 (B) 21 (C) 46 (D) 69

Ans. (B)

Sol. Let r, b, w, g be no. of red, blue, white & green balls respectively & n be the total no. of balls in box.

$$\frac{{}^rC_4}{{}^nC_4} = \frac{{}^wC_1.{}^rC_3}{{}^nC_4} = \frac{{}^wC_1.{}^bC_1.{}^rC_2}{{}^nC_4} = \frac{{}^wC_1.{}^bC_1.{}^rC_1.{}^gC_1}{{}^nC_4}$$

$\Rightarrow$ $r = 2g + 1 = 3b + 2 = 4w + 3$

$\Rightarrow$ min . value of r is 11

$\Rightarrow$ $w_{min} = 2$

$b_{min} = 3$

$g_{min} = 5$

$\Rightarrow$ minimum value of $n = 21$

16. There are 6 boxes labelled B_1, B_2, ..., B_6. In each trial, two fair dice D_1, D_2 are thrown. If D_1 shows j and D_2 shows k, then j balls are put into the box B_k. After n trials, what is the probability that B_1 contains at most one ball?

(A) $\left(\frac{5^{n-1}}{6^{n-1}}\right)+\left(\frac{5^n}{6^n}\right)\left(\frac{1}{6}\right)$ (B) $\left(\frac{5^n}{6^n}\right)+\left(\frac{5^{n-1}}{6^{n-1}}\right)\left(\frac{1}{6}\right)$

(C) $\left(\frac{5^n}{6^n}\right)+n\left(\frac{5^{n-1}}{6^{n-1}}\right)\left(\frac{1}{6}\right)$ (D) $\left(\frac{5^n}{6^n}\right)+n\left(\frac{5^{n-1}}{6^{n-1}}\right)\left(\frac{1}{6^2}\right)$

Ans. (D)

Sol. Required probability

$= P(k \neq 1) + P(k = 1, \text{ when } j = 1)$

$=\left(\frac{5}{6}\right)^n+\left[\left(\frac{5}{6}\right)^{n-1}.\frac{1}{6}\right]\frac{1}{6}$

$=\frac{5^n}{6^n}+\left(\frac{5^{n-1}}{6^{n-1}}\right)\frac{1}{6^2}$

Ans. (D)

17. Let $\vec{a}=6\vec{i}-3\vec{j}-6\vec{k}$ and $\vec{d}=\vec{i}+\vec{j}+\vec{k}$. Suppose that $\vec{a}=\vec{b}+\vec{c}$ where $\vec{b}$ is parallel to $\vec{d}$ and $\vec{c}$ is perpendicular to $\vec{d}$. Then $\vec{c}$ is

(A) $5\vec{i}-4\vec{j}-\vec{k}$ (B) $7\vec{i}-2\vec{j}-5\vec{k}$ (C) $4\vec{i}-5\vec{j}+\vec{k}$ (D) $3\vec{i}+6\vec{j}-9\vec{k}$

Ans. (B)

Sol. As $\vec{b}\,||\,\vec{d}$ & $\vec{c}\perp\vec{d}$, $\vec{b}=\lambda\vec{d}$ & $\vec{c}.\vec{d}=0$

$\Rightarrow$ $6 - \lambda - 3 - \lambda - 6 - \lambda = 0 \Rightarrow \lambda = -1$

$\Rightarrow$ $\vec{c}=(6-\lambda)\hat{i}-(3+l)\hat{j}-(6+\lambda)\hat{k}$

$\Rightarrow$ $\vec{c}=7\hat{i}-2\hat{j}-5\hat{k}$

Ans. (B)

18. If $\log_{(3x-1)}(x-2)=\log_{(9x^2-6x+1)}(2x^2-10x-2)$, then x equals

(A) $9-\sqrt{15}$ (B) $3+\sqrt{15}$ (C) $2+\sqrt{5}$ (D) $6-\sqrt{5}$

Ans. (B)

Sol. $\log_{(3x-1)}(x-2) = \log_{(9x^2-6x+1)}(2x^2 - 10x - 2)$

$\log_{(3x-1)}(x-2) = \log_{(3x-1)^2}(2x^2 - 10x - 2) = \frac{1}{2}\log_{(3x-1)}(2x^2-10x-2)$

$\Rightarrow$ $\log_{3x-1}(x-2) = \log_{3x-1}(2x^2-10x-2)^{1/2}$

$3x - 1 > 0$ & $3x - 1 \neq 1 \Rightarrow x>\frac{1}{3}$ & $x\neq\frac{2}{3}$

also $x - 2 > 0 \Rightarrow x > 2$ & $2x^2 - 10x - 2 > 0$

Then, $(x - 2)^2 = (2x^2 - 10x - 2)$

$\Rightarrow$ $x^2 - 6x - 6 = 0 \Rightarrow x=\frac{6\pm\sqrt{60}}{2}=3\pm\sqrt{15}$

$\Rightarrow$ $x=3+\sqrt{15}$

19. Suppose a, b, c are positive integers such that $2^a + 4^b + 8^c = 328$. Then $\frac{a+2b+3c}{abc}$ is equal to

(A) $\frac{1}{2}$ (B) $\frac{5}{8}$ (C) $\frac{17}{24}$ (D) $\frac{5}{6}$

Ans. (C)

Sol. $2^{a-3} + 2^{2b-3} + 3^{3c-3} = 41$

$2^{a-3} + 2^{2b-3} + 8^{c-1} = 41$

$c - 1 = 1 \Rightarrow c = 2$

$2^{a-3} + 2^{2b-3} = 33$

$a - 3 = 0,\ 2b - 3 = 5$

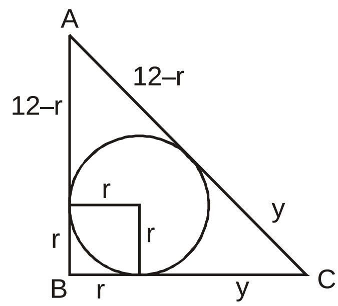
$a = 3,\ b = 4,\ c = 2$

20. The sides of a right-angled triangle are integers. The length of one of the sides is 12. The largest possible radius of the incircle of such a triangle is

(A) 2 (B) 3 (C) 4 (D) 5

Ans. (B)

Sol. $r = \frac{\Delta}{S}$



$\Delta = \frac{1}{2}.12(r+y)$

$\Delta = 6(y + r)$

$S = 12 + r + y + 12 - r + y$

$= \frac{24+2y}{2} \Rightarrow 12 + y$

$r = \frac{6(y+r)}{(12+y)}$

$r(12 + y) - 6r = 6y$

or $(6 + y) = 6y$

$r = \left(\frac{6y}{6+y}\right)$

$r = \frac{6}{\left(\frac{6}{y}+1\right)}$

at $y = 6 \Rightarrow r = \frac{6}{2} = 3$

PHYSICS

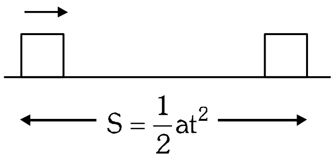
21. A small box resting on one edge of the table is struck in such a way that is slides off the other edge, 1 m away, after 2 seconds. The coefficient of kinetic friction between the box and the table

(A) must be less than 0.05
(B) must be exactly zero
(C) must be more than 0.05
(D) must be exactly 0.05

Ans. (C)

Sol. $S = \frac{1}{2} \times u \times g \times f^2$

$1 = \frac{1}{2} \times u \times 9.8 \times (2)^2$



$\Rightarrow$ u = 0.051

There u > 0.05

Ans. (C)

22. Carbon-11 decays to boron-11 according to the following formula

$$^{11}_{6}C \rightarrow ^{11}_{5}B + e^{+} + v_e + 0.96MeV$$

Assume that positions (e^+) produced in the decay combine with free electrons in the atmosphere and annihilate each other almost immediately. Also assume that the neutrinos (V_e) are massless and do not interact with the environment. At t = 0 we have 1 μg of $^{12}_{6}C$. If the half-life of the decay process is t_0, the net energy produced between time t = 0 and t = $2t_0$ will be nearly

(A) 8×10^{18} MeV
(B) 8×10^{16} MeV
(C) 4×10^{18} MeV
(D) 4×10^{16} MeV

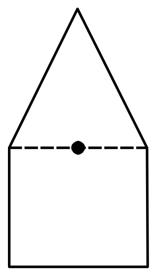
Ans. (D)

Sol. mass left $1\mu g \xrightarrow{t_0} 0.5\mu g \xrightarrow{2t_0} 0.25\mu g$

decayed nuclei N = $\frac{0.75 \times 10^{-6} \times 6.023 \times 10^{23}}{11} = 4.04 \times 10^{16}$

Energy Produced $4.04 \times 10^{16} \times 0.96 = 3.85 \times 10^{16}$ MeV ~ 4×10^{16} MeV

23. Two uniform plates of the same thickness and area but of different materials, one shaped like an isosceles triangle and the other shaped like a rectangle are joined together to form a composite body as shown in the figure. If the centre of mass of the composite body is located at the midpoint of their common side, the ratio between masses of the triangle to that of the rectangle is

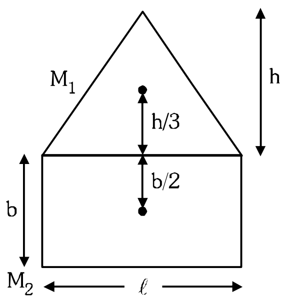


(A) 1 : 1
(B) 4 : 3
(C) 3 : 4
(D) 2 : 1

Ans. (C)

Sol. Area are equal

$bt = \frac{1}{2}h_x t$

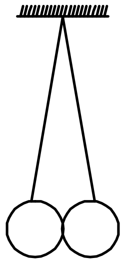


$b = \frac{h}{2}$

$M_1 \frac{h}{3} = M_2 \frac{h}{4}$

$\frac{M_1}{M_2} = \frac{3}{4}$

24. Two spherical objects each of radii R and masses m_1 and m_2 are suspended using two strings of equal length L as shown in the figure (R < < L). The angle, θ which mass m_2 makes with the vertical is approximately



(A) $\dfrac{m_1R}{(m_1+m_2)L}$ (B) $\dfrac{2m_1R}{(m_1+m_2)L}$ (C) $\dfrac{2m_2R}{(m_1+m_2)L}$ (D) $\dfrac{m_2R}{(m_1+m_2)L}$

Ans. (B)

Sol. $\theta = \dfrac{m_1}{(M_1+M_2)}\dfrac{2R}{L}$

(Centre of mass will be directly below the point of suspension)

Total angular separation is $\dfrac{2R}{L}$

Centre of mass will be $\dfrac{m_1}{M_1+M_2}\left(\dfrac{2R}{L}\right)$ angularly separated from m_2.

25. A horizontal disk of moment of inertia 4.25 kg-m^2 with respect to its axis of symmetry is spinning counter clock-wise at 15 revolutions per second about its axis, as viewed from above. A second disk of moment of inertia 1.80 kg-m^2 with respect to its axis of symmetry is spinning clockwise at 25 revolutions per second as viewed from above about the same axis and is dropped on top of the first disk. The two disks stick together and rotate as one about their axis of symmetry. The new angular velocity of the system as viewed from above is close to

(A) 18 revolutions/second and clockwise. (B) 18 revolutions/second and counter clockwise.
(C) 3 revolutions/second and clockwise. (D) 3 revolutions/second and counter clockwise.

Ans. (D)

Sol. From conservation of angular moment

$I_1w_1 - I_2w_2 = (I_1 + I_2)w$

$(4.25 \times 15 - 1.8 \times 25) = 6.05 \times w$

$(63.75 - 45) = 6.05 \times w$

$18.75 = 6.05 \times w$

w = 3.099 rev/sec in counter clockwise.

26. A boy is standing on top of *a* tower of height 85 m and throws a ball in the vertically upward direction with a cenain speed. If 5.25 seconds later he hears the ball hitting the ground, then the speed with which the boy threw the ball is (take g - 10 m/s^2, speed of sound in air, = 340 m/s)

(A) 6 m/s (B) 8 m/s (C) 10 m/s (D) 12 m/s

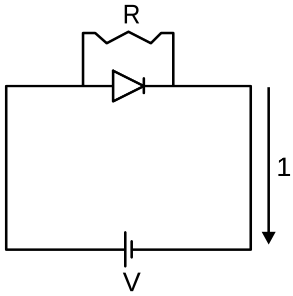
Ans. (B)

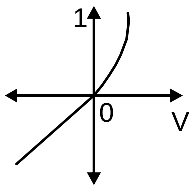
Sol. $t = \dfrac{85}{340}$: Now t + t' = 5.25 $\Rightarrow$ t = 5 sec

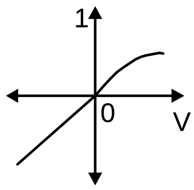
= 0.25

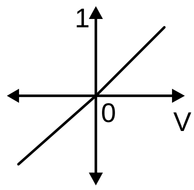
$-h = ut - \dfrac{1}{2}gt^2$

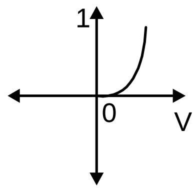
27. For a diode connected in parallel with a resistor, which is the most likely current (I) - voltage (V) characterstitic?



(A) 

(B) 

(C) 

(D) 

Ans. (A)

Sol. If V is positive, current will flow through diode in forward bias.
If V is negative, current will flow through resistance.

28. A beam of monoenergetic electrons, which have been accelerated from rest by a potential U, is used to form an interference pattern in a Young's Double Slit experiment. The electrons are now accelerated by potential 4U. Then the fringe width

(A) remains the same.
(B) is half the original fringe width,
(C) is twice the original fringe width.
(D) is one-fourth the original fringe width,

Ans. (B)

Sol. $\beta \propto \lambda$

Other $\lambda = \frac{h}{\sqrt{2meU}}$

Hence fringe width will become half.

29. A point charge Q (= 3 × 10^{-12} C) rotates uniformly in a vertical circle of radius R = 1 mm. The axis of the circle is aligned along the magnetic axis of the earth. At what value of the angular speed ω, the effective magnetic field at the center of the circle will be reduced to zero? (Horizontal component of Earth's magnetic field is 30 micro Tesla)

(A) 10^{11} rad/s
(B) 10^{9} rad/s
(C) 10^{13} rad/s
(D) 10^{7} rad/s

Ans. (A)

Sol. $\frac{\mu_0 qv}{4\pi R^2} = 30 \times 10^{-6}$

$$\frac{10^{-7} \times 3 \times 10^{-12} \times w}{10^{-3}} = 30 \times 10^{-6}$$

$\omega = 10^{11}$ rad/s

30. A closed bottle containing water at 30°C is open on the surface of the moon, Then

(A) the water will boil.
(B) the water will come out as a spherical ball.
(C) the water will freeze.
(D) the water will decompose into hydrogen and oxygen.

Ans. (A)

Sol. Boiling point of water decreases on decreasing. At moon, atmospheric pressure is zero.
Therefore, pressure when bottle opens, the water will boil.

31. A simple pendulum of length l is made to oscillate with an amplitude of 45 degrees. The acceleration due to gravity is g. Let $T_0 = 2\pi\sqrt{l/g}$. The lime period of oscillation of this pendulum will be

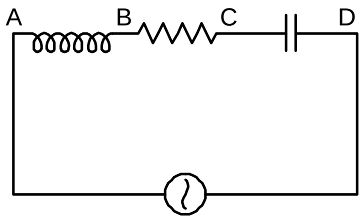
(A) T_0 irrespective of the amplitude.
(B) slightly less than T_0.
(C) slightly more than T_0.
(D) dependent on whether it swings in a plane aligned with the north-south or cast-west directions.

Ans. (A)

Sol. $T_0 = 2\pi\sqrt{\ell/g}$

If θ is greater, then $a < g\theta$
Hence time-period $T' > T_0$

32. An ac voltmeter connected between points A and B in the circuit below reads 36 V. If it is connected between A and C, the reading is 39 V. The reading when it is connected between B and D is 25 V. What will the voltmeter read when it is connected between A and D? (Assume that the voltmeter reads true rms voltage values and that the source generates a pure ac.)



(A) $\sqrt{481}$ V (B) 31 V (C) 61 V (D) $\sqrt{3361}$ V

Ans. (A)

Sol. $V_L = 36$
$V_R^2 + V_L^2 = 39^2$
$V_R^2 + V_C^2 = 25^2$

$V_{AD} = \sqrt{V_R^2 + (V_L - V_C)^2} = \sqrt{481}$

33. A donor atom in a semiconductor has a loosely bound electron. The orbit of this electron is considerably affected by the semiconductor material but behaves in many ways like an electron orbiting a hydrogen nucleus. Given that the electron has an effective mass of $0.07m_e$, (where m_e is mass of the free electron) and the space in which it moves has a permittivity $13\varepsilon_0$, then the radius of the electron's lowermost energy orbit will be close to (The Bohr radius of the hydrogen atom is 0.53 Å)

(A) 0.53 Å (B) 243 Å (C) 10 Å (D) 100 Å

Ans. (D)

Sol. The bohr radius is given by

$$r_n = \frac{n^2h^2}{4\pi^2kzme^2}$$

$$r_n' = \frac{n^2h\ 13}{4\pi^2kz \times 0.07\,m_e \times e^2}$$

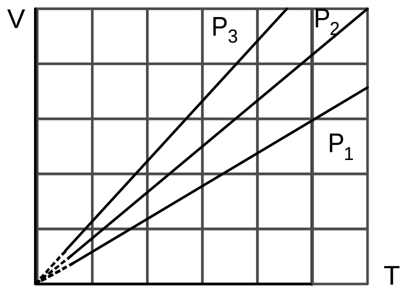
$$r_n' = \frac{13}{0.07} \times r_n$$

$$\Rightarrow \quad \frac{13}{0.07} \times 0.53$$

$$r_n' = 98.43\text{Å}$$

$$r_n' = 100\,\text{Å} \quad \text{(Approximately)}$$

34. The state of an ideal gas was changed isobarically. The graph depicts three such isobaric lines. Which of the following is true about the pressures of the gas?



(A) $P_1 = P_2 = P_3$ (B) $P_1 > P_2 > P_3$ (C) $P_1 < P_2 < P_3$ (D) $P_1/P_2 = P_3/P_1$

Ans. (B)

Sol. $V = \frac{nR}{P}T$

Slope of V and T curve is equal to $\frac{nR}{P}$

$\therefore$ Slope of V and T curve $\propto \frac{1}{P}$

Thus $P_1 > P_2 > P_3$

35. A metallic ring of radius *a* and resistance R is held fixed with its axis along a spatially uniform magnetic field whose magnitude is $B_a\sin(\omega t)$. Neglect gravity. Then,

(A) the current in the ring oscillates with a frequency of 2ω.

(B) the Joule healing loss in the ring is proportional to a^2.

(C) the force per unit length on the ring will be proportional to B_0^2.

(D) the net force on the ring is non-zero.

Ans. (C)

Sol. $I = \frac{(B_0\omega\cos\omega t)\pi a^2}{R}$

$P = \int I^2 R\,dt \propto a^4$

$F = Id\ell B$

$F \propto B_0^2$

36. The dimensions of the area A of a black hole can be written in terms of the universal gravitational constant G, its mass M and the speed of tight c as $A = G^{\alpha}M^{\beta}c^{\gamma}$. Here

(A) $\alpha = -2, \beta = -2$ and $\gamma = 4$ (B) $\alpha = 2, \beta = 2$ and $\gamma = -4$

(C) $\alpha = 3, \beta = 3$ and $\gamma = -2$ (D) $\alpha = -3, \beta = -3$ and $\gamma = 2$

Ans. (B)

Sol. $[M^{\beta-\alpha} L^{3\alpha+\gamma} T^{-2\alpha-\gamma}) = L^2$

$\beta = \alpha$; $3\alpha + \gamma = 2$; $-2\alpha - \gamma = 0 \Rightarrow 2\alpha = -\gamma$

37. A 160 watt infrared source is radiating light of wavelength 50000 Å uniformly in all directions. The photon flux at a distance of 1.8 m is of the order of

(A) $10\ m^{-2}s^{-1}$ (B) $10^{10}\ m^{-2}s^{-1}$ (C) $10^{15}\ m^{-2}s^{-1}$ (D) $10^{20}\ m^{-2}s^{-1}$

Ans. (D)

Sol. Flux = $\frac{P}{4\pi r^2}\frac{\lambda}{hC} \simeq 10^{20} m^2/s$

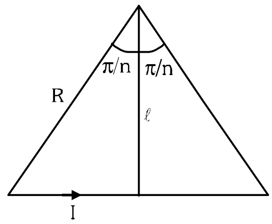
38. A wire bent in the shape of a regular *n* - polygonal loop carries a steady current I, Let *l* be the perpendicular distance of a given segment and R be the distance of a vertex both from the centre of the loop. The magnitude of the magnetic field at the centre of the loop is given by

(A) $\frac{n\mu_0 I}{2\pi l}\sin(\pi/n)$ (B) $\frac{n\mu_0 I}{2\pi R}\sin(\pi/n)$ (C) $\frac{n\mu_0 I}{2\pi l}\cos(\pi/n)$ (D) $\frac{n\mu_0 I}{2\pi R}\cos(\pi/n)$

Ans. (A)

Sol. $B = \left[\frac{\mu_0 I}{4\pi\ell}\sin\left(\frac{\pi}{n}\right)^2\right]n$

$= \frac{n\mu_0 I}{2\pi\ell}\sin\left(\frac{\pi}{n}\right)$



39. The intensity of sound during me festival season increased by 100 times. This could imply a decibel level rise from.

(A) 20 to 120 dB (B) 70 to 72 dB (C) 100 to 10000 dB (D) 80 to 100 dB

Ans. (A)

Sol. $L_2 - L_1 = 10\log_{10}\left(\frac{I_2}{I_1}\right)$

$L_2 - L_1 = 20$

Hence increased of 20 dB

40. One end of a slack wire (Young's modulus Y, length L and cross-sectional area A) is clamped to a rigid wall and the other end to a block (mass m) which rests on a smooth horizontal plane. The block is set in motion with a speed v. What is the maximum distance the block will travel after the wine becomes taut?

(A) $v\sqrt{\frac{mL}{AY}}$ (B) $v\sqrt{\frac{2mL}{AY}}$ (C) $v\sqrt{\frac{mL}{2AY}}$ (D) $L\sqrt{\frac{mv}{AY}}$

Ans. (A)

Sol. $\frac{1}{2}mv^2 = \frac{1}{2}\left(\frac{YA}{L}\right)x^2$

$x = v\sqrt{\frac{mL}{YA}}$

CHEMISTRY

41. The Lewis acid strength of BBr_3, BCl_3 and BF_3 is in the order

(A) $BBr_3 < BCl_3 < BF_3$ (B) $BCl_3 < BF_3 < BBr_3$ (C) $BF_3 < BCl_3 < BBr_3$ (D) $BBr_3 < BF_3 < BC1_3$

Ans. (C)

Sol. Extent of backbond formation B–F > B–Cl > B–Br.

42. O^{2-} isoelectronic with

(A) Zn^{2+} (B) Mg^{2+} (C) K^+ (D) Ni^{2+}

Ans. (B)

Sol. O^{2-} has 10 e^-.

43. The H-C-H, H-N-H, and H-O-H bond angles (in degrees) in methane, ammonia and water are respectively, closest to

(A) 109.5, 104.5, 107.1 (B) 109.5,107.1,104.5 (C) 104.5, 107.1, 109.5 (D) 107.1, 104.5, 109,5

Ans. (B)

Sol. Presence of lone pair decreases bond angle due to greater l.p – b.p. repulsion than b.p.–b.p. repulsion.

44. In alkaline medium, the reaction of hydrogen peroxide with potassium permanganate produces a compound in which the oxidation state of Mn is

(A) 0 (B) +2 (C) +3 (D) +4

Ans. (D)

Sol. $MnO_4^- \xrightarrow[\text{weakly alkaline}]{\text{neutral or}} MnO_2$

45. The rate constant of a chemical reaction at a very high temperature will approach

(A) Arrhenius frequency factor divided by the ideal gas constant

(B) activation energy

(C) Arrhenius frequency factor

(D) activation energy divided by the ideal gas constant

Ans. (C)

Sol. $k = Ae^{-Ea/RT}$

As $T \longrightarrow \infty$

$e^{-Ea/RT} \longrightarrow 1$

46. The standard reduction potentials (in V) of a few metal ion/metal electrodes are given below.

$Cr^{3+}/Cr = -74$; $Cu^{2+}/Cu = +0.34$; $Pb^{2+}/Pb = -0.13$;

$Ag^+/Ag = +0.8$. The reducing strength of the metals follows the order

(A) Ag > Cu > Pb > Cr (B) Cr > Pb > Cu > Ag (C) Pb > Cr > Ag > Cu (D) Cr > Ag > Cu > Pb

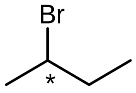
Ans. (B)

Sol. As SRP increases tendency to get reduced increases, hence reducing power decreases.

47. Which of the following molecules can exhibit optical activity?

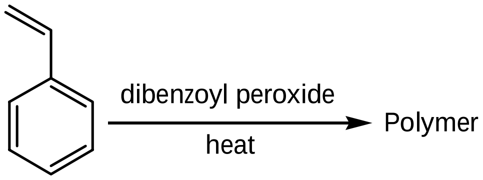
(A) I-bromopropane (B) 2-bromobutane (C) 3-bromopentane (D) bromocyclohexaoe

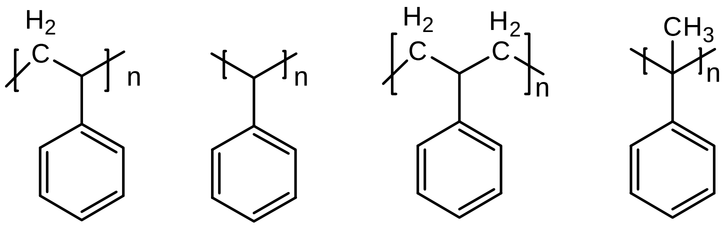
Ans. (B)

Sol. 

Presence of asymmetric carbon and absence of element of symmetry results in optical activity.

48. The structure of the polymer obtained by the following reaction is





I II III IV

(A) I (B) II (C) III (D) IV

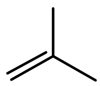
Ans. (A)

Sol. $Ph\text{–}CH=CH_2 \xrightarrow{\text{Polymerize}} \left(\begin{array}{l} CH\text{—}CH_2 \\ | \\ Ph \end{array}\right)_n$

49. The major product of the reaction between CH_3CH_2ONa and $(CH_3)_3CCl$ in ethanol is

(A) $CH_3CH_2OC(CH_3)_3$ (B) $CH_2 = C(CH_3)_2$ (C) $CH_3CH_2(CH_3)_3$ (D) $CH_3CH = CHCH_3$

Ans. (B)

Sol. $EtONa + (CH_3)_3CCl \longrightarrow$  $+$ NaCl $+$ EtOH

E_2 major

50. When H_2S gas is passed through a hot acidic aqueous solution containing Al^{3+}, Cu^{2+}, Pb^{2+} and Ni^{2+} a precipitate is formed which consists of

(A) CuS and Al_2S_3 (B) PbS and NiS (C) CuS and NiS (D) PbS and CuS

Ans. (D)

Sol. In acidic medium group 2 radical will precipitate out.

51. The electronic configuration of an element with the largest difference between the Ist and 2nd utilization energies is

(A) $1s^22s^22p^6$ (B) $1s^22s^22p^63s^1$ (C) $1s^22s^22p^63s^2$ (D) $1s^22s^22p^1$

Ans. (B)

Sol. Formation of noble gas configuration will result in greater increase in IE.

52. The order of electronegativity of carbon tn sp, sp^2 and sp^3 hybridized states follows

(A) $sp > sp^2 > sp^3$ (B) $sp^3 > sp^2 > sp$

(C) $sp > sp^3 > sp^2$ (D) $sp^2 > sp > sp^3$

Ans. (A)

Sol. EN of C $\propto$ % s–character of hybrid C.

53. The most abundant transition metal in human body is

(A) copper (B) iron (C) zinc (D) manganese

Ans. (B)

Sol. Fe is part of haemoglobin.

54. The molar conductivities of HCl, NaCl, CH_3COOH and CH_3COONa at infinite dilution follow the order

(A) HCl > CH_3COOH > NaCl > CH_3COONa

(B) CH_3COONa > HCl > NaCl > CH,COOH

(C) HCl > NaCl > CH_3COOH > CH_3COONa

(D) CH_3COOH > CH_3COONa > HCl > NaCl

Ans. (A)

Sol. H^+ has greatest conductance among cations.

55. The spin only magnetic moment of $[ZCl_4]^{2-}$ is 3.87 BM where Z is

(A) Mn (B) Ni (C) Co (D) Cu

Ans. (C)

Sol. $\vec{\mu}_S = \sqrt{n(n+2)}\text{BM}$

$\therefore$ No. of unpaired electrons = 3.

56. If α-D-glucose is dissolved in water and kept for a few hours, the major constituent(s) present in the solution is (are)

(A) α-D-glucose

(B) mixture of p-D-glucose and open chain D-glucose

(C) open chain D-glucose

(D) mixture of α-D-glucose and β-D-glucose

Ans. (D)

Sol. Mutarotation will occur.

57. The pH of IN aqueous solutions of HCl, CH_3COOH and HCOOH follows the order

(A) HCl > HCOOH > CH_3COOH

(B) HCl = HCOOH > CH_3COOH

(C) CH_3COOH > HCOOH > HCl

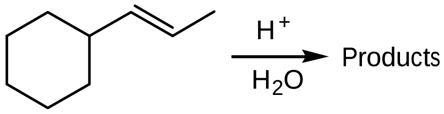
(D) CH_3COOH = HCOOH > HCl

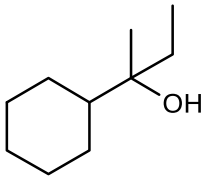
Ans. (C)

Sol. Acidic strength of HCl > HCOOH > CH_3COOH.

For same concentration 1N, $[H^+]$ produced by HCl > HCOOH > CH_3COOH.

58. The major product of the reaction





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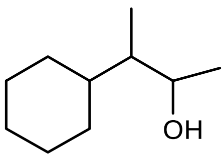
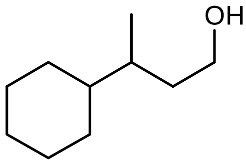
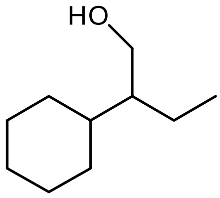


II



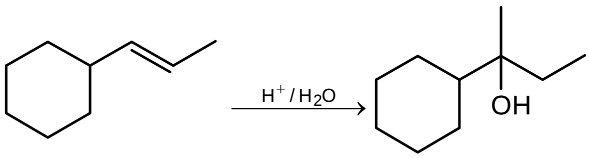
III



IV

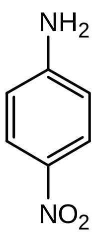
(A) I (B) II (C) III (D) IV

Ans. (A)

Sol. 

Major

59. Reaction of aniline with $NaNO_2^+$ + dil, HCl at 0°C followed by reaction with CuCN yields



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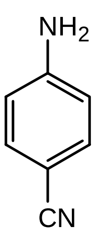

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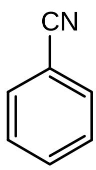
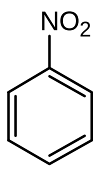


III



IV

(A) I (B) II (C) III (D) IV

Ans. (C)

Sol. $PhNH_2 \xrightarrow[\text{dil.HCl,0°C}]{NaNO_2} Ph\overset{\oplus}{N}_2Cl^- \xrightarrow{CuCN} PhCN$

60. Schottky defect in a crystal arises due to

(A) creation of equal number of cation and anion vacancies

(B) creation of unequal number of cation and anion vacancies

(C) migration of cations to interstitial voids O.

(D) migration of anions to interstitial voids

Ans. (A)

Sol. Creation of equal number of cation and anion vacancies.

BIOLOGY

61. Immunosuppressive drugs like cyclosporin delay the rejection of graft post organ transplantation by
(A) inhibiting T cell infiltration (B) killing B cells
(C) killing macrophages (D) killing dendritic cells
Ans. (A)

62. Which one of these substances will repress the *lac* operon?
(A) Arabinose (B) Glucose (C) Lactose (D) Tryptophan
Ans. (B)

63. Assume a spherical mammalian cell has a diameter of 27 microns. If a polypeptide chain with alpha helical conformation has to stretch across the cell, how many ami no acids should it be comprised of?
(A) 18000 (B) 1800 (C) 27000 (D) 12000
Ans. (B)

64. Which one of the following has phosphoric acid anhydride bonds?
(A) Deoxy ribonuclcic acid (B) Ribonucleic acid
(C) dNTPs (D) Phospholipids
Ans. (C)

65. The two components of autonomous nervous system have antagonistic actions. But in certain cases their effects are mutually helpful. Which of the following statement is correct?
(A) At rest, the control of heart beat is not by the vagus nerve
(B) During exercise the sympathetic control decreases
(C) During exercise the parasympathetic control decreases
(D) Stimulation of sympathetic system results in constriction of the pupil
Ans. (C)

66. In a random DNA sequence, what is the lowest frequency of encountering a stop codon?
(A) 1 in 20 (B) 1 in 3 (C) 1 in 64 (D) 1 in 10
Ans. (A)

67. The two alleles that determine the blood group AB of an individual are located on
(A) two different autosomes
(B) the same autosome
(C) two different sex chromosomes
(D) one on sex chromosome and the other on an autosome
Ans. (D)

68. In biotechnology applications, a selectable marker is incorporated in a plasm id
(A) to increase its copy number (B) to increase the transformation efficiency
(C) to eliminate the non-transformants (D) to increase the expression of the gene of interest
Ans. (C)

69. Spermatids are formed after the second meiotic division from secondary spermatocytes. The ploidy of the secondary' spermatocytes is
(A) n (B) 2n (C) 3n (D) 4n
Ans. (A)

70. Phospholipids are formed by the esterification of
(A) three ethanol molecules with three fatty acid molecules.
(B) one glycerol and two fatty acid molecules.
(C) one glycerol and three fatty acid molecules.
(D) one ethylene glycol and two fatty acids molecules.
Ans. (B)

71. Given the fact that histone binds DNA, it should be rich in
(A) arginine, lysine
(B) cysteine, methionine
(C) glutamate, aspartate
(D) isoleucine, leucine

Ans. (A)

72. If molecular weight of a polypeptide is 15.3 kDa, what would be the minimum number of nucleotides in the mRNA that codes for this polypeptide? Assume that molecular weight of each amino acid is 90 Da.
(A) 510 (B) 663 (C) 123 (D) 170

Ans. (A)

73. Melting temperature for double stranded DNA is the temperature at which 50% of the double stranded molecules are converted into single stranded molecules. Which one of the following DNA will have the highest melting temperature?
(A) DNA with 15% guanine
(B) DNA with 30% cytosine
(C) DNA with 40% thymine
(D) DNA with 50% adenine

Ans. (B)

74. Following arc the types of immunoglobulin and their functions. Which one of the following is INCORRECTLY paired?
(A) IgD: viral pathogen
(B) IgG: phagocytosis
(C) IgE: allergic reaction
(D) IgM: complement Fixation

Ans. (A)

75. Which one of the following can be used to detect amino acids?
(A) Iodine vapour (B) Ninhydrin (C) Ethidium bromide (D) Bromophenol blue

Ans. (B)

76. Mutation in a single gene can lead to changes in multiple traits. This is an example of
(A) Heterotrophy (B) Co-dominance (C) Penetrance (D) Pleiotropy

Ans. (D)

77. Which one of the following is used to treat cancers?
(A) Albumin (B) Cyclosporin A (C) Antibodies (D) Growth hormone

Ans. (C)

78. Which of the following processes leads to DNA ladder formation?
(A) Necrosis (B) Plasmolysis (C) Apoptosis (D) Mitosis

Ans. (D)

79. Co-enzymes are components of an enzyme complex which are necessary for its function. Which of these is a known
(A) Zinc (B) Vitamin B_{12} (C) Chlorophyll (D) Heme

Ans. (B)

80. The peptidoglycans of bacteria consist of
(A) sugars. D-amino acids and L-amino acids
(B) sugars and only D-amino acids
(C) sugars and only L-amino acids
(D) sugars and glycinc

Ans. (A)

PART II
Two Mark Questions
MATHEMATICS

81. Let $x = \left(\sqrt{50}+7\right)^{1/3} - \left(\sqrt{50}-7\right)^{1/3}$. Then

(A) x = 2
(B) x = 3
(C) x is a rational number, but not an integer
(D) x is an irrational number

Ans. (A)

Sol. $x^3 = (\sqrt{50}+7)-(\sqrt{50}-7)-3(x)$

$x^3 + 3x = 14 \Rightarrow x = 2$

82. Let $(1 + x + x^2)^{2014} = a_0 + a_1x + a_2x^2 + a_3x^3 + ... + a_{4025}x^{4028}$, and let

$A = a_0 - a_3 + a_6 - ... + a_{4026}$,
$B = a_1 - a_4 + a_1 - ... - a_{4027}$,
$C = a_2 - a_5 + a_8 - ... + a_{4028}$.

Then

(A) $|A| = |B| > |C|$
(B) $|A| = |B| < |C|$
(C) $|A| = |C| > |B|$
(D) $|A| = |C| < |B|$

Ans. (D)

Sol. $(1+x+x^2)^{2014} = a_0 + a_1x + a_2x^2 + a_3x^3 + ... + a_{4028}x^{4028}$

Put $x = -1, -\omega, -\omega^2$

$(1-1+1)^{2014} + (1-\omega+\omega^2)^{2014} + (1-\omega^2+\omega)^{2014} = 3(a_0 - a_3 + a_6 - a_9 + ...) = 3A$

$x = 0 \qquad a_0 = 1$

$$\frac{(1+x+x^2)^{2014}-1}{x} = a_1 + a_2x + a_3x^2 + ...$$

$$\frac{(1-1+1)^{2014}-1}{-1} + \frac{(1-\omega+\omega^2)^{2014}}{-\omega} + \frac{(1-\omega^2+\omega)^{2014}-1}{-\omega^2} = 3(a_1 - a_4 + a_7 - a_{10} + ...) = 3B$$

$a_1 = 2014$

$$\frac{(1+x+x^2)^{2014}-1}{x^2} = \frac{a_1}{x} = a_2 + a_3x + a_4x^2 + ...$$

$$\left(\frac{(1-1+1)^{2014}}{1} + \frac{a_1}{1}\right) + \left(\frac{(1-\omega+\omega^2}{\omega^2} + \frac{a_1}{\omega}\right) + \left(\frac{1-\omega^2+\omega-1}{\omega} + \frac{a_1}{\omega^2}\right) = 3(a_2 - a_5 + a_8 ....) = 3C$$

$3A = 1-2^{2014}$
$3B = -1-2^{2015}$ $\qquad |3A| = |3C| < |3B|$
$3C = 1-2^{2014}$ $\qquad |A| = |C| > |B|$ $\qquad$ Ans. (D)

83. A mirror in the first quadrant is in the shape of a hyperbola whose equation is xy = 1. A light source in the second quadrant emits a beam of light that hits the mirror at the point (2, 1/2). If the reflected ray is parallel to the y-axis, the slope of the incident beam is

(A) $\frac{13}{8}$
(B) $\frac{7}{4}$
(C) $\frac{15}{8}$
(D) 2

Ans. (C)

Sol. Slope of normal at $\left(2, \frac{1}{2}\right) = 4$

Let slope of incident ray = m
and slope of reflected ray = ∞

$$\frac{4-m}{1+4m} = \frac{1}{4} \Rightarrow m = \frac{15}{8}$$

84. Let $C(\theta)=\sum_{n=0}^{\infty}\frac{\cos(n\theta)}{n!}$ which of the following statements is FALSE?

(A) C(0). C(π) = 1
(B) C(0) + C(π) > 2
(C) C(θ) > 0 for all $\theta \in R$
(D) $C'(\theta) \neq 0$ for all $\theta \in R$

Ans. (D)

Sol. $$C_{(Q)} = \sum_{n=0}^{\infty}\frac{\cos(n\theta)}{ni}$$

$$= R_e\left(\sum_{n=0}^{\infty}\frac{e^{(i\theta)^n}}{n!}\right)$$

$$= R_e\,(e^{e^{i\theta}}) = e^{\cos\theta}\,.\cos(\sin\theta)$$

$$C(0) = e\,,\quad C(\pi) = \frac{1}{e}$$

85. Let a > 0 be a real number. Then the limit $\lim_{x\to 2}\frac{a^x+a^{3-x}-(a^2+a)}{a^{3-x}-a^{x/2}}$ is

(A) 2 log a
(B) $-\frac{4}{3}a$
(C) $\frac{a^2+a}{2}$
(D) $\frac{2}{3}(1-a)$

Ans. (D)

Sol. $$\lim_{x\to 2}\frac{a^x+a^{3-x}-(a^2+a)}{a^{3-x}-a^{x/2}}$$

$$\lim_{x\to 2}\left[\frac{(a^{x-2}-1)a^2}{x-2}-\frac{a(a^{2-x}-1)}{x-2}\right]\frac{\left[\frac{3}{2}(x-2)\right].\frac{2}{3}}{a^{x/2}\left[a^{\frac{3}{2}(2-x)}-1\right]}$$

$$(a^2-a)-\frac{2}{3a}=\frac{2}{3}(1-a)$$

86. Let f(x) = $ax^2 - 2 + \frac{1}{x}$ where α is a real constant. The smallest α for which f(x) $\geq$ 0 for all x > 0 is

(A) $\frac{2^2}{3^3}$
(B) $\frac{2^3}{3^3}$
(C) $\frac{2^4}{3^3}$
(D) $\frac{2^5}{3^3}$

Ans. (D)

Sol. f(x) = $f(x)=\alpha x^2-2+\frac{1}{x}\geq 0$ for if α > 0

$$f(x) = \alpha x^2-2+\frac{1}{x}=\left(\alpha x^2+\frac{1}{2x}+\frac{1}{2x}\right)-2 \;\geq 3\left(\alpha x^2.\frac{1}{2x}.\frac{1}{2x}\right)-2$$

$$f(x)_{min} = 3\left(\frac{\alpha}{4}\right)^{1/3}-2\geq 0$$

$$\Rightarrow \left(\frac{\alpha}{4}\right)\geq\left(\frac{2}{3}\right)^3$$

$$\alpha \geq \frac{2^5}{3^3}$$

87. Let $f : R \to R$ be a continuous function satisfying

$$f(x)+\int_0^x tf(t)dt + x^2 = 0,$$

for all $x \in R$. Then

(A) $\lim\limits_{x\to\infty} f(x) = 2$

(B) $\lim\limits_{x\to-\infty} f(x) = -2$

(C) f(x) has more than one point in common with the x-axis

(D) f(x) is an odd function

Ans. (B)

Sol. $f(0) = 0$ Putting $x = 0$

$f'(x) + xf(x) + 2x = 0$

$\Rightarrow \quad \dfrac{f'(x)}{f(x)+2} = -x$

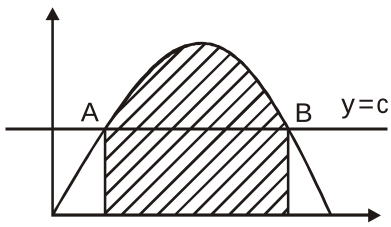
$\Rightarrow \quad \ell n(f(x)+2 = = -\dfrac{x^2}{2}+c$

$f(x) = Ae^{-x^2/2} - 2$

$f(0) = 0 \Rightarrow A = 2$

$f(x) = 2\left(e^{-x^2/2} - 1\right)$

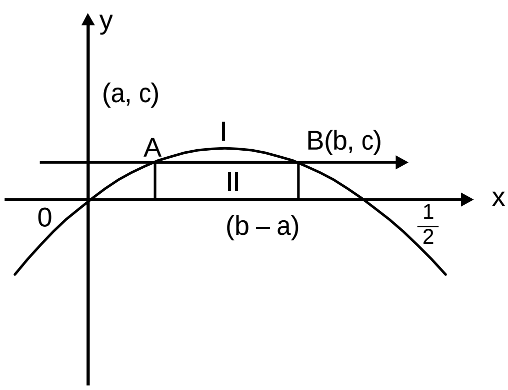
88. The figure shows a portion of the graph $y = 2x - 4x^3$. The line $y = c$ is such that the areas of the regions marked I and II are equal. If a, b are the x-coordinates of A, B respectively, then a + b equals



(A) $\dfrac{2}{\sqrt{7}}$ (B) $\dfrac{3}{\sqrt{7}}$ (C) $\dfrac{4}{\sqrt{7}}$ (D) $\dfrac{5}{\sqrt{7}}$

Ans. (A)

Sol. $y = 2x(1-2^{x^2})$



Area of II region = $(b - 0)c$

$$\text{I + II} = \int_a^b (2x-4x^3)dx = x^2 - x^4 |_a^b$$

$= b^2 - b^4 - a^2 + a^4$

$= (b^2 - a^2) - (b^2 - a^2)(b^2 - a^2)$

$= (b^2 - a^2)(1 - b^2 - a^2)$

Area of region I $= (b^2 - a^2)(1 - b^2 - a^2) - (b - a)c$

Area I = Area II

$2(b - a)c = (b^2 - a^2)(1 - b^2 - a^2)$

$2c = (b + a)(1 - b^2 - a^2)$

$c = 2a - 4a^2$

$c = 2b - 4b^2$

$$0 = 2(a-b) - 41a^2 - b^2$$
$$0 = 1 - 2(a^2 + b^2 + ab)$$
$$2((a+b)^2 - ab) = 1$$
$$2(a+b)^2 = 1 + 2ab$$
$$2c = (a+b)(1 - (a+b)^2 + 2ab)$$
$$2c = (a+b)(a+b)^2 \Rightarrow (a+b)^3 = 2c$$
$$2c = (a+b)^3$$
$$c = 2a - 4a^3$$
$$c = 2b - 4b^3$$
$$2c = 2(a+b) - 4(a+b)(a^2+b^2-ab)$$
$$c = (a+b) - 2(a+b)((a+b)^2 - 3ab)$$

$$\frac{(a+b)^3}{2} + 2(a+b)^3 = (a+b) + 6ab(a+b)$$

$$\frac{5}{2}(a+b)^2 = 2 + 12ab$$
$$5(a+b)^2 = 2 + 12\,ab$$
$$12(a+b)^2 = 6 + 12\,ab$$
$$7(a+2)^2 = 4 \Rightarrow (a+b)^2 = \frac{4}{7} \Rightarrow (a+b) = \frac{2}{\sqrt{7}}$$

89. Let $X_n = \{1, 2, 3, ..., n\}$ and let a subset A of X_n be chosen so that every pair of elements of A differ by at least 3. (For example, if n = 5, A can be ϕ, $\{2\}$ or $\{1, 5\}$ among others). When n = 10, let the probability that $1 \in A$ be p and let the probability that $2 \in A$ be q. Then

(A) $p > q$ and $p - q = \frac{1}{6}$ (B) $p < q$ and $q - p = \frac{1}{6}$

(C) $p > q$ and $p - q = \frac{1}{10}$ (D) $p < q$ and $q - p = \frac{1}{10}$

Ans. ()

Sol. $1 \in A$

cases $\{1, 4, 7, 10\}$

Let 3 element case be

$\{1, 1+x, 1+x_1+x_2\}\ \forall x_1, x_2 \geq 3 \quad 1+x_1+x_2 \leq 10$

90. The remainder when the determinant

$$\begin{vmatrix} 2014^{2014} & 2015^{2015} & 2016^{2016} \\ 2017^{2017} & 2018^{2018} & 2019^{2019} \\ 2020^{2020} & 2021^{2021} & 2022^{2022} \end{vmatrix}$$

is divided by 5 is

(A) 1 (B) 2 (C) 3 (D) 4

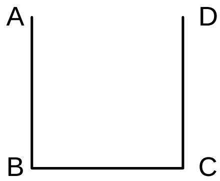
Ans. (D)

Sol. $\begin{vmatrix} 2014^{2014} & 2015^{2015} & 2016^{2016} \\ 2017^{2017} & 2018^{2018} & 2019^{2019} \\ 2020^{2020} & 2021^{2021} & 2022^{2022} \end{vmatrix} \bmod 5$

$= \begin{vmatrix} 1 & 0 & 1 \\ 2 & 4 & 4 \\ 0 & 1 & 4 \end{vmatrix} \bmod 5 \quad = (12 + 2) \bmod 5 = 4$

PHYSICS

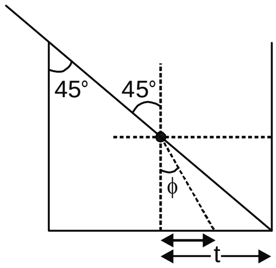
91. A cubical vessel has opaque walls. An observer (dark circle in figure below) is located such that she can sec only the wall CD but not the bottom. Nearly to what height tshould water be poured so that she can see an object placed at the bottom at a distance of 10 cm from the comer C? Refractive index of water is 1.33



(A) 10 cm (B) 16 cm (C) 27 cm (D) 45 cm

Ans. (C)

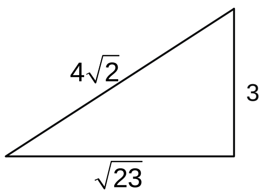
Sol. $1 \sin 45^\circ = \frac{4}{3} \cdot \sin\phi$



$\frac{1}{\sqrt{2}} \frac{3}{4} = \sin\phi a$

$\tan\theta = 1$

$\tan\phi = \frac{h-10}{h} = \frac{3}{\sqrt{23}}$



$3h = \sqrt{23}h - 10\sqrt{23}$

$h = 26.67 \approx 27$

92. The moments of inertia of a non-uniform circular disc (of mass M and radius R) about four mutually perpendicular tangents AB, BC, CD, DA are I_1, I_2, I_3 and I_4, respectively (the square ABCD circumscribes the circle) The distance of the center of mass of the disc from its geometrical center is given by

(A) $\frac{1}{4MR}\sqrt{(I_1 - I_3)^2 + (I_2 - I_4)^2}$ (B) $\frac{1}{12MR}\sqrt{(I_1 - I_3)^2 + (I_2 - I_4)^2}$

(C) $\frac{1}{3MR}\sqrt{(I_1 - I_2)^2 + (I_3 - I_4)^2}$ (D) $\frac{1}{2MR}\sqrt{(I_1 - I_3)^2 + (I_2 - I_4)^2}$

Ans. (A)

Sol. $I_1 = I_x + M(R - y)^2$

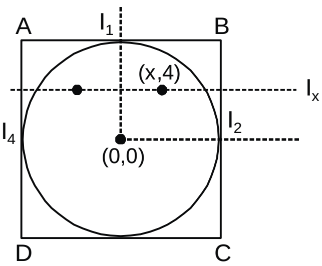
$I_3 = I_x + M(R + y)^2$

$I_3 - I_1 = M[(2R)(2y)] = 4MRy$

Similarly $I_2 - I_4 = 4MRx$

$(I_3 - I_1)^2 + (I_4 - I_2)^2 = 16M^2R^2d^2$

$d = \frac{\sqrt{(I_3 - I_1)^2 + (I_4 - I_2)^2}}{4MR}$



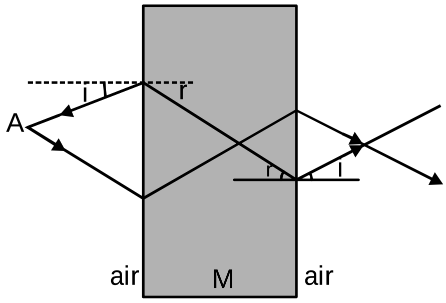
93. A horizontal steel railroad track has a length of 100 m when the temperature is 25°(C). The track is constrained from expanding or bending. The stress on the track on a hot summer day. when the temperature is 40°C, is (Note : the linear coefficient of thermal expansion for steel is 1.1×10^{-5}/°C and the Young's modulus of steel is 2×10^{11}Pa)

(A) 6.6×10^7 Pa (B) 8.8×10^7 Pa (C) 3.3×10^7 Pa (D) 5.5×10^7 Pa

Ans. (C)

Sol. Stress $= y(\alpha\Delta T) = 2 \times 10^{11} \times 1.1 \times 10^{-5} \times 15 = 3.3 \times 10^7$ Pa

94. Electromagnetic waves emanating from a point A (in air) are incident on a rectangular block of material M and emerge from the other side as shown. The angles i and r are angles of incidence and refraction when the wave travels from air to the medium. Such paths for the rays are possible



(A) if the material has a refractive index very nearly equal to zero,
(B) only with gamma rays with a wavelength smaller than the atomic nuclei of the material.
(C) if the material has a refractive index less than zero.
(D) only if the wave travels in M with a speed faster than the speed of light in vacuum.

Ans. (C)

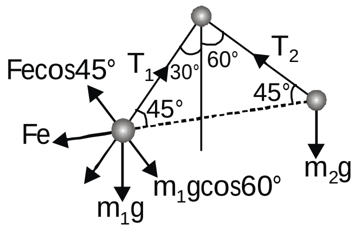
Sol. If refractive index less than zero r in negative.

95. Two small metal balls of different mass m_1 and m_2 are connected by strings of equal length to a fixed point. When the balls are given equal charges, the angles that the two strings make with the vertical are 30° and 60°, respectively. The ratio m_1/m_2 is close to

(A) 1.7 (B) 3.0 (C) 0.58 (D) 2.0

Ans. (A)

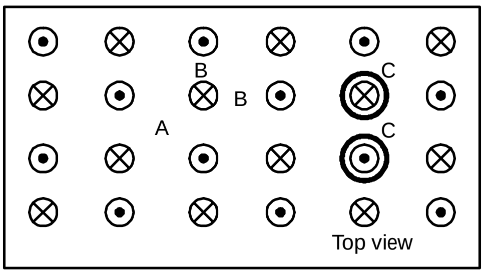
Sol. $m_1g\cos 60° = Fe\cos 45°$ ....(1)
$m_2g\cos 30 = Fe\cos 45°$ ....(2)
(1) ÷ (2)



$$\frac{m_1 g\cos 60°}{m_2 g\cos 30°} = 1$$

$$\frac{m_1}{m_2} = \sqrt{3} = 1.7$$

96. Consider the regular array of vertical identical current carrying wires (with direction of current flow as indicated in the figure below) protruding through a horizontal table. If we scatter some diarnagnetic particles on the table, they are likely to accumulate

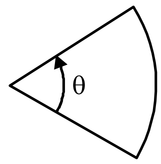


(A) around regions such as A.
(B) around regions such as B.
(C) in circular regions around individual wires such as C.
(D) uniformly everywhere.

Ans. (A)

Sol. Diamagnetic particle moves from strong to week magnetic field, that is why they will accumulate around region such as (A)

97. The distance between the vertex and the center of mass of a uniform solid planar circular segment of angular size θ and radius R is given by



(A) $\frac{4}{3}R\frac{\sin(\theta/2)}{\theta}$ (B) $R\frac{\sin(\theta/2)}{\theta}$ (C) $\frac{4}{3}R\cos\left(\frac{\theta}{2}\right)$ (D) $\frac{2}{3}R\cos(\theta)$

Ans. (A)

Sol. By checking methed.

CM for half dbc $= \frac{4R}{3\pi}$

By putting in options $\theta = \pi$

in option (A) $\frac{4}{3}R\frac{\sin\pi/2}{\pi} = \frac{4R}{3\pi}$

(A) is correct

98. An object is propelled vertically to a maximum height of 4R from the surface of a planet of radius R and mass M. The speed of object when it returns to the surface of the planet is

(A) $2\sqrt{\frac{2GM}{5R}}$ (B) $\sqrt{\frac{GM}{2R}}$ (C) $\sqrt{\frac{3GM}{2R}}$ (D) $\sqrt{\frac{GM}{5R}}$

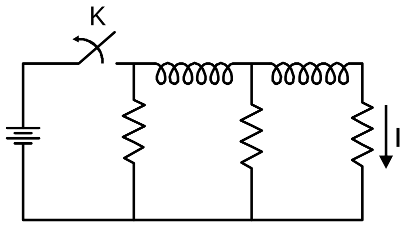
Ans. (A)

Sol. $\frac{GMem}{Re} - \frac{GMem}{SRe} = \frac{1}{2}mv^2$

$2 \times \frac{4}{5}\frac{GMe}{Re} = v^2$

$v = 2\sqrt{\frac{2}{5}\frac{GMe}{R}}$

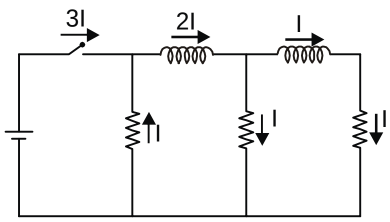
99. In the circuit shown below, all the inductors (assumed ideal) and resistors are identical. The current through the resistance on the right is / after the key K has been switched on for a long time. The currents through the three resistors (in order, from left to right) immediately after the key is switched off are



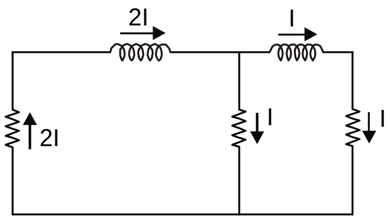
(A) 2I upwards, I downwards and I downwards.
(B) 2I downwards, I downwards and I downwards.
(C) I downwards, I downwards and I downwards.
(D) 0 downwards and I downwards

Ans. (A)

Sol.

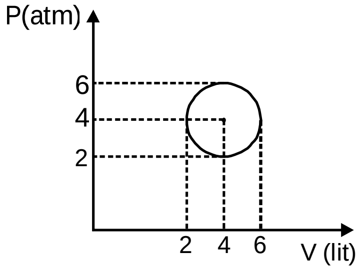


After switch is opened, current through inductors remain same.



Hence 2I upwards, I downwards, I downwards.

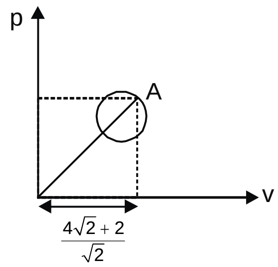
100. An ideal gas undergoes a circular cycle centered at 4 atm, 4 lit as shown in the diagram. The maximum temperature attained in this process is close to



(A) 30/R (B) 36/R (C) 24/R (D) 16/R

Ans. (A)

Sol. Temperature will be maximum at point A



$$pv = nRT$$

$$\frac{\left(\frac{4\sqrt{2}+2}{\sqrt{2}}\right)^2}{R} = T$$

$$\Rightarrow \frac{(4+\sqrt{2})^2}{R} = T$$

$$\Rightarrow \frac{(5\cdot 414)^2}{R} = T = \frac{30}{R}$$

CHEMISTRY

101. For the reaction $N_2 + 3X_2 \rightarrow 2NX_3$ where X = F, Cl (the average bond energies are F-F = 155 kJ mol^{-1}, N-F = 272 kJ mol^{-1}, Cl-Cl = 242 U mol^{-1}, N-Cl = 200 kJ mol^{-1} and N = N = 941 kJ mol^{-1}), the heats of formation of NF_3 and NCl_3 in kJ mol^{-1}, respectively, are closest to

(A) – 226 and + 467 (B) +226 and –467 (C) –151 and + 311 (D) + 151 and –311

Ans. (A)

Sol. For 2 mol NF_3 $\Delta H_f = 941 + 3(155) - 6(272) = -272\,kJ$

For 2 mol NCl_3 $\Delta H_f = 941 + 3(242) - 6(200) = +467\,kJ$

102. The equilibrium constants for the reactions X = 2Y and Z = P + Q are K_1 and K_2 respectively. If the initial concentrations and the degree of dissociation of X and Z arc the same, the ratio K_1/K_2 is

(A) 4 (B) 1 (C) 0.5 (D) 2

Ans. (A)

Sol. $$\frac{K_1}{K_2} = \frac{\frac{4C^2\alpha^2}{C(1-\alpha)}}{\frac{C^2\alpha^2}{C(1-\alpha)}} = 4$$

103. The geometry and the number of unpaired clectron(s) of $[MnBr_4]^{2-}$, respectively, are

(A) tetrahedral and 1 (B) square planar and 1 (C) tetrahedral and 5 (D) square planar and 5

Ans. (C)

Sol. Mn^{2+} : $[Ar]3d^5$

Br^- is weak field ligand.

104. The standard cell potential for $Zn|Zn^{2+}||(Cu^{2+}|Cu$ is 1.10 V. When the cell is completely discharged, log $[Zn^{2+}]/[Cu^{2+}]$ is closest to

(A) 37.3 (B) 0.026 (C) 18.7 (D) 0.052

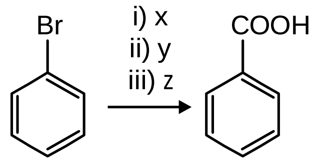
Ans. (A)

Sol. For complete discharging

$E_{Cell} = 0$

$$\therefore \quad E^{\circ}_{Cell} = \frac{0.059}{2}\log\frac{[Zn^{2+}]}{[Cu^{2+}]}$$

105. In the reaction



x,y and z are

(A) x = Mg, dry ether; y = CH_3Cl; z = H_2O

(B) x = Mg, dry methanol; y = CO_2; z = dil. HCl

(C) x = Mg, dry ether, y = CO_2; z = dil. HCl

(D) x = Mg, dry methanol; y = CH_3Cl; z = H_2O

Ans. (C)

Sol. $PhBr \xrightarrow[Ether]{Mg} PhMgBr \xrightarrow{CO_2} PhCOO^-MgBr \xrightarrow{H_3O^+} PhCOOH$

106. An organic compound having molecular formula C_2H_6O undergoes oxidation with $K_2Cr_2O_7/H_2SO_4$ to produce X which contains 40 % carbon, 6.7 % hydrogen and 53.3 % oxygen. The molecular formula of the compound X is

(A) CH_2O (B) $C2H_4O_2$ (C) C_2H_4O (D) $C_2H_6O_2$

Ans. (B)

Sol. $C_{\frac{40}{12}}H_{\frac{6.7}{1}}O_{\frac{53.3}{16}}$

$= C_1H_2O_1$
(e.f. of X)

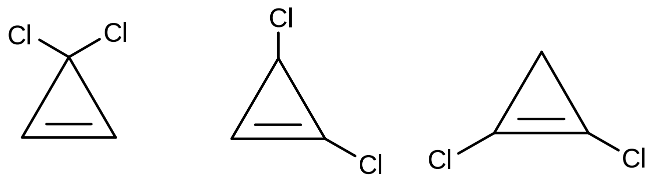
$CH_3CH_2OH \xrightarrow{K_2Cr_2O_7/H_2SO_4} CH_3COOH$

107. The maximum number of cyclic isomers (positional and optical) of a compound having molecular formula $C_3H_2Cl_2$ is

(A) 2 (B) 3 (C) 4 (D) 5

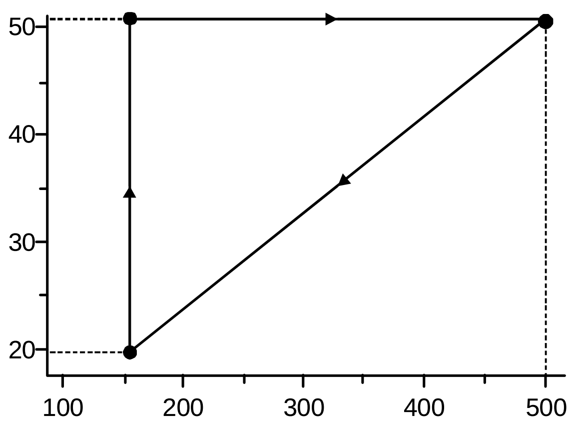
Ans. (C)

Sol. DBE = 2



Shows optical isomerism

108. The volume vs. temperature graph of 1 mole of an ideal gas is given below



The pressure of the gas (in atm) at X, Y and Z, respectively, are

(A) 0.328, 0.820, 0.820 (B) 3.28, 8.20, 3.28

(C) 0.238, 0.280, 0.280 (D) 32.8, 0.280, 82.0

Ans. (A)

Sol. $P_X = \frac{1\times 0.082\times 200}{50} = 0.328\,\text{atm}$

$P_Y = \frac{1\times 0.082\times 500}{50} = 0.82\,\text{atm}$

$P_Z = \frac{1\times 0.082\times 200}{20} = 0.82\,\text{atm}$

109. MnO_2 when fused with KOH and oxidized in air gives a dark green compound X. In acidic solution. X undergoes disproportionation to give an intense purple compound Y and MnO_2. The compounds X and Y, respectively, are

(A) K_2MnO_4 and $KMnO_4$ (B) Mn_2O_7 and $KMnO_4$ (C) $K.MnO_4$ and Mn,O_7 (D) $KMnO_4$ and K_2MnO_4

Ans. (A)

Sol. $MnO_2 + KOH \xrightarrow[\Delta]{O_2} \underset{'X'}{K_2MnO_4} \xrightarrow{H^+} \underset{\text{purple}}{KMnO_4} + MnO_2$

110. A metal (X) dissolves both in dilute HCI and dilute NaOH to liberate H_2. Addition of NH_4Cl and excess NH_4OH to an HCI solution of X produces Y as a precipitate. Y is also produced by adding NH_4Cl to the NaOH solution of X. The species X and Y. respectively, are

(A) Zn and $Zn(OH)_2$ (B) Al and $Al(OH)_3$ (C) Zn and Na_2ZnO_2 (D) Al and $NaAlO_2$

Ans. (B)

Sol. Both Zn and Al react with dil. HCl and dil. NaOH to release H_2.
$Al(OH)_3$ is insoluble in NH_4OH solution.

BIOLOGY

111. How many bands are seen when immunoglobulin G molecules are analysed on a sodium dodecyl sulphate-polyacrylamidc gel electrophoresis (SDS-PAGE) under reducing conditions?

(A) 6 (B) I (C) 2 (D) 4

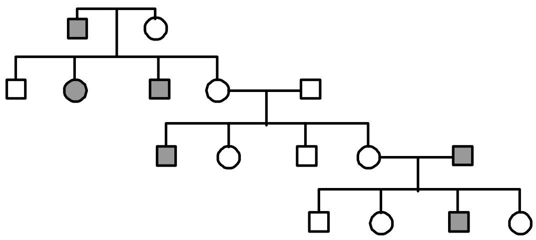
Ans. (C)

112. In a mixed culture of slow and fast growing bacteria, penicillin will

(A) kill the fast growing bacteria more than the slow growing
(B) kill slow growing bacteria more than the fast growing
(C) kill both the fast and slow growing bacteria equally
(D) will not kill bacteria at ail

Ans. (C)

113. Consider the following pedigree over four generations and mark the correct answer below about the inheritance of haemophilia.



□ Normal male
■ Haemophtilic male
○ Normal female
■ Hacmophilic female

(A) Haemophilia is X-linked dominant (B) Haemophilia is autosomal dominant
(C) Haemophilia is X-linked recessive (D) Haemophilia is Y-linked dominant

Ans. (C)

114. A person has 400 million alveoli per lung with an average radius of 0.1 mm for each alveolus. Considering the alveoli arc spherical in shape, the total respiratory- surface of that person is closest to

(A) 500 mm^2 (B) 200 mm^2 (C) 100 mm^2 (D) 1000 mm^2

Ans. (A)

115. A mixture of equal numbers of fast and slow dividing cells is cultured in a medium containing a trace amount of radioactively labeled thymidine for one hour. The cells are then transferred to regular (unlabelled) medium. After 24 hrs of growth in regular media

(A) fast dividing cells will have maximum radioactivity

(B) slow dividing cells will have maximum radioactivity

(C) both will have same amount of radioactivity

(D) there will be no radioactivity in either types of cells

Ans. (A)

116. If a double stranded DNA has 15% cytosine, what is the % of adenine in the DNA?

(A) 15 % (B) 70 % (C) 35 % (D) 30 %

Ans. (C)

117. The mitochondrial inner membrane consists of a number of infoldings called cristae. The increased surface area due to cristae helps in:

(A) Increasing the volume of mitochondria

(B) Incorporating more of the protein complexes essential for electron transport chain

(C) Changing the pH

(D) Increasing diffusion of ions

Ans. (B)

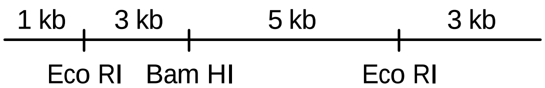
118. The activity of a certain protein is dependent on its phosphorylation. A mutation in its gene changed a single amino acid which affected the function of the molecule. Which amino acid change is most likely to account for this observation?

(A) Tyrosine to Tryptophan (B) Lysine to valine

(C) Leucine to isoleucine (D) Valine to alanine

Ans. (A)

119. Consider the linear double stranded DNA shown below. The restriction enzyme sites and the lengths demarcated are shown. This DNA is completely digested with both EcoRI and *Bam*HI restriction enzymes. If the product is analyzed by gel electrophoresis. How many distinct bands would be observed?



(A) 5 (B) 2 (C) 3 (D) 4

Ans. (A)

120. Enzyme X catalyzes hydrolysis of GTP into GDP. The GTP-bound form of X transmits a signal that leads to cell proliferation. The GDP-bound form does not transmit any such signal. Mutations in X arc found in many cancers. Which of the following alterations of X are most likely to contribute to cancer?

(A) Mutations that increase the affinity of X for GDP.

(B) Mutations that decrease the affinity of X for GTP.

(C) Mutations that decrease the rate of GTP hydrolysis.

(D) Mutations that prevent expression of enzyme X.

Ans. (D)