WAVE MOTION

1. Which of the following equations represents a travelling wave ?

(1) $y = A\sin(15x - 2t)$ (2) $y = Ae^{-x^2}(vt+\theta)$

(3) $y = Ae^{x}\cos(\omega t - \theta)$ (4) $y = A\sin x \cos\omega t$

2. Two cars are approaching each other at an equal speed of 7.2 km/hr. When they see each other, both blow horns having frequency of 676 Hz. The beat frequency heard by each driver will be _____ Hz. [Velocity of sound in air is 340 m/s.]

3. A student is performing the experiment of resonance column. The diameter of the column tube is 6 cm. The frequency of the tuning fork is 504 Hz. Speed of the sound at the given temperature is 336 m/s. The zero of the meter scale coincides with the top end of the resonance column tube. The reading of the water level in the column when the first resonance occurs is:

(1) 13 cm (2) 16.6 cm
(3) 18.4 cm (4) 14.8 cm

4. The percentage increase in the speed of transverse waves produced in a stretched string if the tension is increased by 4%, will be __________%.

5. The mass per unit length of a uniform wire is 0.135 g/cm. A transverse wave of the form $y = -0.21 \sin(x + 30t)$ is produced in it, where x is in meter and t is in second. Then, the expected value of tension in the wire is $x \times 10^{-2}$ N. Value of x is. (Round-off to the nearest integer)

6. A tuning fork A of unknown frequency produces 5 beats/s with a fork of known frequency 340 Hz. When fork A is filed, the beat frequency decreases to 2 beats/s. What is the frequency of fork A ?

(1) 342 Hz (2) 345 Hz
(3) 335 Hz (4) 338 Hz

7. A closed organ pipe of length L and an open organ pipe contain gases of densities ρ_1 and ρ_2 respectively. The compressibility of gases are equal in both the pipes. Both the pipes are vibrating in their first overtone with same frequency. The length of the open pipe is $\frac{x}{3}L\sqrt{\frac{\rho_1}{\rho_2}}$ where x is ________. (Round off to the Nearest Integer)

8. A sound wave of frequency 245 Hz travels with the speed of 300 ms^{-1} along the positive x-axis. Each point of the wave moves to and fro through a total distance of 6 cm. What will be the mathematical expression of this travelling wave ?

(1) $Y(x,t) = 0.03 [\sin 5.1 x - (0.2 \times 10^3)t]$
(2) $Y(x,t) = 0.06 [\sin 5.1 x - (1.5 \times 10^3)t]$
(3) $Y(x,t) = 0.06 [\sin 0.8 x - (0.5 \times 10^3)t]$
(4) $Y(x,t) = 0.03 [\sin 5.1 x - (1.5 \times 10^3)t]$

9. The amplitude of wave disturbance propagating in the positive x-direction is given by $y = \frac{1}{(1+x)^2}$ at time t = 0 and $y = \frac{1}{1+(x-2)^2}$ at t = 1s, where x and y are in meres. The shape of wave does not change during the propagation. The velocity of the wave will be ____m/s.

10. The frequency of a car horn encountered a change from 400 Hz to 500 Hz. When the car approaches a vertical wall. If the speed of sound is 330 m/s. Then the speed of car is ______ km/h.

11. With what speed should a galaxy move outward with respect to earth so that the sodium-D line at wavelength 5890 Å is observed at 5896 Å ?

(1) 306 km/sec (2) 322 km/sec
(3) 296 km/sec (4) 336 km/sec

12. A source and a detector move away from each other in absence of wind with a speed of 20 m/s with respect to the ground. If the detector detects a frequency of 1800 Hz of the sound coming from the source, then the original frequency of source considering speed of sound in air 340 m/s will be .... Hz.

13. Two travelling waves produces a standing wave represented by equation,
$y = 1.0\text{ mm} \cos(1.57\text{ cm}^{-1})\ x \sin(78.5\text{ s}^{-1})t$.
The node closest to the origin in the region x > 0 will be at x = ... cm.

14. Two waves are simultaneously passing through a string and their equations are :
$y_1 = A_1 \sin k(x-vt)$, $y_2 = A_2 \sin k(x-vt + x_0)$. Given amplitudes A_1 = 12 mm and A_2 = 5 mm, x_0 = 3.5 cm and wave number k = 6.28 cm^{-1}. The amplitude of resulting wave will be .... mm.

15. Two cars X and Y are approaching each other with velocities 36 km/h and 72 km/h respectively. The frequency of a whistle sound as emitted by a passenger in car X, heard by the passenger in car Y is 1320 Hz. If the velocity of sound in air is 340 m/s, the actual frequency of the whistle sound produced is .... Hz.

16. A tuning fork is vibrating at 250 Hz. The length of the shortest closed organ pipe that will resonate with the tuning fork will be _____ cm. (Take speed of sound in air as 340 ms^{-1})

17. A wire having a linear mass density 9.0×10^{-4} kg/m is stretched between two rigid supports with a tension of 900 N. The wire resonates at a frequency of 500 Hz. The next higher frequency at which the same wire resonates is 550 Hz. The length of the wire is ______m.

SOLUTION

1. Official Ans. by NTA (1)

Sol. y = F(x, t)

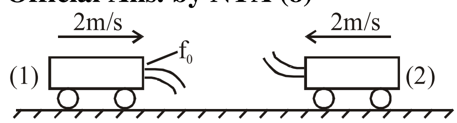
For travelling wave y should be linear function of x and t and they must exist as (x ± vt)

y = Asin(15*x* – 2*t*)→linear function in x and t

Option (1) is correct.

2. Official Ans. by NTA (8)

Sol.



Frequency of sound heard by car-1, which comes by reflection from car-2

$$f_1 = f_0\left(\frac{340+2}{340-2}\right)\left(\frac{340+2}{340-2}\right) = f_0\left(\frac{342}{338}\right)^2$$

Frequency of sound coming directly from car-2

$$f_2 = f_0\left(\frac{340+2}{340-2}\right)$$

$$\therefore\ f_1 - f_2 = f_0\left(\frac{342}{338}\right)\left(\frac{342}{338}-1\right) = 8.09 \simeq 8$$

3. Official Ans. by NTA (4)

Sol. d = 6cm, f = 504, v = 336 m/s

e = 0.3d

$$l + e = \frac{\lambda}{4} = \frac{V}{4f}$$

$l = 16.66 - 0.3 \times 6$

$l = 14.866$ cm

$l = 14.8$ cm

4. Official Ans. by NTA (2)

Sol. $v = \sqrt{\frac{T}{\mu}}$

$$\frac{\Delta V}{V} = \frac{1}{2}\frac{\Delta T}{T}$$

5. Official Ans. by NTA (1215)

Sol. μ = 0.135 gm/cm = 0.0135 kg/m

y = –0.21 sin (x + 30t)

(x in meter & t in sec)

$$v = \frac{\omega}{k} = \frac{30}{1} = 30\text{m/s}$$

$v = \sqrt{\frac{T}{\mu}} \Rightarrow T = v^2\mu = (30)^2\ (0.0135)$

= 12.15

$= x \times 10^{-2}$ N ⇒ x = 1215

6. Official Ans. by NTA (3)

Sol. Initially beat frequency = 5 Hz

so, ρ_A = 340 ± 5 = 345 Hz, or 335 Hz

after filing frequency increases slightly

so, new value of frequency of A > ρ_A

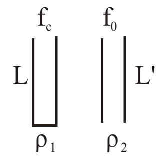
Now, beat frequency = 2Hz

⇒ new ρ_A = 340 ± 2 = 342 Hz, or 338 Hz

hence, original frequency of A is ρ_A = 335 Hz

7. Official Ans. by NTA (4)

Sol. Ans. (4)



$f_c = f_0$

$$\frac{3V_C}{4L} = \frac{2V_0}{2L'}$$

$$\frac{3V_C}{4L} = \frac{V_0}{L'}$$

$$L' = \frac{4L}{3}\frac{V_0}{V_C} = \frac{4L}{3}\sqrt{\frac{B.\rho_1}{\rho_2.B}}$$ (B is bulk modulus)

$$= \frac{4L}{3}\sqrt{\frac{\rho_1}{\rho_2}}$$ x = 4

8. Official Ans. by NTA (4)

Sol. (4) ω = 2π f

$= 1.5 \times 10^3$

$A = \frac{6}{2}$ = 3 cm = 0.03 m

9. Official Ans. by NTA (2)

Sol. At t = 0, $y = \frac{1}{1+x^2}$

At time t = t, $y = \frac{1}{1+(x-vt)^2}$

At t = 1, $y = \frac{1}{1+(x-v)^2}$ … (i)

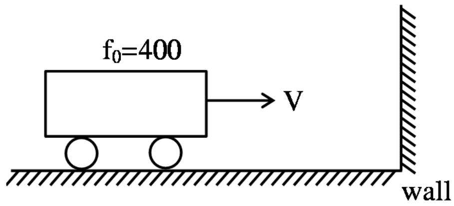
At t = 1, $y = \frac{1}{1+(x-2)^2}$ … (ii)

Comparing (i) & (ii)

v = 2m/s

10. Official Ans. by NTA (132)

Sol.



Wall as an observer

Frequency received by wall

$$f_1 = f_0\left(\frac{C}{C-V}\right)$$

Again wall as a source

Frequency received by observer on car

$$f_2 = f_1\left(\frac{C+V}{C}\right)$$

$$f_2 = f_0\left(\frac{C+V}{C-V}\right); \quad 500 = 400\left(\frac{C+V}{C-V}\right)$$

$$\frac{5}{4} = \frac{C+V}{C-V}$$

C = 9V

$$V = \frac{C}{9} = \frac{330}{9}\,m/s$$

$$V = \frac{330}{9}\times\frac{18}{5} = 132\,km/hr$$

11. Official Ans. by NTA (1)

Sol. $f = f_0\sqrt{\frac{1+\beta}{1-\beta}}$; $\beta = \frac{v}{c}$

$$\frac{f}{f_0}\sqrt{\frac{1+\beta}{1-\beta}}$$

$$\left(1+\frac{\Delta f}{f_0}\right)^2 = (1+\beta)(1-\beta)^{-1}$$

β is small compared to 1

$$\left(1+\frac{2\Delta f}{f_0}\right) = (1+2\beta)$$

$$\beta = \frac{\Delta f}{f_0} = \frac{v}{c}$$

$$v = 6\times\frac{c}{5890} = 305.6\text{ km/s}$$

12. Official Ans. by NTA (2025)

Sol.

$$f' = f\left(\frac{C-V_0}{C+V_s}\right)$$

$$1800 = f\left(\frac{340-20}{340+20}\right)$$

f = 2025 Hz

Ans. 2025

13. Official Ans. by NTA (1)

Sol. For node

$$\cos\left(1.57cm^{-1}\right)x = 0$$

$$\left(1.57cm^{-1}\right)x = \frac{\pi}{2}$$

$$x = \frac{\pi}{2(1.57)}cm = 1cm$$

Ans. 1.00

14. Official Ans. by NTA (7)

Sol. $y_1 = A_1 \sin k(x - vt)$

$y_1 = 12 \sin 6.28\ (x - vt)$

$y_2 = 5 \sin 6.28\ (x - vt + 3.5)$

$$\Delta\phi = \frac{2\pi}{\lambda}(\Delta x)$$

$= K(\Delta x)$

$$= 6.28 \times 3.5 = \frac{7}{2}\times 2\pi = 7\pi$$

$$A_{net} = \sqrt{A_1^2 + A_2^2 + 2A_1A_2\cos\phi}$$

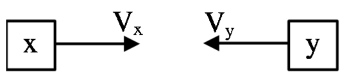
$$A_{net} = \sqrt{(12)^2 + (5)^2 + 2(12)(5)\cos(7\pi)}$$

$$= \sqrt{144+25-120}$$

Ans. 7

15. Official Ans. by NTA (1210)

Sol.



V_x = 36 km/hr = 10 m/s

V_y = 72 km/hr = 20 m/s

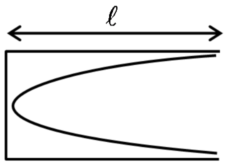
by doppler's effect

$$F' = F_0\left(\frac{V\pm V_0}{V\pm V_s}\right)$$

$$1320 = F_0\left(\frac{340+20}{340-10}\right) \Rightarrow F_0 = 1210\text{ Hz}$$

16. Official Ans. by NTA (34)

Sol.



$\frac{\lambda}{4} = \ell \Rightarrow \lambda = 4\ell$

$f = \frac{V}{\lambda} = \frac{V}{4\ell}$

$\Rightarrow 250 = \frac{340}{4\ell}$

$\Rightarrow \quad \ell = \frac{34}{4 \times 25} = 0.34\text{m}$

$\ell = 34\text{cm}$

17. Official Ans. by NTA (10)

Sol. $\mu = 9.0 \times 10^{-4} \frac{\text{kg}}{\text{m}}$

T = 900 N

$V = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{900}{9 \times 10^{-4}}} = 1000\text{m/s}$

f_1 = 500 Hz

f = 550

$\frac{nV}{2\ell} = 500$ …. (i)

$\frac{(n+1)V}{2\ell} = 500$ … (ii)

(ii) (i) $\frac{V}{2\ell} = 50$

$\ell = \frac{1000}{2 \times 50} = 10$