UNIT & DIMENSION

1. The workdone by a gas molecule in an isolated system is given by, $W = \alpha\beta^2 e^{-\frac{x^2}{\alpha kT}}$, where x is the displacement, k is the Boltzmann constant and T is the temperature, α and β are constants. Then the dimension of β will be :
(1) $[M L^2 T^{-2}]$ (2) $[M L T^{-2}]$
(3) $[M^2 L T^2]$ (4) $[M^0 L T^0]$

2. Match List-I with List-II :

List-I	List-II
(a) h (Planck's constant)	(i) $[M L T^{-1}]$
(b) E (kinetic energy)	(ii) $[M L^2 T^{-1}]$
(c) V (electric potential)	(iii) $[M L^2 T^{-2}]$
(d) P (linear momentum)	(iv) $[M L^2 I^{-1} T^{-3}]$

Choose the correct answer from the options given below :
(1) (a)→(iii), (b)→(iv), (c)→(ii), (d)→(i)
(2) (a)→(ii), (b)→(iii), (c)→(iv), (d)→(i)
(3) (a)→(i), (b)→(ii), (c)→(iv), (d)→(iii)
(4) (a)→(iii), (b)→(ii), (c)→(iv), (d)→(i)

3. If e is the electronic charge, c is the speed of light in free space and h is Planck's constant, the quantity $\frac{1}{4\pi\varepsilon_0}\frac{|e|^2}{hc}$ has dimensions of :
(1) $[M^0 L^0 T^0]$ (2) $[L C^{-1}]$
(3) $[M L T^{-1}]$ (4) $[M L T^0]$

4. In a typical combustion engine the work done by a gas molecule is given $W = \alpha^2\beta e^{\frac{-\beta x^2}{kT}}$, where x is the displacement, k is the Boltzmann constant and T is the temperature. If α and β are constants, dimensions of α will be :
(1) $[MLT^{-2}]$ (2) $[M^0LT^0]$
(3) $[M^2LT^{-2}]$ (4) $[MLT^{-1}]$

5. If 'C' and 'V' represent capacity and voltage respectively then what are the dimensions of λ, where $\frac{C}{V} = \lambda$?
(1) $[M^{-2}L^{-3}I^2T^6]$ (2) $[M^{-3}L^{-4}I^3T^7]$
(3) $[M^{-1}L^{-3}I^{-2}T^{-7}]$ (4) $[M^{-2}L^{-4}I^3T^7]$

6. If time (*t*), velocity (*v*), and angular momentum (*l*) are taken as the fundamental units. Then the dimension of mass (m) in terms of *t*, *v* and *l* is :
(1) $[t^{-1} v^1 l^{-2}]$ (2) $[t^1 v^2 l^{-1}]$
(3) $[t^{-2} v^{-1} l^1]$ (4) $[t^{-1} v^{-2} l^1]$

7. The force is given in terms of time t and displacement x by the equation
F = A cos Bx + C sin Dt
The dimensional formula of $\frac{AD}{B}$ is :
(1) $[M^0 L T^{-1}]$ (2) $[M L^2 T^{-3}]$
(3) $[M^1 L^1 T^{-2}]$ (4) $[M^2 L^2 T^{-3}]$

8. If E, L, M and G denote the quantities as energy, angular momentum, mass and constant of gravitation respectively, then the dimensions of P in the formula $P = EL^2M^{-5}G^{-2}$ are :-
(1) $[M^0 L^1 T^0]$ (2) $[M^{-1} L^{-1} T^2]$
(3) $[M^1 L^1 T^{-2}]$ (4) $[M^0 L^0 T^0]$

9. Match List–I with List–II.

List-I		List-II	
(a)	Magnetic Induction	(i)	$ML^2T^{-2}A^{-1}$
(b)	Magnetic Flux	(ii)	$M^0L^{-1}A$
(c)	Magnetic Permeability	(iii)	$MT^{-2}A^{-1}$
(d)	Magnetization	(iv)	$MLT^{-2}A^{-2}$

Choose the most appropriate answer from the options given below :
(1) (a)-(ii), (b)-(iv), (c)-(i), (d)-(iii)
(2) (a)-(ii), (b)-(i), (c)-(iv), (d)-(iii)
(3) (a)-(iii), (b)-(ii), (c)-(iv), (d)-(i)
(4) (a)-(iii), (b)-(i), (c)-(iv), (d)-(ii)

10. Which of the following is not a dimensionless quantity ?
(1) Relative magnetic permeability (μ_r)
(2) Power factor
(3) Permeability of free space (μ_0)
(4) Quality factor

11. If E and H represents the intensity of electric field and magnetising field respectively, then the unit of E/H will be :
(1) ohm (2) mho
(3) joule (4) newton

12. Match List-I with List-II.

List-I	List-II
(a) R_H (Rydberg constant)	(i) $kg\ m^{-1} s^{-1}$
(b) h(Planck's constant)	(ii) $kg\ m^2 s^{-1}$
(c) μ_B (Magnetic field energy density)	(iii) m^{-1}
(d) η(coefficient of viscocity)	(iv) $kg\ m^{-1} s^{-2}$

Choose the most appropriate answer from the options given below :
(1) (a)–(ii), (b)–(iii), (c)–(iv), (d)–(i)
(2) (a)–(iii), (b)–(ii), (c)–(iv), (d)–(i)
(3) (a)–(iv), (b)–(ii), (c)–(i), (d)–(iii)
(4) (a)–(iii), (b)–(ii), (c)–(i), (d)–(iv)

13. If force (F), length (L) and time (T) are taken as the fundamental quantities. Then what will be the dimension of density :

(1)$[FL^{-4}T^{2}]$ (2) $[FL^{-3}T^{2}]$

(3) $[FL^{-5}T^{2}]$ (4) $[FL^{-3}T^{3}]$

14. Match **List-I** with **List-II**.

List-I	**List-II**
(a) Torque	(i) MLT^{-1}
(b) Impulse	(ii) MT^{-2}
(c) Tension	(iii) $ML^{2}T^{-2}$
(d) Surface Tension	(iv) MLT^{-2}

Choose the **most appropriate** answer from the option given below :

(1) (a)–(iii), (b)–(i), (c)–(iv), (d)–(ii)

(2) (a)–(ii), (b)–(i), (c)–(iv), (d)–(iii)

(3) (a)–(i), (b)–(iii), (c)–(iv), (d)–(ii)

(4) (a)–(iii), (b)–(iv), (c)–(i), (d)–(ii)

15. Which of the following equations is dimensionally incorrect ?

Where t = time, h = height, s = surface tension, θ = angle, ρ = density, a, r = radius, g = acceleration due to gravity, v = volume , p = pressure, W = work done, Γ = torque, $\in$= permittivity, E = electric field, J = current density, L = length.

(1) $v = \frac{\pi p a^{4}}{8\eta L}$ (2) $h = \frac{2s\cos\theta}{\rho r g}$

(3) $J = \in\frac{\partial E}{\partial t}$ (4) $W = \Gamma\theta$

16. If velocity [V], time [T] and force [F] are chosen as the base quantities, the dimensions of the mass will be :

(1) $[FT^{-1}V^{-1}]$ (2) $[FTV^{-1}]$

(3) $[FT^{2}V]$ (4) $[FVT^{-1}]$

SOLUTION

1. Official Ans. by NTA (2)

Sol. $\frac{x^2}{\alpha kT} \rightarrow$ dimensionless

$\Rightarrow [\alpha] = \frac{[x^2]}{[kT]} = \frac{L^2}{ML^2T^{-2}} = M^{-1}T^2$

Now $[W] = [\alpha][\beta]^2$

$[\beta] = \sqrt{\frac{ML^2T^{-2}}{M^{-1}T^2}} = M^1L^1T^{-2}$

2. Official Ans. by NTA (2)

Sol. By dimensional analysis.

3. Official Ans. by NTA (1)

Sol. $F = \frac{1}{4\pi \in_0}\frac{e^2}{r^2}$; $E = \frac{hc}{\lambda}$

$\left[\frac{e^2}{4\pi\varepsilon_0} \times \frac{1}{hc}\right] = \frac{Fr^2}{E\lambda} = \left(M^0L^0T^0\right)$

4. Official Ans. by NTA (2)

Sol. kT has dimension of energy

$\frac{\beta x^2}{kT}$ is dimensionless

$[\beta][L^2] = [ML^2T^{-2}]$

$[\beta] = [MT^{-2}]$

$\alpha^2\beta$ has dimensions of work

$[\alpha^2][MT^{-2}] = [ML^2T^{-2}]$

$[\alpha] = [M^0LT^0]$

Ans. 2

5. Official Ans. by NTA (4)

Sol. $\lambda = \frac{C}{V} = \frac{Q/V}{V} = \frac{Q}{V^2}$

$V = \frac{\text{work}}{Q}$

$\lambda = \frac{Q^3}{(\text{work})^2} = \frac{(It)^3}{(F.s)^2}$

$= \frac{[I^3T^3]}{[ML^2T^{-2}]^2} = [M^{-2}L^{-4}I^3T^7]$

6. Official Ans. by NTA (4)

Sol. $m \propto t^a v^b \ell^c$

$m \propto [T]^a[LT^{-1}]^b[ML^2T^{-1}]^c$

$M^1L^0T^0 = M^cL^{b+2c}T^{a-b-c}$

comparing powers

$c = 1, b = -2, a = -1$

$m \propto t^{-1}v^{-2}\ell^1$

7. Official Ans. by NTA (2)

Sol. $[A] = [MLT^{-2}]$

$[B] = [L^{-1}]$

$[D] = [T^{-1}]$

$\left[\frac{AD}{B}\right] = \frac{[MLT^{-2}][T^{-1}]}{[L^{-1}]}$

$\left[\frac{AD}{B}\right] = [ML^2T^{-3}]$

8. Official Ans. by NTA (4)

Sol. $E = ML^2T^{-2}$

$L = ML^2T^{-1}$

$m = M$

$G = M^{-1}L^{+3}T^{-2}$

$P = \frac{EL^2}{M^5G^2}$

$[P] = \frac{(ML^2T^{-2})(M^2L^4T^{-2})}{M^5(M^{-2}L^6T^{-4})} = M^0L^0T^0$

Option (4)

9. Official Ans. by NTA (4)

Sol. (a) Magnetic Induction = $MT^{-2}A^{-1}$

(b) Magnetic Flux = $ML^2T^{-2}A^{-1}$

(c) Magnetic Permeability = $MLT^{-2}A^{-2}$

(d) Magnetization = $M^0L^{-1}A$

Ans. 4

10. Official Ans. by NTA (3)

Sol. $[\mu_r] = 1$ as $\mu_r = \frac{\mu}{\mu_m}$

[power factor ($\cos\phi$)] = 1

$\mu_0 = \frac{B_0}{H}$ (unit = NA^{-2}) : Not dimensionless

$[\mu_0] = [MLT^{-2}A^{-2}]$

quality factor (Q) = $\frac{\text{Energy stored}}{\text{Energy dissipated per cycle}}$

So Q is unitless & dimensionless.

11. Official Ans. by NTA (1)

Sol. Unit of $\frac{E}{H}$ is $\frac{\text{volt / metre}}{\text{Ampere / metre}}$

$= \frac{\text{volt}}{\text{Ampere}} = \text{ohm}$

12. Official Ans. by NTA (2)

Sol. SI unit of Rydberg const. = m^{-1}

SI unit of Plank's const. = kg m^2s^{-1}

SI unit of Magnetic field energy density= kg $m^{-1}s^{-2}$

SI unit of coeff. of viscosity = kg $m^{-1}s^{-1}$

13. Official Ans. by NTA (1)

Sol. Density = $[F^aL^bT^c]$

$[ML^{-3}] = [M^aL^aT^{-2a}L^bT^c]$

$[M^1L^{-3}] = [M^aL^{a+b}T^{-2a+c}]$

a = 1; a + b = –3; –2a + c = 0

1 + b = –3 c = 2a

b = – 4 c = 2

So, density = $[F^1L^{-4}T^2]$

14. Official Ans. by NTA (1)

Sol. torque $\tau \rightarrow ML^2T^{-2}$ (III)

Impulse $I \Rightarrow MLT^{-1}$ (I)

Tension force $\Rightarrow MLT^{-2}$ (IV)

Surface tension $\Rightarrow MT^{-2}$ (II)

Option (1)

15. Official Ans. by NTA (1)

Sol. (i) $\frac{\pi p a^4}{8\eta L} = \frac{dv}{dt}$ = Volumetric flow rate (poiseuille's law)

(ii) $h\rho g = \frac{2s}{r}\cos\theta$

(iii) RHS $\Rightarrow \varepsilon \times \frac{1}{4\pi\varepsilon_0}\frac{a}{r^2}\times\frac{1}{\varepsilon} = \frac{q}{t}\times\frac{1}{r^2}$

$= \frac{I}{L^2} = IL^{-2}$

LHS

$T = \frac{I}{A} = IL^{-2}$

(iv) $W = \tau\theta$

Option (1)

16. Official Ans. by NTA (2)

Sol. $[M] = K[F]^a [T]^b [V]^c$

$[M^1] = [M^1L^1T^{-2}]^a [T^1]^b [L^1T^{-1}]^c$

a = 1, b = 1, c = –1

$\therefore$ $[M] = [FTV^{-1}]$