GRAVITATION

1. Two stars of masses m and 2m at a distance d rotate about their common centre of mass in free space. The period of revolution is :

(1) $\frac{1}{2\pi}\sqrt{\frac{d^3}{3Gm}}$ (2) $2\pi\sqrt{\frac{d^3}{3Gm}}$

(3) $\frac{1}{2\pi}\sqrt{\frac{3Gm}{d^3}}$ (4) $2\pi\sqrt{\frac{3Gm}{d^3}}$

2. Four identical particles of equal masses 1kg made to move along the circumference of a circle of radius 1 m under the action of their own mutual gravitational attraction. The speed of each particle will be :

(1) $\sqrt{\frac{G}{2}(1+2\sqrt{2})}$ (2) $\sqrt{G(1+2\sqrt{2})}$

(3) $\sqrt{\frac{G}{2}(2\sqrt{2}-1)}$ (4) $\sqrt{\frac{(1+2\sqrt{2})G}{2}}$

3. Consider two satellites S_1 and S_2 with periods of revolution 1 hr. and 8hr. respectively revolving around a planet in circular orbits. The ratio of angular velocity of satellite S_1 to the angular velocity of satellites S_2 is :

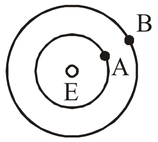
(1) 8 : 1 (2) 1 : 4
(3) 2 : 1 (4) 1 : 8

4. A body weighs 49 N on a spring balance at the north pole. What will be its weight recorded on the same weighing machine, if it is shifted to the equator ?

(Use $g = \frac{GM}{R^2} = 9.8\ ms^{-2}$ and radius of earth, R = 6400 km.]

(1) 49 N (2) 48.83 N
(3) 49.83 N (4) 49.17 N

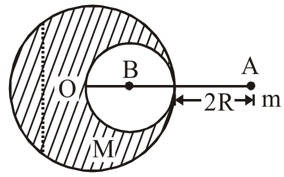
5. Two satellites A and B of masses 200kg and 400kg are revolving round the earth at height of 600 km and 1600 km respectively. If T_A and T_B are the time periods of A and B respectively then the value of $T_B - T_A$:



[Given : radius of earth = 6400km, mass of earth = 6×10^{24} kg]

(1) 1.33×10^3 s (2) 3.33×10^2 s
(3) 4.24×10^3 s (4) 4.24×10^2 s

6. A solid sphere of radius R gravitationally attracts a particle placed at 3R form its centre with a force F_1. Now a spherical cavity of radius $\left(\frac{R}{2}\right)$ is made in the sphere (as shown in figure) and the force becomes F_2. The value of $F_1 : F_2$ is :



(1) 25 : 36 (2) 36 : 25
(3) 50 : 41 (4) 41 : 50

7. Given below are two statements : one is labelled as Assertion A and the other is labelled as Reason R.

Assertion A : The escape velocities of planet A and B are same. But A and B are of unequal mass.

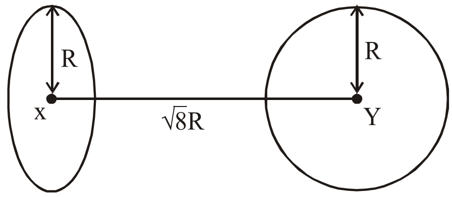
Reason R : The product of their mass and radius must be same, $M_1R_1 = M_2R_2$

In the light of the above statements, choose the most appropriate answer from the options given below :

(1) Both A and R are correct but R is NOT the correct explanation of A
(2) A is correct but R is not correct
(3) Both A and R are correct and R is the correct explanation of A
(4) A is not correct but R is correct

8. The initial velocity v_i required to project a body vertically upward from the surface of the earth to reach a height of 10R, where R is the radius of the earth, may be described in terms of escape velocity v_e such that $v_i = \sqrt{\frac{x}{y}} \times v_e$. The value of x will be ________.

9. Find the gravitational force of attraction between the ring and sphere as shown in the diagram, where the plane of the ring is perpendicular to the line joining the centres. If $\sqrt{8}R$ is the distance between the centres of a ring (of mass 'm') and a sphere (mass 'M') where both have equal radius 'R'.



(1) $\frac{\sqrt{8}}{9}\cdot\frac{GmM}{R}$ (2) $\frac{2\sqrt{2}}{3}\cdot\frac{GMm}{R^2}$

(3) $\frac{1}{3\sqrt{8}}\cdot\frac{GMm}{R^2}$ (4) $\frac{\sqrt{8}}{27}\cdot\frac{GmM}{R^2}$

10. Assume that a tunnel is dug along a chord of the earth, at a perpendicular distance (R/2) from the earth's centre, where 'R' is the radius of the Earth. The wall of the tunnel is frictionless. If a particle is released in this tunnel, it will execute a simple harmonic motion with a time period :

(1) $\frac{2\pi R}{g}$ (2) $\frac{g}{2\pi R}$

(3) $\frac{1}{2\pi}\sqrt{\frac{g}{R}}$ (4) $2\pi\sqrt{\frac{R}{g}}$

11. A planet revolving in elliptical orbit has :

(A) a constant velocity of revolution.

(B) has the least velocity when it is nearest to the sun.

(C) its areal velocity is directly proportional to its velocity.

(D) areal velocity is inversely proportional to its velocity.

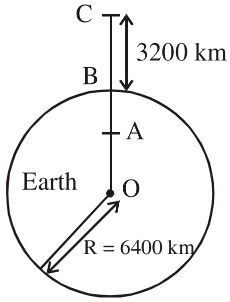
(E) to follow a trajectory such that the areal velocity is constant.

Choose the correct answer from the options given below :

(1) A only (2) D only

(3) C only (4) E only

12. In the reported figure of earth, the value of acceleration due to gravity is same at point A and C but it is smaller than that of its value at point B (surface of the earth). The value of OA : AB will be x : y. The value of x is ....



13. The maximum and minimum distances of a comet from the Sun are 1.6×10^{12} m and 8.0×10^{10} m respectively. If the speed of the comet at the nearest point is 6×10^4 ms^{-1}, the speed at the farthest point is :

(1) 1.5×10^3 m/s (2) 6.0×10^3 m/s

(3) 3.0×10^3 m/s (4) 4.5×10^3 m/s

14. If one wants to remove all the mass of the earth to infinity in order to break it up completely. The amount of energy that needs to be supplied will be $\frac{x}{5}\frac{GM^2}{R}$ where x is ____ (Round off to the Nearest Integer)

(M is the mass of earth, R is the radius of earth, G is the gravitational constant)

15. A geostationary satellite is orbiting around an arbitary planet 'P' at a height of 11R above the surface of 'P', R being the radius of 'P'. The time period of another satellite in hours at a height of 2R from the surface of 'P' is _________. 'P' has the time period of 24 hours.

(1) $6\sqrt{2}$ (2) $\frac{6}{\sqrt{2}}$ (3) 3 (4) 5

16. The time period of a satellite in a circular orbit of radius R is T. The period of another satellite in a circular orbit of radius 9R is :

(1) 9 T (2) 27 T

(3) 12 T (4) 3 T

17. If the angular velocity of earth's spin is increased such that the bodies at the equator start floating, the duration of the day would be approximately :

(Take : g = 10 ms^{-2}, the radius of earth, R = 6400×10^3 m, Take π = 3.14)

(1) 60 minutes (2) does not change

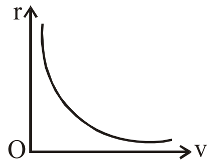
(3) 1200 minutes (4) 84 minutes

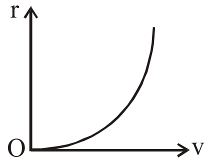
18. The angular momentum of a planet of mass M moving around the sun in an elliptical orbit is $\vec{L}$. The magnitude of the areal velocity of the planet is :

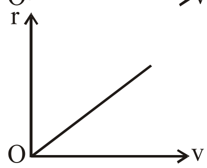
(1) $\frac{4L}{M}$ (2) $\frac{L}{M}$

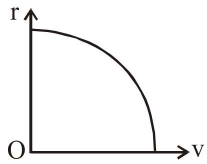
(3) $\frac{2L}{M}$ (4) $\frac{L}{2M}$

19. A particle of mass m moves in a circular orbit under the central potential field, $U(r)=\frac{-C}{r}$, where C is a positive constant.
The correct radius – velocity graph of the particle's motion is :

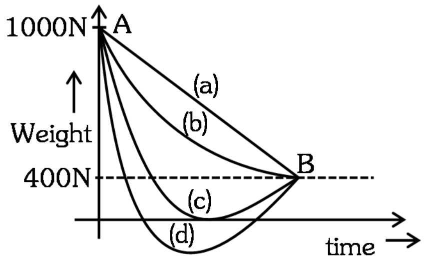
(1)


(2)


(3)


(4)


20. A person whose mass is 100 kg travels from Earth to Mars in a spaceship. Neglect all other objects in sky and take acceleration due to gravity on the surface of the Earth and Mars as 10 m/s^2 and 4 m/s^2 respectively. Identify from the below figures, the curve that fits best for the weight of the passenger as a function of time.



(1) (c) (2) (a) (3) (d) (4) (b)

21. A satellite is launched into a circular orbit of radius R around earth, while a second satellite is launched into a circular orbit of radius 1.02 R. The percentage difference in the time periods of the two satellites is :

(1) 1.5 (2) 2.0 (3) 0.7 (4) 3.0

22. Consider a binary star system of star A and star B with masses m_A and m_B revolving in a circular orbit of radii r_A and r_B, respectively. If T_A and T_B are the time period of star A and star B, respectively, then :

(1) $\frac{T_A}{T_B}=\left(\frac{r_A}{r_B}\right)^{\frac{3}{2}}$ (2) $T_A = T_B$

(3) $T_A > T_B$ (if $m_A > m_B$)

(4) $T_A > T_B$ (if $r_A > r_B$)

23. A body is projected vertically upwards from the surface of earth with a velocity sufficient enough to carry it to infinity. The time taken by it to reach height h is ______ S.

(1) $\sqrt{\frac{R_e}{2g}}\left[\left(1+\frac{h}{R_e}\right)^{3/2}-1\right]$

(2) $\sqrt{\frac{2R_e}{g}}\left[\left(1+\frac{h}{R_e}\right)^{3/2}-1\right]$

(3) $\frac{1}{3}\sqrt{\frac{R_e}{2g}}\left[\left(1+\frac{h}{R_e}\right)^{3/2}-1\right]$

(4) $\frac{1}{3}\sqrt{\frac{2R_e}{g}}\left[\left(1+\frac{h}{R_e}\right)^{3/2}-1\right]$

24. The minimum and maximum distances of a planet revolving around the Sun are x_1 and x_2. If the minimum speed of the planet on its trajectory is v_0 then its maximum speed will be :

(1) $\frac{v_0x_1^2}{x_2^2}$ (2) $\frac{v_0x_2^2}{x_1^2}$

(3) $\frac{v_0x_1}{x_2}$ (4) $\frac{v_0x_2}{x_1}$

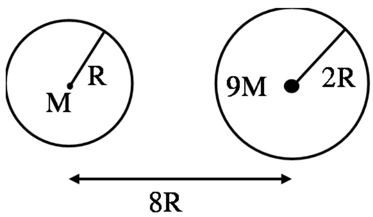
25. Consider a planet in some solar system which has a mass double the mass of earth and density equal to the average density of earth. If the weight of an object on earth is W, the weight of the same object on that planet will be :

(1) 2W (2) W

(3) $2^{\frac{1}{3}}$W (4) $\sqrt{2}$ W

26. Suppose two planets (spherical in shape) of radii R and 2R, but mass M and 9 M respectively have a centre to centre separation 8 R as shown in the figure. A satellite of mass 'm' is projected from the surface of the planet of mass 'M' directly towards the centre of the second planet. The minimum speed 'v' required for the satellite to reach the surface of the second planet is $\sqrt{\frac{a}{7}\frac{GM}{R}}$ then the value of 'a' is ________.

[Given : The two planets are fixed in their position]



27. Two identical particles of mass 1 kg each go round a circle of radius R, under the action of their mutual gravitational attraction. The angular speed of each particle is :

(1) $\sqrt{\frac{G}{2R^3}}$ (2) $\frac{1}{2}\sqrt{\frac{G}{R^3}}$

(3) $\frac{1}{2R}\sqrt{\frac{1}{G}}$ (4) $\sqrt{\frac{2G}{R^3}}$

28. The planet Mars has two moons, if one of them has a period 7 hours, 30 minutes and an orbital radius of 9.0×10^3 km. Find the mass of Mars.

$\left\{\text{Given } \frac{4\pi^2}{G} = 6\times10^{11}\ N^{-1}\ m^{-2}\ kg^2\right\}$

(1) 5.96×10^{19} kg (2) 3.25×10^{21} kg

(3) 7.02×10^{25} kg (4) 6.00×10^{23} kg

29. Inside a uniform spherical shell :

(a) the gravitational field is zero

(b) the gravitational potential is zero

(c) the gravitational field is same everywhere

(d) the gravitation potential is same everywhere

(e) all of the above

Choose the most appropriate answer from the options given below :

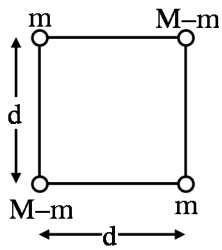
(1) (a), (c) and (d) only

(2) (e) only

(3) (a), (b) and (c) only

(4) (b), (c) and (d) only

30. A body of mass (2M) splits into four masses {m, M – m, m, M – m}, which are rearranged to form a square as shown in the figure. The ratio of $\frac{M}{m}$ for which, the gravitational potential energy of the system becomes maximum is x : 1. The value of x is ....



31. A mass of 50 kg is placed at the centre of a uniform spherical shell of mass 100 kg and radius 50 m. If the gravitational potential at a point, 25 m from the centre is V kg/m. The value of V is :

(1) – 60 G (2) + 2 G

(3) – 20 G (4) – 4 G

32. The masses and radii of the earth and moon are (M_1, R_1) and (M_2, R_2) respectively. Their centres are at a distance 'r' apart. Find the minimum escape velocity for a particle of mass 'm' to be projected from the middle of these two masses:

(1) $V = \frac{1}{2}\sqrt{\frac{4G(M_1+M_2)}{r}}$

(2) $V = \sqrt{\frac{4G(M_1+M_2)}{r}}$

(3) $V = \frac{1}{2}\sqrt{\frac{2G(M_1+M_2)}{r}}$

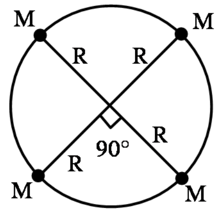
(4) $V = \frac{\sqrt{2G}(M_1+M_2)}{r}$

33. If R_E be the radius of Earth, then the ratio between the acceleration due to gravity at a depth 'r' below and a height 'r' above the earth surface is : (Given : $r < R_E$)

(1) $1-\frac{r}{R_E}-\frac{r^2}{R_E^2}-\frac{r^3}{R_E^3}$ (2) $1+\frac{r}{R_E}+\frac{r^2}{R_E^2}+\frac{r^3}{R_E^3}$

(3) $1+\frac{r}{R_E}-\frac{r^2}{R_E^2}+\frac{r^3}{R_E^3}$ (4) $1+\frac{r}{R_E}-\frac{r^2}{R_E^2}-\frac{r^3}{R_E^3}$

34. Four particles each of mass M, move along a circle of radius R under the action of their mutual gravitational attraction as shown in figure. The speed of each particle is :



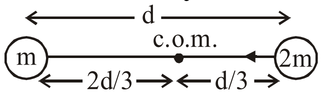
(1) $\frac{1}{2}\sqrt{\frac{GM}{R(2\sqrt{2}+1)}}$ (2) $\frac{1}{2}\sqrt{\frac{GM}{R}(2\sqrt{2}+1)}$

(3) $\frac{1}{2}\sqrt{\frac{GM}{R}(2\sqrt{2}-1)}$ (4) $\sqrt{\frac{GM}{R}}$

SOLUTION

1. Official Ans. by NTA (2)

Sol.



$$F = \frac{G(2m)m}{d^2} = (2m)\omega^2 (d/3)$$

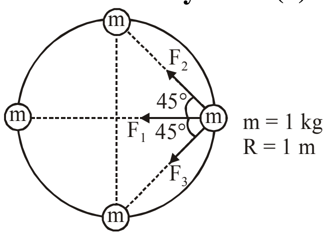
$$\frac{Gm}{d^2} = \omega^2 \frac{d}{3}$$

$$\Rightarrow \omega^2 = \frac{3Gm}{d^3} \Rightarrow \omega = \sqrt{\frac{3Gm}{d^3}}$$

$$\Rightarrow T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{d^3}{3Gm}}$$

2. Official Ans. by NTA (4)

Sol.



$$F_1 = \frac{Gmm}{(2R)^2} = \frac{Gm^2}{4R^2}$$

$$F_2 = \frac{Gmm}{(\sqrt{2}R)^2} = \frac{Gm^2}{2R^2}$$

$$F_3 = \frac{Gmm}{(\sqrt{2}R)^2} = \frac{Gm^2}{2R^2}$$

$$\Rightarrow F_{net} = F_1 + F_2 \cos 45° + F_3 \cos 45°$$

$$= \frac{Gm^2}{4R^2} + \frac{Gm^2}{2R^2}\frac{1}{\sqrt{2}} + \frac{Gm^2}{2R^2}\frac{1}{\sqrt{2}}$$

$$= \frac{Gm^2}{R^2}\left(\frac{1}{4} + \frac{1}{2\sqrt{2}} + \frac{1}{2\sqrt{2}}\right)$$

$$= \frac{Gm^2}{R^2}\left(\frac{1}{4} + \frac{1}{\sqrt{2}}\right) = \frac{Gm^2}{4R^2}\left(1 + 2\sqrt{2}\right)$$

$$F_{net} = \frac{Gm^2}{4R^2}\left(1 + 2\sqrt{2}\right) = \frac{mv^2}{R}$$

$$\Rightarrow v = \frac{\sqrt{G(1+2\sqrt{2})}}{2}$$

3. Official Ans. by NTA (1)

Sol. $\frac{T_1}{T_2} = \frac{1}{8}$

$$\frac{2\pi/\omega_1}{2\pi/\omega_2} = \frac{1}{8}$$

$$\frac{\omega_1}{\omega_2} = \frac{8}{1}$$

4. Official Ans. by NTA (2)

Sol. Weight of pole = mg = 49 N

At equator due to rotation = $g_e = g - R\omega^2$

so $W = mg_e = m(g - R\omega^2)$

$\therefore W_P > W_e$ $\qquad$ $W_P = 49$ N

So, $W_e = 48.83$ N. $\qquad$ $W_e < 49$ N

Option (2) is correct.

5. Official Ans. by NTA (1)

Sol. $T = 2\pi\sqrt{\frac{r^3}{GM}}$

$$T_A = 2\pi\sqrt{\frac{(6400+600)\times 10^3}{GM}}$$

$$T_A = 2\pi \times 10^9 \sqrt{\frac{7^3}{GM}}$$

$$T_B = 2\pi \times 10^9 \sqrt{\frac{8^3}{GM}}$$

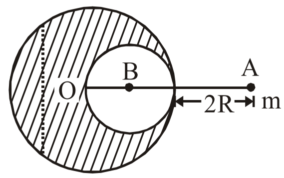
$$T_B - T_A = \frac{2\pi 10^9}{\sqrt{GM}}\left[8\sqrt{8} - 7\sqrt{7}\right]$$

$= 314 \times 4.107 = 1289.64 = 1.289 \times 10^3$ s

6. Official Ans. by NTA (3)

Sol. Let initial mass of sphere is m'. Hence mass of removed portion will be m'/8

$$F_1 = m.E. = \frac{m.Gm'}{9R^2}$$



$$F_2 = m\left[\frac{G.m'}{(3R)^2} - \frac{G.m'/8}{(5R/2)^2}\right]$$

$$= \frac{Gm'}{9R^2} - \frac{Gm'\times 4}{8\times 25} = \left(\frac{1}{9} - \frac{1}{50}\right)\frac{Gm'}{R^2}$$

$$F_2 = \frac{41}{50\times 9}.\frac{Gm'}{R^2}$$

$$\frac{F_1}{F_2} = \frac{1}{9}\times\frac{50\times 9}{41} = \frac{50}{41}$$

7. Official Ans. by NTA (2)

Sol. $V_e = \sqrt{\frac{2GM}{R}}$

$$\frac{M_1}{R_1} = \frac{M_2}{R_2}$$

$M_1R_2 = M_2R_1$

Hence reason R is not correct.

8. Official Ans. by NTA (10)

Sol. $\frac{-GMm}{11R} = \frac{-GMm}{R} + \frac{1}{2}mv^2$

$v = \sqrt{\frac{20GM}{11R}}$

9. Official Ans. by NTA (4)

Sol. Gravitational field of ring

$= -\frac{Gmx}{\left(R^2 + x^2\right)^{3/2}}$

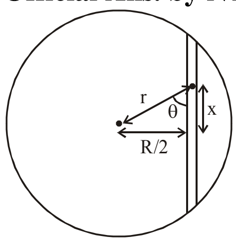
Force between sphere & ring

$= \frac{GmM\left(\sqrt{8}R\right)}{\left(R^2 + 8R^2\right)^{3/2}} = \frac{GmM}{R^2} \times \frac{\sqrt{8}}{27}$

Ans. (4)

10. Official Ans. by NTA (4)

Sol.



Force along the tunnel

$F = -\left(\frac{GMmr}{R^3}\right)\cos\theta$

$F = -\frac{gm}{R}x \quad \left(\frac{GM}{R^2} = g, r\cos\theta = x\right)$

$a = -\frac{g}{R}x$

$\omega^2 = \frac{g}{R} \qquad T = 2\pi\sqrt{\frac{R}{g}}$

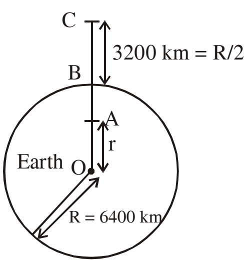
Ans. (4)

11. Official Ans. by NTA (4)

Sol. As per Keppler's 2nd law, Areal velocity is constant.

12. Official Ans. by NTA (4)

Sol. $g_A = \frac{GM(r)}{R^3}$



$g_C = \frac{GM}{\left(R + \frac{R}{2}\right)^2}$

$g_A = g_C$

$\frac{r}{R^3} = \frac{1}{\frac{9}{4}R^2} \Rightarrow r = \frac{4R}{9}$

so OA = $\frac{4R}{9}$; AB = R – r = $\frac{5R}{9}$

OA : AB = 4 : 5

13. Official Ans. by NTA (3)

Sol. By angular momentum conservation :

$mv_1r_1 = mv_2r_2$

$v_1 = \frac{48 \times 10^{14}}{1.6 \times 10^{12}}$ = 3000 m/sec

= 3×10^3 m/sec.

14. Official Ans. by NTA (3)

Sol. **Ans. (3)**

Energy given = $U_f - U_i$

$= 0 - \left(-\frac{3}{5}\frac{GM^2}{R}\right)$

$= \frac{3}{5}\frac{GM^2}{R} \qquad x = 3$

15. Official Ans. by NTA (3)

Sol. (3) $T \propto R^{3/2}$

$\frac{24}{T} = \left(\frac{12R}{3R}\right)^{3/2} \Rightarrow T = 3hr$

16. Official Ans. by NTA (2)

Sol. $T^2 \propto R^3$

$\left(\frac{T'}{T}\right)^2 = \left(\frac{9R}{R}\right)^3$

$T'^2 = T^2 \times 9^3$

$T' = T \times 3^3$

T' = 27 T

17. Official Ans. by NTA (4)

Sol. For objects to float

$mg = m\omega^2R$

ω = angular velocity of earth.

R = Radius of earth

$\omega = \sqrt{\frac{g}{R}}$ ... (1)

Duration of day = T

$T = \frac{2\pi}{\omega}$ ... (2)

$\Rightarrow T = 2\pi\sqrt{\frac{R}{g}}$

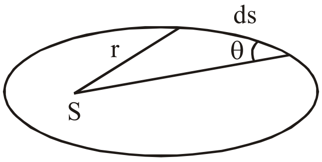
$= 2\pi\sqrt{\frac{6400 \times 10^3}{10}}$

$\Rightarrow \frac{T}{60} = 83.775$ minutes

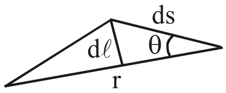
$\simeq$ 84 minutes

18. Official Ans. by NTA (4)

Sol.



For small displacement ds of the planet its area can be written as



$$dA = \frac{1}{2} r d\ell$$

$$= \frac{1}{2} r ds \sin\theta$$

$$\text{A.vel} = \frac{dA}{dt} = \frac{1}{2} r \sin\theta \frac{ds}{dt} = \frac{Vr\sin\theta}{2}$$

$$\frac{dA}{dt} = \frac{1}{2}\frac{mVr\sin\theta}{m} = \frac{L}{2m}$$

19. Official Ans. by NTA (1)

Sol. $U = -\frac{C}{r}$

$$F = -\frac{dU}{dr} = -\frac{C}{r^2}$$

$$|F| = \frac{mv^2}{r}$$

$$\frac{C}{r^2} = \frac{mv^2}{r}$$

$$v^2 \propto \frac{1}{r}$$

20. Official Ans. by NTA (1)

Sol. At neutral point g = 0 so graph (C) is correct Hence option (1).

21. Official Ans. by NTA (4)

Sol. $T^2 \propto R^3$

$$T = kR^{3/2}$$

$$\frac{dT}{T} = \frac{3}{2}\frac{dR}{R} = \frac{3}{2} \times 0.02 = 0.03$$

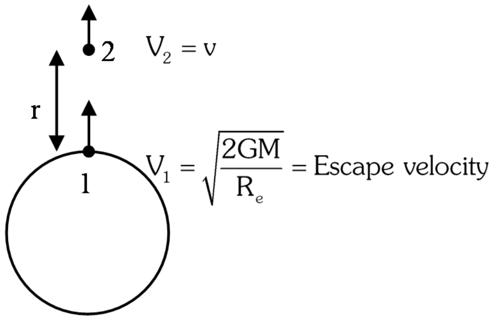
% Change = 3%

22. Official Ans. by NTA (2)

Sol. $T_A = T_B$ (since $\omega_A = \omega_B$)

23. Official Ans. by NTA (4)

Sol.



Applying energy conservation from (1) to (2)

$$\frac{1}{2}m.\left(\frac{2GM}{R_e}\right) - \frac{GMm}{R_e} = \frac{1}{2}mv^2 - \frac{GMm}{R+r}$$

$$\Rightarrow \quad \frac{1}{2}mv^2 = \frac{GMm}{R+r}$$

$$\Rightarrow \quad v = \sqrt{\frac{2GM}{R+r}} = \frac{dr}{dt}$$

$$\Rightarrow \quad \sqrt{2GM}\int_0^t dt = \int_{R_e}^{R_e+h} \left(\sqrt{R+r}\right) dr$$

$$\sqrt{2GM}.\ t = \frac{2}{3}\left[(R+r)^{3/2}\right]_{R_e}^{R_e+h}$$

$$t = \frac{2}{3}\sqrt{\frac{R_e^3}{2GM}}\left[\left(1+\frac{h}{R_e}\right)^{3/2} - 1\right]$$

$$\frac{GM}{R_e^2} = g$$

$$t = \frac{1}{3}\sqrt{\frac{2R_e}{g}}\left[\left(1+\frac{h}{R_e}\right)^{3/2} - 1\right]$$

24. Official Ans. by NTA (4)

Sol. Angular momentum conservation equation

$v_0 x_2 = v_1 x_1$

$$v_1 = \frac{v_0 x_2}{x_1}$$

25. Official Ans. by NTA (3)

Sol. Density is same

$$M = \frac{4}{3}\pi R^3 \rho,\ 2m = \frac{4}{3}\pi R'^3 \rho$$

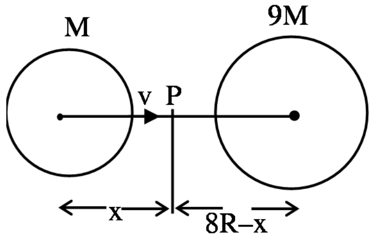
$$R' = 2^{1/3} R$$

$$\omega = \frac{GMm}{R^2}\ ;\ \omega_2 = \frac{G2Mm}{R'^2}$$

$$\omega_2 = 2^{1/3}\omega$$

26. Official Ans. by NTA (4)

Sol.



Acceleration due to gravity will be zero at P therefore,

$$\frac{GM}{x^2} = \frac{G9M}{(8R-x)^2}$$

$8R - x = 3x$

$x = 2R$

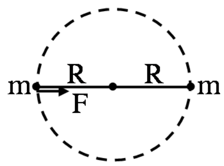
Apply conservation of energy and consider velocity at P is zero.

$$\frac{1}{2}mv^2 - \frac{GMm}{R} - \frac{G9Mm}{7R} = 0 - \frac{GMm}{2R} - \frac{G9Mm}{6R}$$

$$\therefore V = \sqrt{\frac{4}{7}\frac{GM}{R}}$$

27. Official Ans. by NTA (2)

Sol.



$$F = \frac{Gm^2}{(2R)^2} = mR\omega^2$$

$$\omega = \frac{1}{2}\sqrt{\frac{G}{R^3}}$$

28. Official Ans. by NTA (4)

Sol. Option D is correct

$$T^2 = \frac{4\pi^2}{GM} \cdot r^3$$

$$M = \frac{4\pi^2}{G} \cdot \frac{r^3}{T^2}$$

by putting values

$M = 6 \times 10^{23}$

29. Official Ans. by NTA (1)

Sol. Inside a spherical shell, gravitational field is zero and hence potential remains same everywhere

Hence option (1)

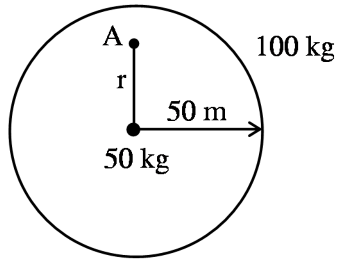
30. Official Ans. by NTA (2)

Sol. Energy is maximum when mass is split equally

so $\frac{M}{m} = 2$

31. Official Ans. by NTA (4)

Sol.



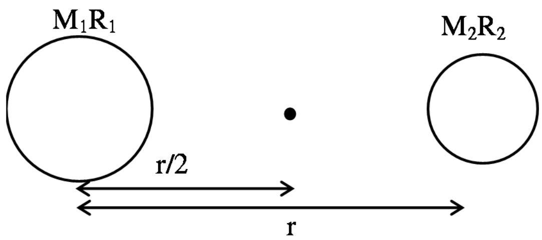
$$V_A = \left[-\frac{GM_1}{r} - \frac{GM_2}{R}\right]$$

$$= \left[-\frac{50}{25}G - \frac{100}{50}G\right]$$

$= -4G$

32. Official Ans. by NTA (2)

Sol.



$$\frac{1}{2}mV^2 - \frac{GM_1m}{r/2} - \frac{GM_2m}{r/2} = 0$$

$$\frac{1}{2}mV^2 = \frac{2Gm}{r}(M_1 + M_2)$$

$$V = \sqrt{\frac{4G(M_1+M_2)}{r}}$$

Option (2)

33. Official Ans. by NTA (4)

Sol.

$$g_{up} = \frac{g}{\left(1+\frac{r}{R}\right)^2}$$

$$g_{down} = g\left(1-\frac{r}{R}\right)$$

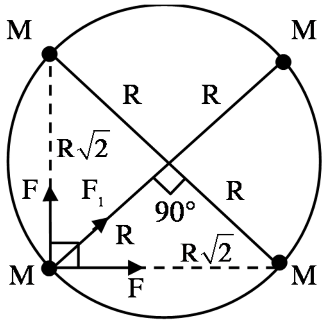
$$\frac{g_{down}}{g_{up}} = \left(1-\frac{r}{R}\right)\left(1+\frac{r}{R}\right)^2$$

$$= \left(1-\frac{r}{R}\right)\left(1+\frac{2r}{R}+\frac{r^2}{R^2}\right)$$

$$= 1 + \frac{r}{R} - \frac{r^2}{R^2} - \frac{r^3}{R^3}$$

34. Official Ans. by NTA (2)

Sol.



$$F_{net} = \frac{MV^2}{R}$$

$$\sqrt{2}F + F_1 = \frac{MV^2}{R}$$

$$\sqrt{2}\frac{GMM}{\left(\sqrt{2}R\right)^2} + \frac{GMM}{(2R)^2} = \frac{MV^2}{R}$$

$$\frac{GM}{R}\left(\frac{1}{\sqrt{2}} + \frac{1}{4}\right) = V^2$$

$$\frac{GM}{R}\left(\frac{4+\sqrt{2}}{4\sqrt{2}}\right) = V^2$$

$$V = \sqrt{\frac{GM\left(4+\sqrt{2}\right)}{R4\sqrt{2}}}$$

$$V = \frac{1}{2}\sqrt{\frac{GM\left(2\sqrt{2}+1\right)}{R}}$$

Option (2)