# ELASTICITY

1. If Y, K and η are the values of Young's modulus, bulk modulus and modulus of rigidity of any material respectively. Choose the correct relation for these parameters.

(1) $Y=\frac{9K\eta}{3K-\eta}N/m^2$ (2) $\eta=\frac{3YK}{9K+Y}N/m^2$

(3) $Y=\frac{9K\eta}{2\eta+3K}N/m^2$ (4) $K=\frac{Y\eta}{9\eta-3Y}N/m^2$

2. A uniform metallic wire is elongated by 0.04 m when subjected to a linear force F. The elongation, if its length and diameter is doubled and subjected to the same force will be _____ cm.

3. The normal density of a material is ρ and its bulk modulus of elasticity is K. The magnitude of increase in density of material, when a pressure P is applied uniformly on all sides, will be :

(1) $\frac{\rho K}{P}$ (2) $\frac{\rho P}{K}$ (3) $\frac{K}{\rho P}$ (4) $\frac{PK}{\rho}$

4. The length of metallic wire is $\ell_1$ when tension in it is $T_1$. It is $\ell_2$ when the tension is $T_2$. The original length of the wire will be –

(1) $\frac{\ell_1+\ell_2}{2}$ (2) $\frac{T_2\ell_1+T_1\ell_2}{T_1+T_2}$

(3) $\frac{T_2\ell_1-T_1\ell_2}{T_2-T_1}$ (4) $\frac{T_1\ell_1-T_2\ell_2}{T_2-T_1}$

5. An object is located at 2 km beneath the surface of the water. If the fractional compression $\frac{\Delta V}{V}$ is 1.36% , the ratio of hydraulic stress to the corresponding hydraulic strain will be _______.
[Given : density of water is 1000 kg $m^{-3}$ and $g = 9.8\ ms^{-2}$.]

(1) $1.96 \times 10^7\ Nm^{-2}$ (2) $1.44 \times 10^7\ Nm^{-2}$

(3) $2.26 \times 10^9\ Nm^{-2}$ (4) $1.44 \times 10^9\ Nm^{-2}$

6. Two separate wires A and B are stretched by 2 mm and 4 mm respectively, when they are subjected to a force of 2 N. Assume that both the wires are made up of same material and the radius of wire B is 4 times that of the radius of wire A. The length of the wires A and B are in the ratio of a : b. Then a/b can be expressed as 1/x where x is _______ .

7. The value of tension in a long thin metal wire has been changed from $T_1$ to $T_2$. The lengths of the metal wire at two different values of tension $T_1$ and $T_2$ are $\ell_1$ and $\ell_2$ respectively. The actual length of the metal wire is :

(1) $\frac{T_1\ell_2-T_2\ell_1}{T_1-T_2}$ (2) $\frac{T_1\ell_1-T_2\ell_2}{T_1-T_2}$

(3) $\frac{\ell_1+\ell_2}{2}$ (4) $\sqrt{T_1T_2\ell_1\ell_2}$

8. The length of a metal wire is $\ell_1$, when the tension in it is $T_1$ and is $\ell_2$ when the tension is $T_2$. The natural length of the wire is :

(1) $\sqrt{\ell_1\ell_2}$ (2) $\frac{\ell_1T_2-\ell_2T_1}{T_2-T_1}$

(3) $\frac{\ell_1T_2+\ell_2T_1}{T_2+T_1}$ (4) $\frac{\ell_1+\ell_2}{2}$

9. The area of cross-section of a railway track is 0.01 $m^2$. The temperature variation is 10°C. Coefficient of linear expansion of material of track is $10^{-5}$/°C. The energy stored per meter in the track is _______ J/m.
(Young's modulus of material of track is $10^{11}$ $Nm^{-2}$)

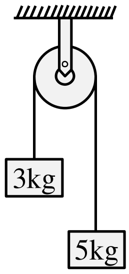
10. Two wires of same length and radius are joined end to end and loaded. The Young's modulii of the materials of the two wires are $Y_1$ and $Y_2$. The combination behaves as a single wire then its Young's modulus is :

(1) $Y=\frac{2Y_1Y_2}{3(Y_1+Y_2)}$ (2) $Y=\frac{2Y_1Y_2}{Y_1+Y_2}$

(3) $Y=\frac{Y_1Y_2}{2(Y_1+Y_2)}$ (4) $Y=\frac{Y_1Y_2}{Y_1+Y_2}$

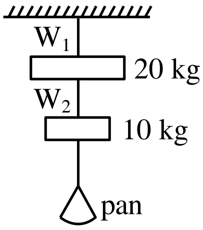
11. A stone of mass 20 g is projected from a rubber catapult of length 0.1 m and area of cross section $10^{-6}$ $m^2$ stretched by an amount 0.04 m. The velocity of the projected stone is __________ m/s. (Young's modulus of rubber = $0.5 \times 10^9$ N/$m^2$)

**12.** Two blocks of masses 3 kg and 5 kg are connected by a metal wire going over a smooth pulley. The breaking stress of the metal is $\frac{24}{\pi} \times 10^2$ $Nm^{-2}$. What is the minimum radius of the wire? (Take g = 10 $ms^{-2}$)




(1) 125 cm (2) 1250 cm
(3) 12.5 cm (4) 1.25 cm

**13.** Wires $W_1$ and $W_2$ are made of same material having the breaking stress of $1.25 \times 10^9$ N/$m^2$. $W_1$ and $W_2$ have cross-sectional area of $8 \times 10^{-7}$ $m^2$ and $4 \times 10^{-7}$ $m^2$, respectively. Masses of 20 kg and 10 kg hang from them as shown in the figure. The maximum mass that can be placed in the pan without breaking the wires is ____ kg. (Use g = 10 m/$s^2$)




**14.** A uniform heavy rod of weight 10 kg $ms^{-2}$, cross-sectional area 100 $cm^2$ and length 20 cm is hanging from a fixed support. Young modulus of the material of the rod is $2 \times 10^{11}$ $Nm^{-2}$. Neglecting the lateral contraction, find the elongation of rod due to its own weight.

(1) $2 \times 10^{-9}$ m (2) $5 \times 10^{-8}$ m
(3) $4 \times 10^{-8}$ m (4) $5 \times 10^{-10}$ m

**15.** When a rubber ball is taken to a depth of __________m in deep sea, its volume decreases by 0.5%. (The bulk modulus of rubber = $9.8 \times 10^8$ $Nm^{-2}$ Density of sea water = $10^3$ $kgm^{-3}$ (g = 9.8 m/$s^2$)

**16.** Four identical hollow cylindrical columns of mild steel support a big structure of mass $50 \times 10^3$ kg, The inner and outer radii of each column are 50 cm and 100 cm respectively. Assuming uniform local distribution, calculate the compression strain of each column.
[Use Y = $2.0 \times 10^{11}$ Pa, g = 9.8 m/$s^2$]

(1) $3.60 \times 10^{-8}$ (2) $2.60 \times 10^{-7}$
(3) $1.87 \times 10^{-3}$ (4) $7.07 \times 10^{-4}$

## SOLUTION

**1. Official Ans. by NTA (4)**

**Sol.** Y- Younge modulus, K- Bulk modulus,

η- modulus of rigidity

We know that

$y = 3k(1 - 2\sigma)$

$\sigma = \frac{1}{2}\left(1 - \frac{y}{3k}\right)$ ....(i)

$y = 2\eta(1 + \sigma)$

$\sigma = \frac{y}{2\eta} - 1$ ....(ii)

From Eq.(i) and Eq. (ii)

$\frac{1}{2}\left(1 - \frac{Y}{3k}\right) = \frac{y}{2\eta} - 1$

$1 - \frac{y}{3k} = \frac{y}{\eta} - 2$

$\frac{y}{3k} = 3 - \frac{y}{\eta}$

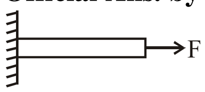
$\frac{y}{3k} = \frac{3\eta - y}{\eta}$

$\frac{\eta y}{3k} = 3\eta - y$

$k = \frac{\eta y}{9\eta - 3y}$

**2. Official Ans. by NTA (2)**

**Sol.**




$F = Y.A.\frac{\Delta\ell}{\ell}$

$\Delta\ell = \frac{F}{Y.A.}.\ell$

$\Delta\ell = \frac{F.\ell}{Y.\pi r^2}$

$\Delta\ell \propto \frac{\ell}{r^2}$

$\frac{\Delta\ell_2}{\Delta\ell_1} = \left(\frac{\ell_2}{\ell_1}\right)\left(\frac{r_1}{r_2}\right)^2 = (2)\left(\frac{1}{2}\right)^2$

$\frac{\Delta\ell_2}{\Delta\ell_1} = \frac{1}{2}$

$\Delta\ell_2 = \frac{\Delta\ell_1}{2}$

$= \frac{0.04}{2} = 0.02$ m

$\boxed{\Delta\ell_2 = 2\text{cm}}$ $\boxed{\text{Ans.} = 2}$

**3. Official Ans. by NTA (2)**

**Sol.** $\rho = \frac{M}{V}$

$\frac{d\rho}{\rho} = -\frac{dV}{V}$

$k = -\frac{P}{\frac{dV}{V}}$

$-\frac{dV}{V} = \frac{P}{k}$

$\frac{d\rho}{\rho} = \frac{P}{k} \Rightarrow d\rho = \frac{\rho P}{k}$

**4. Official Ans. by NTA (3)**

**Sol.** Assuming Hooke's law to be valid.

$T \propto (\Delta\ell)$

$T = k(\Delta\ell)$

Let, $\ell_0$ = natural length (original length)

$\Rightarrow T = k(\ell - \ell_0)$

so, $T_1 = k(\ell_1 - \ell_0)$ & $T_2 = k(\ell_2 - \ell_0)$

$\Rightarrow \frac{T_1}{T_2} = \frac{\ell_1 - \ell_0}{\ell_2 - \ell_0} \Rightarrow \ell_0 = \frac{T_2\ell_1 - T_1\ell_2}{T_2 - T_1}$

**5. Official Ans. by NTA (4)**

**Sol.** (4) $P = h\rho g$

$\beta = \frac{p}{\frac{\Delta V}{V}} = \frac{2\times10^3\times10^3\times9.8}{1.36\times10^{-2}}$

$= 1.44 \times 10^9$ N/m²

**6. Official Ans. by NTA (32)**

**Sol.** For A $\frac{E}{\pi r^2} = y\frac{2mm}{a}$ ....(1)

For B $\frac{E}{\pi.16r^2} = y\frac{4mm}{b}$ ....(2)

$\therefore$ (1)/(2)

$16 = \frac{2b}{4a}$

$\frac{a}{b} = \frac{1}{32}$ $\therefore$ Answer = 32

**7. Official Ans. by NTA (1)**

**Sol.** $Y = \dfrac{FL}{A\Delta L}$

$\Rightarrow Y = \dfrac{T_1\ell_0}{A(\ell_1 - \ell_0)} = \dfrac{T_2\ell_0}{A(\ell_2 - \ell_0)}$

$1 = \dfrac{T_1(\ell_2 - \ell_0)}{T_2(\ell_1 - \ell_0)}$

$T_2\ell_1 - T_2\ell_0 = T_1\ell_2 - T_1\ell_0$

$(T_1 - T_2)\ell_0 = T_1\ell_2 - T_2\ell_1$

$\ell_0 = \left(\dfrac{T_1\ell_2 - T_2\ell_1}{T_1 - T_2}\right)$

**8. Official Ans. by NTA (2)**

**Sol.** $T_1 = k(\ell_1 - \ell_0)$

$T_2 = k(\ell_2 - \ell_0)$

$\dfrac{T_1}{T_2} = \dfrac{\ell_1 - \ell_0}{\ell_2 - \ell_0}$

$\dfrac{T_1\ell_2 - T_2\ell_1}{T_1 - T_2} = \ell_0$

**9. Official Ans. by NTA (5)**

**Sol.** Elastic energy = $\dfrac{Y}{2}(\text{strain})^2 \times \text{Area} \times \text{length}$

$\Rightarrow$ Elastic energy per unit length

= $\dfrac{Y}{2}(\text{strain})^2 \times \text{Area}$

$\left(\text{strain} = \dfrac{\Delta\ell}{\ell} = \alpha\Delta T = 10^{-5} \times 10 = 10^{-4}\right)$

$= \dfrac{10^{11}}{2} \times (10^{-4})^2 \times 10^{-2} = 5\text{J/m}$

**10. Official Ans. by NTA (2)**

**Sol.** In series combination $\Delta l = \ell_1 + \ell_2$

$Y = \dfrac{F/A}{\Delta\ell/\ell} \Rightarrow \Delta\ell = \dfrac{F\ell}{AY}$

$\Rightarrow \Delta\ell \propto \dfrac{\ell}{Y}$

Equivalent length of rod after joing is = $2\ell$

As, lengths are same and force is also same in series

$\Delta\ell = \Delta\ell_1 + \Delta\ell_2$

$\dfrac{\ell_{eq}}{Y_{eq}} = \dfrac{\ell}{Y_1} + \dfrac{\ell}{Y_2} \Rightarrow \dfrac{2\ell}{Y} = \dfrac{\ell}{Y_1} + \dfrac{\ell}{Y_2}$

$\therefore\ Y = \dfrac{2Y_1Y_2}{Y_1 + Y_2}$

**11. Official Ans. by NTA (20)**

Sol. By energy conservation

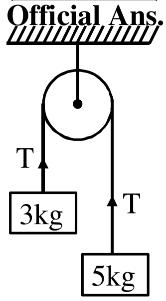
$\dfrac{1}{2}.\dfrac{YA}{L}.x^2 = \dfrac{1}{2}mv^2$

$\dfrac{0.5 \times 10^9 \times 10^{-6} \times (0.04)^2}{0.1} = \dfrac{20}{1000}v^2$

$\therefore\ v^2 = 400$

$\boxed{v = 20\ \text{m/s}}$

**12. Official Ans. by NTA (3)**




**Sol.**

$T = \dfrac{2m_1m_2g}{m_1 + m_2} = \dfrac{2 \times 3 \times 5 \times 10}{8}$

$= \dfrac{75}{2}$

Stress = $\dfrac{T}{A}$

$\dfrac{24}{\pi} \times 10^2 = \dfrac{75}{2 \times \pi R^2}$

$R^2 = \dfrac{75}{2 \times 24 \times 100} = \dfrac{3}{8 \times 24}$

$\Rightarrow R = 0.125$ m

R = 12.5 cm

**13. Official Ans. by NTA (40)**

**Sol.** $B.S_1 = \dfrac{T_{1max}}{8 \times 10^{-7}} \Rightarrow T_{1\,max} = 8 \times 1.25 \times 100$

= 1000 N

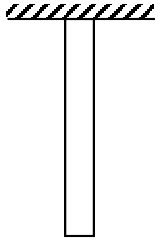
$B.S_2 = \dfrac{T_{2max}}{4 \times 10^{-7}} \Rightarrow T_{2max} = 4 \times 1.25 \times 100$

= 500 N

$m = \dfrac{500 - 100}{10} = 40\ \text{kg}$

**14. Official Ans. by NTA (4)**

**Sol.**



We know,

$$\Delta\ell = \frac{WL}{2AY}$$

$$\Delta\ell = \frac{10\times1}{2\times5}\times100\times10^{-4}\times2\times10^{11}$$

$$\Delta\ell = \frac{1}{2}\times10^{-9} = 5\times10^{-10}\,m$$

Option (4)

**15. Official Ans. by NTA (500)**

**Sol.** $B = -\dfrac{\Delta P}{\left(\dfrac{\Delta V}{V}\right)} = -\dfrac{\rho gh}{\left(\dfrac{\Delta V}{V}\right)}$

$$-\frac{B\frac{\Delta V}{V}}{\rho g} = h$$

$$\frac{9.8\times10^{8}\times0.5}{100\times10^{3}\times9.8} = h$$

h = 500

**16. Official Ans. by NTA (2)**

**Sol.** Force on each column = $\dfrac{mg}{4}$

Strain = $\dfrac{mg}{4AY}$

$$= \frac{50\times10^{3}\times9.8}{4\times\pi(1-0.25)\times2\times10^{11}}$$

$= 2.6\times10^{-7}$