CIRCULAR MOTION

1. If $\vec{P}\times\vec{Q}=\vec{Q}\times\vec{P}$, the angle between $\vec{P}$ and $\vec{Q}$ is θ (0° < θ < 360°). The value of 'θ' will be _________°.

2. A particle is moving with uniform speed along the circumference of a circle of radius R under the action of a central fictitious force F which is inversely proportional to R^3. Its time period of revolution will be given by :

(1) $T \propto R^2$ (2) $T \propto R^{\frac{3}{2}}$

(3) $T \propto R^{\frac{5}{2}}$ (4) $T \propto R^{\frac{4}{3}}$

3. A block of 200 g mass moves with a uniform speed in a horizontal circular groove, with vertical side walls of radius 20 cm. If the block takes 40 s to complete one round, the normal force by the side walls of the groove is :

(1) 0.0314 N (2) 9.859×10^{-2} N

(3) 6.28×10^{-3} N (4) 9.859×10^{-4} N

4. **Statement I :** A cyclist is moving on an unbanked road with a speed of 7 kmh^{-1} and takes a sharp circular turn along a path of radius of 2m without reducing the speed. The static friction coefficient is 0.2. The cyclist will not slip and pass the curve ($g = 9.8\ m/s^2$)

Statement II : If the road is banked at an angle of 45°, cyclist can cross the curve of 2m radius with the speed of 18.5 kmh^{-1} without slipping.

In the light of the above statements, choose the correct answer from the options given below.

(1) Statement I is incorrect and statement II is correct

(2) Statement I is correct and statement II is incorrect

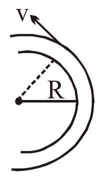
(3) Both statement I and statement II are false

(4) Both statement I and statement II are true

5. The angular speed of truck wheel is increased from 900 rpm to 2460 rpm in 26 seconds. The number of revolutions by the truck engine during this time is _______.

(Assuming the acceleration to be uniform).

6. A modern grand-prix racing car of mass m is travelling on a flat track in a circular arc of radius R with a speed v. If the coefficient of static friction between the tyres and the track is μ_s, then the magnitude of negative lift F_L acting downwards on the car is : (Assume forces on the four tyres are identical and g = acceleration due to gravity)



(1) $m\left(\frac{v^2}{\mu_s R}+g\right)$ (2) $m\left(\frac{v^2}{\mu_s R}-g\right)$

(3) $m\left(g-\frac{v^2}{\mu_s R}\right)$ (4) $-m\left(g+\frac{v^2}{\mu_s R}\right)$

7. The normal reaction 'N' for a vehicle of 800 kg mass, negotiating a turn on a 30° banked road at maximum possible speed without skidding is _______ $\times 10^3$ kg m/s^2.

[Take : cos 30° = 0.87, μ_s = 0.2]

(1) 10.2 (2) 7.2 (3) 12.4 (4) 6.96

8. A body rotating with an angular speed of 600 rpm is uniformly accelerated to 1800 rpm in 10 sec. The number of rotations made in the process is____.

9. A particle of mass m is suspended from a ceiling through a string of length L. The particle moves in a horizontal circle of radius r such that $r = \frac{L}{\sqrt{2}}$. The speed of particle will be :

(1) $\sqrt{rg}$ (2) $\sqrt{2rg}$ (3) $2\sqrt{rg}$ (4) $\sqrt{\frac{rg}{2}}$

10. A huge circular arc of length 4.4 ly subtends an angle '4s' at the centre of the circle. How long it would take for a body to complete 4 revolution if its speed is 8 AU per second ?

Given : 1 ly = 9.46×10^{15} m; AU = 1.5×10^{11} m

(1) 4.1×10^{8} s (2) 4.5×10^{10} s

(3) 3.5×10^{6} s (4) 7.2×10^{8} s

11. Two satellites revolve around a planet in coplanar circular orbits in anticlockwise direction. Their period of revolutions are 1 hour and 8 hours respectively. The radius of the orbit of nearer satellite is 2×10^{3} km. The angular speed of the farther satellite as observed from the nearer satellite at the instant when both the satellites are closest is $\frac{\pi}{x}$ rad h^{-1} where x is ....

SOLUTION

1. Official Ans. by NTA (180)

Sol. $-PQ \sin\theta$

$= PQ \sin\theta$

$\Rightarrow \theta = 180°$

2. Official Ans. by NTA (1)

Sol. $F \propto \frac{1}{R^3}$

$\frac{K}{R^3} = m\omega^2 R$

$\omega^2 = \frac{K}{m} \times \frac{1}{R^4}$

$\left(\frac{2\pi}{T}\right)^2 = \frac{K}{m} \times \frac{1}{R^4}$

$T^2 \propto R^4$

$T \propto R^2$

3. Official Ans. by NTA (4)

Sol. $N = m\omega^2 R$

$N = m\left[\frac{4\pi^2}{T^2}\right]R$ .....(1)

Given m = 0.2 kg, T = 40 S, R = 0.2 m

Put values in equation (1)

$N = 9.859 \times 10^{-4}$ N

4. Official Ans. by NTA (4)

Sol. Statement I :

$v_{max} = \sqrt{\mu R g} = \sqrt{(0.2) \times 2 \times 9.8}$

$v_{max} = 1.97$ m/s

7 km/h = 1.944 m/s

Speed is lower than v_{max}, hence it can take safe turn.

Statement II

$v_{max} = \sqrt{Rg\left[\frac{\tan\theta + \mu}{1 - \mu\tan\theta}\right]}$

$= \sqrt{2 \times 9.8\left[\frac{1+0.2}{1-0.2}\right]} = 5.42$ m / s

18.5 km/h = 5.14 m/s

Speed is lower than v_{max}, hence it can take safe turn.

5. Official Ans. by NTA (728)

Sol. We know, $\theta = \left(\frac{\omega_1 + \omega_2}{2}\right)t$

Let number of revolutions be N

$\therefore 2\pi N = 2\pi\left(\frac{900+2460}{60 \times 2}\right) \times 26$

N = 728

6. Official Ans. by NTA (2)

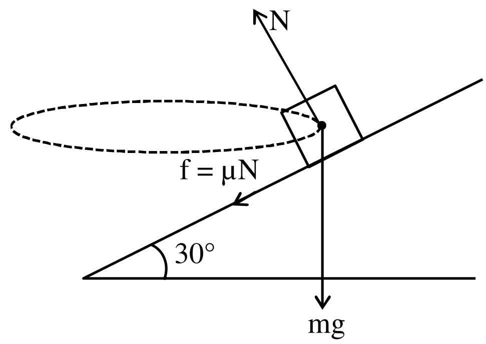
Sol. $\mu_s N = \frac{mv^2}{R}$

$N = \frac{mv^2}{\mu_s R} = mg + F_L$

$F_L = \frac{mv^2}{\mu_s R} - mg$

7. Official Ans. by NTA (1)

Sol.



At v_{max}, f will be limiting in nature.

$\therefore$ Balancing force in vertical direction,

$N\cos30° - mg - \mu N\cos60° = 0$

$\Rightarrow N[\cos30° - \mu\cos60°] = mg$

$\therefore N = \frac{800 \times 10}{(0.87 - 0.1)} \approx 10.2 \times 10^3 \text{ kg m/s}^2$

Hence option 1.

8. Official Ans. by NTA (200)

Sol. $\omega_f = \omega_0 + \alpha t$

$\alpha = 1200 \times 6$

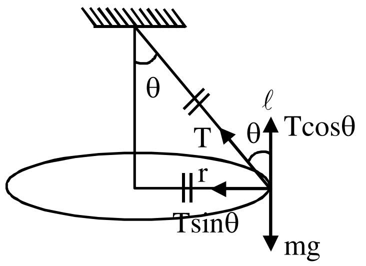
$\theta = \omega_0 t + \frac{1}{2}\alpha t^2$

$= 600 \times \frac{10}{60} + \frac{1}{2} \times 1200 \times 6 \times \frac{1}{36}$

$\theta = 200$

9. Official Ans. by NTA (1)

Sol. Conical pendulum



$r = \frac{\ell}{\sqrt{2}}$

$\sin\theta = \frac{r}{\ell} = \frac{1}{\sqrt{2}}$

$\theta = 45°$

$T\sin\theta = \frac{mv^2}{r}$

$T\cos\theta = mg$

$\tan\theta = \frac{v^2}{rg} \Rightarrow v = \sqrt{rg}$

Ans. 1

10. Official Ans. by NTA (2)

Sol. $R = \frac{\ell}{\theta}$

Time $= \frac{4\times 2\pi R}{v} = \frac{4\times 2\pi}{v}\left(\frac{\ell}{\theta}\right)$

put $\ell = 4.4 \times 9.46 \times 10^{15}$

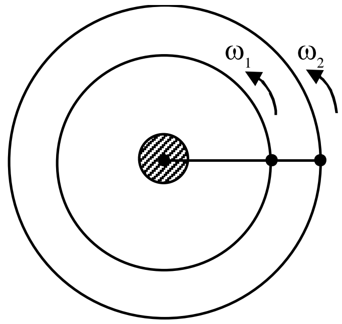
$v = 8 \times 1.5 \times 10^{11}$

$\theta = \frac{4}{3600} \times \frac{\pi}{180}$ rad.

we get time $= 4.5 \times 10^{10}$ sec

11. Official Ans. by NTA (3)

Sol.



$T_1 = 1$ hour

$\Rightarrow \omega_1 = 2\pi$ rad/hour

$T_2 = 8$ hours

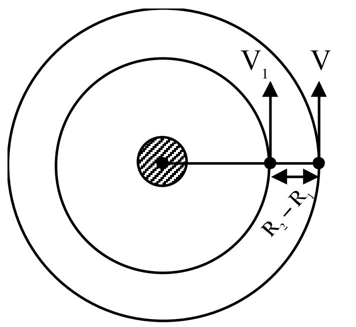
$\Rightarrow \omega_2 = \frac{\pi}{4}$ rad / hour

$R_1 = 2 \times 10^3$ km

As $T^2 \propto R^3$

$\Rightarrow \left(\frac{R_2}{R_1}\right)^3 = \left(\frac{T_2}{T_1}\right)^2$

$\Rightarrow \frac{R_2}{R_1} = \left(\frac{8}{1}\right)^{2/3} = 4 \Rightarrow R_2 = 8\times 10^3$ km



$V_1 = \omega_1 R_1 = 4\pi\times 10^3$ km / h

$V_2 = \omega_2 R_2 = 2\pi\times 10^3$ km / h

Relative $\omega = \frac{V_1 - V_2}{R_2 - R_1} = \frac{2\pi\times 10^3}{6\times 10^3}$

$= \frac{\pi}{3}$ rad / hour x = 3