VECTORS, BASIC MATHS & CALCULUS

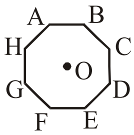
1. A current through a wire depends on time as $i = \alpha_0 t + \beta t^2$ where $\alpha_0 = 20$ A/s and $\beta = 8$ As^{-2}. Find the charge crossed through a section of the wire in 15 s.

(1) 2250 C (2) 11250 C
(3) 2100 C (4) 260 C

2. In an octagon ABCDEFGH of equal side, what is the sum of

$\overrightarrow{AB}+\overrightarrow{AC}+\overrightarrow{AD}+\overrightarrow{AE}+\overrightarrow{AF}+\overrightarrow{AG}+\overrightarrow{AH}'$

if, $\overrightarrow{AO}=2\hat{i}+3\hat{j}-4\hat{k}$



(1) $-16\hat{i}-24\hat{j}+32\hat{k}$ (2) $16\hat{i}+24\hat{j}-32\hat{k}$
(3) $16\hat{i}+24\hat{j}+32\hat{k}$ (4) $16\hat{i}-24\hat{j}+32\hat{k}$

3. If $\vec{A}$ and $\vec{B}$ are two vectors satisfying the relation $\vec{A}\cdot\vec{B}=|\vec{A}\times\vec{B}|$. Then the value of $|\vec{A}-\vec{B}|$ will be :

(1) $\sqrt{A^2+B^2}$ (2) $\sqrt{A^2+B^2+\sqrt{2}AB}$
(3) $\sqrt{A^2+B^2+2AB}$ (4) $\sqrt{A^2+B^2-\sqrt{2}AB}$

4. Two vectors $\vec{P}$ and $\vec{Q}$ have equal magnitudes. If the magnitude of $\vec{P}+\vec{Q}$ is *n* times the magnitude of $\vec{P}-\vec{Q}$, then angle between $\vec{P}$ and $\vec{Q}$ is :

(1) $\sin^{-1}\left(\frac{n-1}{n+1}\right)$ (2) $\cos^{-1}\left(\frac{n-1}{n+1}\right)$
(3) $\sin^{-1}\left(\frac{n^2-1}{n^2+1}\right)$ (4) $\cos^{-1}\left(\frac{n^2-1}{n^2+1}\right)$

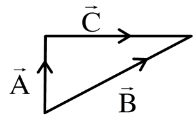
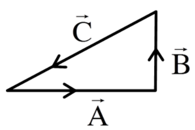
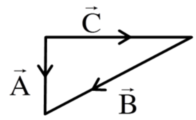
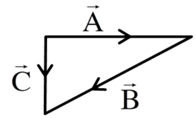
5. What will be the projection of vector $\vec{A}=\hat{i}+\hat{j}+\hat{k}$ on vector $\vec{B}=\hat{i}+\hat{j}$?

(1) $\sqrt{2}(\hat{i}+\hat{j}+\hat{k})$ (2) $2(\hat{i}+\hat{j}+\hat{k})$
(3) $\sqrt{2}(\hat{i}+\hat{j})$ (4) $(\hat{i}+\hat{j})$

6. Three particles P, Q and R are moving along the vectors $\vec{A}=\hat{i}+\hat{j}$, $\vec{B}=\hat{j}+\hat{k}$ and $\vec{C}=-\hat{i}+\hat{j}$ respectively. They strike on a point and start to move in different directions. Now particle P is moving normal to the plane which contains vector $\vec{A}$ and $\vec{B}$. Similarly particle Q is moving normal to the plane which contains vector $\vec{A}$ and $\vec{C}$. The angle between the direction of motion of P and Q is $\cos^{-1}\left(\frac{1}{\sqrt{x}}\right)$.

Then the value of x is ________.

7. Match **List I** with **List II**.

	List I		List II
(a)	$\vec{C}-\vec{A}-\vec{B}=0$	(i)	
(b)	$\vec{A}-\vec{C}-\vec{B}=0$	(ii)	
(c)	$\vec{B}-\vec{A}-\vec{C}=0$	(iii)	
(d)	$\vec{A}+\vec{B}=-\vec{C}$	(iv)	

Choose the correct answer from the options given below :

(1) (a) → (iv), (b) → (i) , (c) → (iii), (d) → (ii)
(2) (a) → (iv), (b) → (iii) , (c) → (i), (d) → (ii)
(3) (a) → (iii), (b) → (ii) , (c) → (iv), (d) → (i)
(4) (a) → (i), (b) → (iv) , (c) → (ii), (d) → (iii)

8. Two vectors $\vec{X}$ and $\vec{Y}$ have equal magnitude. The magnitude of $(\vec{X}-\vec{Y})$ is n times the magnitude of $(\vec{X}+\vec{Y})$. The angle between $\vec{X}$ and $\vec{Y}$ is :

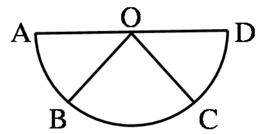
(1) $\cos^{-1}\left(\frac{-n^2-1}{n^2-1}\right)$ (2) $\cos^{-1}\left(\frac{n^2-1}{-n^2-1}\right)$
(3) $\cos^{-1}\left(\frac{n^2+1}{-n^2-1}\right)$ (4) $\cos^{-1}\left(\frac{n^2+1}{n^2-1}\right)$

9. **Assertion A :** If A, B, C, D are four points on a semi-circular arc with centre at 'O' such that $\left|\overrightarrow{AB}\right| = \left|\overrightarrow{BC}\right| = \left|\overrightarrow{CD}\right|$, then

$\overrightarrow{AB} + \overrightarrow{AC} + \overrightarrow{AD} = 4\,\overrightarrow{AO} + \overrightarrow{OB} + \overrightarrow{OC}$

Reason R : Polygon law of vector addition yields

$\overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CD} + \overrightarrow{AD} = 2\,\overrightarrow{AO}$



In the light of the above statements, choose the **most appropriate** answer from the options given below :

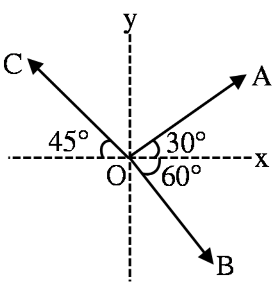
(1) **A** is correct but **R** is not correct.

(2) **A** is not correct but **R** is correct.

(3) Both **A** and **R** are correct and **R** is the correct explanation of **A**.

(4) Both **A** and **R** are correct but **R** is not the correct explanation of **A**.

10. The magnitude of vectors $\overrightarrow{OA}$, $\overrightarrow{OB}$ and $\overrightarrow{OC}$ in the given figure are equal. The direction of $\overrightarrow{OA} + \overrightarrow{OB} - \overrightarrow{OC}$ with x-axis will be :-



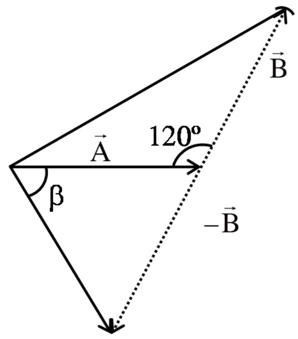
(1) $\tan^{-1}\dfrac{(1-\sqrt{3}-\sqrt{2})}{(1+\sqrt{3}+\sqrt{2})}$

(2) $\tan^{-1}\dfrac{\left(\sqrt{3}-1+\sqrt{2}\right)}{\left(1+\sqrt{3}-\sqrt{2}\right)}$

(3) $\tan^{-1}\dfrac{\left(\sqrt{3}-1+\sqrt{2}\right)}{\left(1-\sqrt{3}+\sqrt{2}\right)}$

(4) $\tan^{-1}\dfrac{\left(1+\sqrt{3}-\sqrt{2}\right)}{\left(1-\sqrt{3}-\sqrt{2}\right)}$

11. The angle between vector $\left(\vec{A}\right)$ and $\left(\vec{A}-\vec{B}\right)$ is :

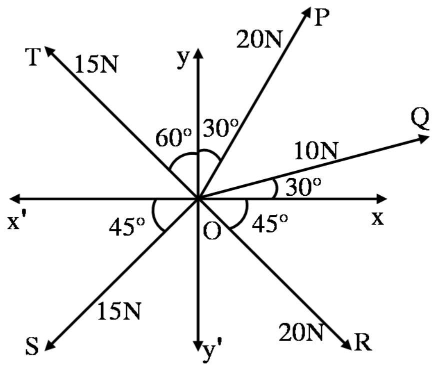


(1) $\tan^{-1}\left(\dfrac{-\dfrac{B}{2}}{A-B\dfrac{\sqrt{3}}{2}}\right)$ (2) $\tan^{-1}\left(\dfrac{A}{0.7B}\right)$

(3) $\tan^{-1}\left(\dfrac{\sqrt{3}B}{2A-B}\right)$ (4) $\tan^{-1}\left(\dfrac{B\cos\theta}{A-B\sin\theta}\right)$

12. The resultant of these forces $\overrightarrow{OP}, \overrightarrow{OQ}, \overrightarrow{OR}, \overrightarrow{OS}$ and $\overrightarrow{OT}$ is approximately ...... N.

[Take $\sqrt{3}=1.7, \sqrt{2}=1.4$ Given $\hat{i}$ and $\hat{j}$ unit vectors along x, y axis]



(1) $9.25\hat{i} + 5\hat{j}$ (2) $3\hat{i} + 15\hat{j}$

(3) $2.5\hat{i} - 14.5\hat{j}$ (4) $-1.5\hat{i} - 15.5\hat{j}$

13. Statement I :

Two forces $(\vec{P}+\vec{Q})$ and $(\vec{P}-\vec{Q})$ where $\vec{P}\perp\vec{Q}$, when act at an angle θ_1 to each other, the magnitude of their resultant is $\sqrt{3(P^2+Q^2)}$, when they act at an angle θ_2, the magnitude of their resultant becomes $\sqrt{2(P^2+Q^2)}$. This is possible only when $\theta_1 < \theta_2$.

Statement II :

In the situation given above.

$\theta_1 = 60°$ and $\theta_2 = 90°$

In the light of the above statements, choose the most appropriate answer from the options given below :-

(1) Statement-I is false but Statement-II is true
(2) Both Statement-I and Statement-II are true
(3) Statement-I is true but Statement-II is false
(4) Both Statement-I and Statement-II are false.

14. Statement : I

If three forces $\vec{F}_1, \vec{F}_2$ and $\vec{F}_3$ are represented by three sides of a triangle and $\vec{F}_1+\vec{F}_2=-\vec{F}_3$, then these three forces are concurrent forces and satisfy the condition for equilibrium.

Statement : II

A triangle made up of three forces $\vec{F}_1, \vec{F}_2$ and $\vec{F}_3$ as its sides taken in the same order, satisfy the condition for translatory equilibrium.

In the light of the above statements, choose the most appropriate answer from the options given below:

(1) Statement-I is false but Statement-II is true
(2) Statement-I is true but Statement-II is false
(3) Both Statement-I and Statement-II are false
(4) Both Statement-I and Statement-II are true.

SOLUTION

1. Official Ans. by NTA (2)

Sol. $i = 20t + 8t^2$

$i = \frac{dq}{dt} \Rightarrow \int dq = \int idt$

$\Rightarrow q = \int_0^{15} (20t + 8t^2)dt$

$q = \left(\frac{20t^2}{2} + \frac{8t^3}{3}\right)_0^{15}$

$q = 10 \times (15)^2 + \frac{8(15)^3}{3}$

$q = 2250 + 9000$

$q = 11250$ C

2. Official Ans. by NTA (2)

Sol. We know,

$\because \overrightarrow{OA} + \overrightarrow{OB} + \overrightarrow{OC} + \overrightarrow{OD} + \overrightarrow{OE} + \overrightarrow{OF} + \overrightarrow{OG} + \overrightarrow{OH} = \vec{0}$

By triangle law of vector addition, we can write

$\overrightarrow{AB} = \overrightarrow{AO} + \overrightarrow{OB}$; $\overrightarrow{AC} = \overrightarrow{AO} + \overrightarrow{OC}$

$\overrightarrow{AD} = \overrightarrow{AO} + \overrightarrow{OD}$; $\overrightarrow{AE} = \overrightarrow{AO} + \overrightarrow{OE}$

$\overrightarrow{AF} = \overrightarrow{AO} + \overrightarrow{OF}$; $\overrightarrow{AG} = \overrightarrow{AO} + \overrightarrow{OG}$

$\overrightarrow{AH} = \overrightarrow{AO} + \overrightarrow{OH}$

Now

$\overrightarrow{AB} + \overrightarrow{AC} + \overrightarrow{AD} + \overrightarrow{AE} + \overrightarrow{AF} + \overrightarrow{AG} + \overrightarrow{AH}$

$= (7\overrightarrow{AO}) + \overrightarrow{OB} + \overrightarrow{OC} + \overrightarrow{OD} + \overrightarrow{OE} + \overrightarrow{OF} + \overrightarrow{OG} + \overrightarrow{OH}$

$= (7\overrightarrow{AO}) + \vec{0} - \overrightarrow{OA}$

$= (7\overrightarrow{AO}) + \overrightarrow{AO}$

$= 8\overrightarrow{AO} = 8(2\hat{i} + 3\hat{j} - 4\hat{k})$

$= 16\hat{i} + 24\hat{j} - 32\hat{k}$

3. Official Ans. by NTA (4)

Sol. $\vec{A} \cdot \vec{B} = |\vec{A} \times \vec{B}|$

$AB\cos\theta = AB\sin\theta \Rightarrow \theta = 45°$

$|\vec{A} - \vec{B}| = \sqrt{A^2 + B^2 - 2AB\cos 45°}$

$= \sqrt{A^2 + B^2 - \sqrt{2}AB}$

Hence option (4).

4. Official Ans. by NTA (4)

Sol. $|\vec{P}| = |\vec{Q}| = x$ …(i)

$|\vec{P} + \vec{Q}| = n|\vec{P} - \vec{Q}|$

$\mathbf{P^2 + Q^2 + 2PQ\cos\theta = n^2(P^2 + Q^2 - 2PQ\cos\theta)}$

Using (i) in above equation

$\cos\theta = \frac{n^2 - 1}{1 + n^2}$

$\theta = \cos^{-1}\left(\frac{n^2 - 1}{n^2 + 1}\right)$

5. Official Ans. by NTA (4)

Sol. $(A\cos\theta)\hat{B} = A\left(\frac{\vec{A}.\vec{B}}{AB}\right)\hat{B} = \frac{\vec{A}.\vec{B}}{B}\hat{B}$

$= \frac{2}{\sqrt{2}}\left(\frac{\hat{i} + \hat{j}}{\sqrt{2}}\right) = \hat{i} + \hat{j}$

6. Official Ans. by NTA (3)

Sol. Direction of P $\hat{v}_1 = \pm\frac{\vec{A} \times \vec{B}}{|\vec{A} \times \vec{B}|} = \pm\frac{\hat{i} - \hat{j} + \hat{k}}{\sqrt{3}}$

Direction of Q $\hat{v}_2 = \pm\frac{\vec{A} \times \vec{C}}{|\vec{A} \times \vec{C}|} = \pm\frac{2\hat{k}}{2} = \pm\hat{k}$

Angle between $\hat{v}_1$ and $\hat{v}_2$

$\frac{\hat{v}_1.\hat{v}_2}{|\hat{v}_1||\hat{v}_2|} = \frac{\pm 1/\sqrt{3}}{(1)(1)} = \pm\frac{1}{\sqrt{3}}$

$\Rightarrow x = 3$

7. Official Ans. by NTA (2)

Sol. (a) $\vec{C} = \vec{A} + \vec{B}$

Option (iv)

(b) $\vec{A} = \vec{B} + \vec{C} = \vec{C} + \vec{B}$

Option (iii)

(c) $\vec{B} = \vec{A} + \vec{C}$

Option (i)

(d) $\vec{A} + \vec{B} + \vec{C} = 0$

Option (ii)

8. Official Ans. by NTA (2)

Sol. Given X = Y

$\sqrt{X^2 + Y^2 - 2\times Y\cos\theta}$

$= n\sqrt{X^2 + Y^2 + 2\times Y\cos\theta}$

Square both sides

$2X^2(1-\cos\theta) = n^2.2X^2(1+\cos\theta)$

$1 - \cos\theta = n^2 + n^2\cos\theta$

$\cos\theta = \dfrac{1-n^2}{1+n^2}$

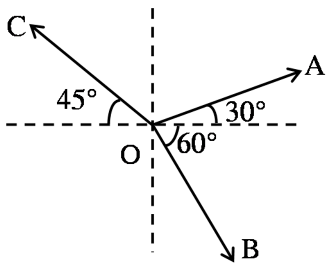
$\theta = \cos^{-1}\left[\dfrac{n^2-1}{-n^2-1}\right]$

9. Official Ans. by NTA (4)

Sol. Polygon law is applicable in both but the equation given in the reason is not useful in explaining the assertion.

10. Official Ans. by NTA (1)

Sol.



Let magnitude be equal to λ.

$\overrightarrow{OA} = \lambda\left[\cos 30°\hat{i} + \sin 30\hat{j}\right] = \lambda\left[\dfrac{\sqrt{3}}{2}\hat{i} + \dfrac{1}{2}\hat{j}\right]$

$\overrightarrow{OB} = \lambda\left[\cos 60°\hat{i} - \sin 60\hat{j}\right] = \lambda\left[\dfrac{1}{2}\hat{i} - \dfrac{\sqrt{3}}{2}\hat{j}\right]$

$\overrightarrow{OC} = \lambda\left[\cos 45°\left(-\hat{i}\right) + \sin 45\hat{j}\right] = \lambda\left[-\dfrac{1}{\sqrt{2}}\hat{i} + \dfrac{1}{\sqrt{2}}\hat{j}\right]$

$\therefore \overrightarrow{OA} + \overrightarrow{OB} - \overrightarrow{OC}$

$= \lambda\left[\left(\dfrac{\sqrt{3}+1}{2} + \dfrac{1}{\sqrt{2}}\right)\hat{i} + \left(\dfrac{1}{2} - \dfrac{\sqrt{3}}{2} - \dfrac{1}{\sqrt{2}}\right)\hat{j}\right]$

∴ Angle with x-axis

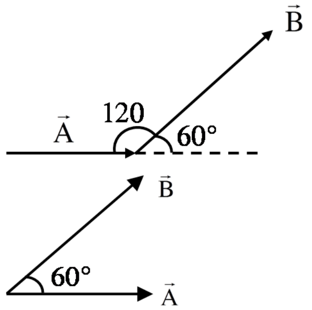
$\tan^{-1}\left[\dfrac{\dfrac{1}{2} - \dfrac{\sqrt{3}}{2} - \dfrac{1}{\sqrt{2}}}{\dfrac{\sqrt{3}}{2} + \dfrac{1}{2} + \dfrac{1}{\sqrt{2}}}\right] = \tan^{-1}\left[\dfrac{\sqrt{2}-\sqrt{6}-2}{\sqrt{6}+\sqrt{2}+2}\right]$

$= \tan^{-1}\left[\dfrac{1-\sqrt{3}-\sqrt{2}}{\sqrt{3}+1+\sqrt{2}}\right]$

Hence option (1)

11. Official Ans. by NTA (3)

Sol.



Angle between $\vec{A}$ and $\vec{B}$, θ = 60°

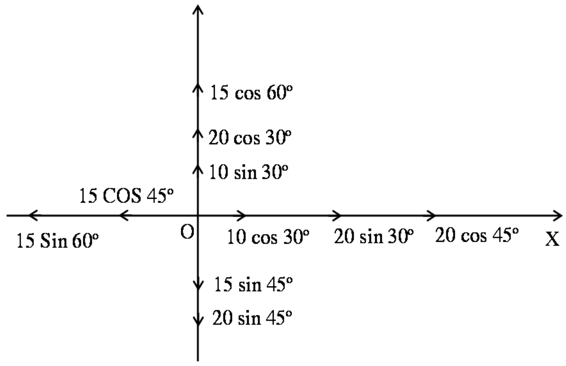
Angle betwenn $\vec{A}$ and $\vec{A} - \vec{B}$

$\tan\alpha = \dfrac{B\sin\theta}{A - B\cos\theta} = \dfrac{B\sqrt{\dfrac{3}{2}}}{A - B\times\dfrac{1}{2}2}$

$\tan\alpha = \dfrac{\sqrt{3}B}{2A - B}$ Ans 3

12. Official Ans. by NTA (1)

Sol.



$\vec{F}_x = \left(10\times\dfrac{\sqrt{3}}{2} + 20\left(\dfrac{1}{2}\right) + 20\left(\dfrac{1}{\sqrt{2}}\right) - 15\left(\dfrac{1}{\sqrt{2}}\right) - 15\left(\dfrac{\sqrt{3}}{2}\right)\right)\hat{i}$

$= 9.25\ \hat{i}$

$\vec{F}_y = \left(15\left(\dfrac{1}{2}\right) + 20\left(\dfrac{\sqrt{3}}{2}\right) + 10\left(\dfrac{1}{2}\right) - 15\left(\dfrac{1}{\sqrt{2}}\right) - 20\left(\dfrac{1}{\sqrt{2}}\right)\right)\hat{j}$

$= 5\ \hat{j}$

13. Official Ans. by NTA (2)

Sol. $\vec{A} = \vec{P} + \vec{Q}$

$\vec{B} = \vec{P} - \vec{Q}$ $\vec{P} \perp \vec{Q}$

$|\vec{A}| = |\vec{B}| = \sqrt{P^2 + Q^2}$

$|\vec{A} + \vec{B}| = \sqrt{2(P^2 + Q^2)(1+\cos\theta)}$

For $|\vec{A} + \vec{B}| = \sqrt{3(P^2 + Q^2)}$

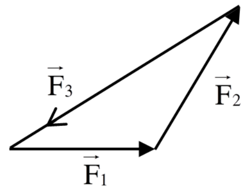
$\theta_1 = 60°$

For $|\vec{A} + \vec{B}| = \sqrt{2(P^2 + Q^2)}$

$\theta_2 = 90°$

14. Official Ans. by NTA (4)

Sol.



Here $\vec{F}_1 + \vec{F}_2 + \vec{F}_3 = 0$

$\vec{F}_1 + \vec{F}_2 = -\vec{F}_3$

Since $\vec{F}_{net} = 0$ (equilibrium)

Both statements correct