

TRIGONOMETRIC EQUATION

- समीकरण,  $\sin^7 x + \cos^7 x = 1$  के  $x \in [0, 4\pi]$  में हलों की संख्या है -  
 (1) 11 (2) 7  
 (3) 5 (4) 9
- $[0, 2\pi]$  में  $x$  के सभी मानों, जिनके लिए  $\sin x + \sin 2x + \sin 3x + \sin 4x = 0$  है, का योग है :  
 (1)  $8\pi$  (2)  $11\pi$   
 (3)  $12\pi$  (4)  $9\pi$
- यदि  $\sin \theta + \cos \theta = \frac{1}{2}$ , है, तो  $16(\sin(2\theta) + \cos(4\theta) + \sin(6\theta))$  बराबर है :  
 (1) 23 (2) -27 (3) -23 (4) 27
- समीकरण  $\frac{\cos x}{1 + \sin x} = |\tan 2x|$ ,  
 $x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right) - \left\{\frac{\pi}{4}, -\frac{\pi}{4}\right\}$  के हलों का योग है :  
 (1)  $-\frac{11\pi}{30}$  (2)  $\frac{\pi}{10}$   
 (3)  $-\frac{7\pi}{30}$  (4)  $-\frac{\pi}{15}$
- माना  $[0, 4\pi]$  में समीकरण  $\sin^4 \theta + \cos^4 \theta - \sin \theta \cos \theta = 0$  के सभी हलों (रेडियन में) का योग  $S$  है। तो  $\frac{8S}{\pi}$  बराबर है \_\_\_\_\_।
- समीकरण  $32^{\tan^2 x} + 32^{\sec^2 x} = 81$ ,  $0 \leq x \leq \frac{\pi}{4}$  के हलों की संख्या है :  
 (1) 3 (2) 1  
 (3) 0 (4) 2

- $\theta \in [0, 2\pi]$  के सभी संभव मान, जिनके लिए  $\sin 2\theta + \tan 2\theta > 0$  है, निम्न में से किस में हैं ?  
 (1)  $\left(0, \frac{\pi}{2}\right) \cup \left(\pi, \frac{3\pi}{2}\right)$   
 (2)  $\left(0, \frac{\pi}{2}\right) \cup \left(\frac{\pi}{2}, \frac{3\pi}{4}\right) \cup \left(\pi, \frac{7\pi}{6}\right)$   
 (3)  $\left(0, \frac{\pi}{4}\right) \cup \left(\frac{\pi}{2}, \frac{3\pi}{4}\right) \cup \left(\frac{3\pi}{2}, \frac{11\pi}{6}\right)$   
 (4)  $\left(0, \frac{\pi}{4}\right) \cup \left(\frac{\pi}{2}, \frac{3\pi}{4}\right) \cup \left(\pi, \frac{5\pi}{4}\right) \cup \left(\frac{3\pi}{2}, \frac{7\pi}{4}\right)$
- यदि  $0 < x, y < \pi$  तथा  $\cos x + \cos y - \cos(x+y) = \frac{3}{2}$ , है, तो  $\sin x + \cos y$  बराबर है :  
 (1)  $\frac{1}{2}$  (2)  $\frac{1+\sqrt{3}}{2}$  (3)  $\frac{\sqrt{3}}{2}$  (4)  $\frac{1-\sqrt{3}}{2}$
- 'k' के पूर्णाकीय मानों, जिनके लिए समीकरण  $3\sin x + 4\cos x = k + 1$  का एक हल है,  $k \in \mathbb{R}$  है, की संख्या है \_\_\_\_\_।
- समीकरण  $\sqrt{3}(\cos^2 x) = (\sqrt{3} - 1)\cos x + 1$ , जबकि  $x \in \left[0, \frac{\pi}{2}\right]$ , के हलों की संख्या है \_\_\_\_\_।
- यदि  $x \in \left(0, \frac{\pi}{2}\right)$  के लिए,  $\log_{10} \sin x + \log_{10} \cos x = -1$  तथा  $\log_{10}(\sin x + \cos x) = \frac{1}{2}$  ( $\log_{10} n - 1$ ),  $n > 0$  हैं, तो  $n$  का मान बराबर है :  
 (1) 20 (2) 12 (3) 9 (4) 16
- अंतराल  $[0, \pi]$  में समीकरण  $(81)^{\sin^2 x} + (81)^{\cos^2 x} = 30$  के मूलों की संख्या है :  
 (1) 3 (2) 4 (3) 8 (4) 2
- अंतराल  $[0, 2\pi]$  में समीकरण  $|\cot x| = \cot x + \frac{1}{\sin x}$  के हलों की संख्या है \_\_\_\_\_।

### SOLUTION

#### 1. Official Ans. by NTA (3)

**Sol.**  $\sin^7 x \leq \sin^2 x \leq 1 \quad \dots(1)$

and  $\cos^7 x \leq \cos^2 x \leq 1 \quad \dots(2)$

also  $\sin^2 x + \cos^2 x = 1$

$\Rightarrow$  equality must hold for (1) & (2)

$\Rightarrow \sin^7 x = \sin^2 x$  &  $\cos^7 x = \cos^2 x$

$\Rightarrow \sin x = 0$  &  $\cos x = 1$

or

$\cos x = 0$  &  $\sin x = 1$

$\Rightarrow x = 0, 2\pi, 4\pi, \frac{\pi}{2}, \frac{5\pi}{2}$

$\Rightarrow$  5 solutions

#### 2. Official Ans. by NTA (4)

**Sol.**  $(\sin x + \sin 4x) + (\sin 2x + \sin 3x) = 0$

$\Rightarrow 2 \sin \frac{5x}{2} \left\{ \cos \frac{3x}{2} + \cos \frac{x}{2} \right\} = 0$

$\Rightarrow 2 \sin \frac{5x}{2} \left\{ 2 \cos x \cos \frac{x}{2} \right\} = 0$

$2 \sin \frac{5x}{2} = 0 \Rightarrow \frac{5x}{2} = 0, \pi, 2\pi, 3\pi, 4\pi, 5\pi$

$\Rightarrow x = 0, \frac{2\pi}{5}, \frac{4\pi}{5}, \frac{6\pi}{5}, \frac{8\pi}{5}, 2\pi$

$\cos \frac{x}{2} = 0 \Rightarrow \frac{x}{2} = \frac{\pi}{2} \Rightarrow x = \pi$

$\cos x = 0 \Rightarrow x = \frac{\pi}{2}, \frac{3\pi}{2}$

So sum =  $6\pi + \pi + 2\pi = 9\pi$

#### 3. Official Ans. by NTA (3)

**Sol.**  $\sin \theta + \cos \theta = \frac{1}{2}$

$\sin^2 \theta + \cos^2 \theta + 2 \sin \theta \cos \theta = \frac{1}{4}$

$\sin 2\theta = -\frac{3}{4}$

**Now :**

$\cos 4\theta = 1 - 2 \sin^2 2\theta$

$= 1 - 2 \left( -\frac{3}{4} \right)^2$

$= 1 - 2 \times \frac{9}{16} = -\frac{1}{8}$

$\sin 6\theta = 3 \sin 2\theta - 4 \sin^3 2\theta$

$= (3 - 4 \sin^2 2\theta) \cdot \sin 2\theta$

$= \left[ 3 - 4 \left( \frac{9}{16} \right) \right] \cdot \left( -\frac{3}{4} \right)$

$\Rightarrow \left[ \frac{3}{4} \right] \times \left( -\frac{3}{4} \right) = -\frac{9}{16}$

$16[\sin 2\theta + \cos 4\theta + \sin 6\theta]$

$16 \left( -\frac{3}{4} - \frac{1}{8} - \frac{9}{16} \right) = -23$

#### 4. Official Ans. by NTA (1)

**Sol.**  $\frac{\cos x}{1 + \sin x} = |\tan 2x|$

$\Rightarrow \frac{\cos^2 x / 2 - \sin^2 x / 2}{(\cos x / 2 + \sin x / 2)} = |\tan 2x|$

$\Rightarrow \tan^2 \left( \frac{\pi}{4} - \frac{x}{2} \right) = \tan^2 2x$

$\Rightarrow 2x = n\pi \pm \left( \frac{\pi}{4} - \frac{x}{2} \right)$

$\Rightarrow x = \frac{-3\pi}{10}, \frac{-\pi}{6}, \frac{\pi}{10}$

or sum =  $\frac{-11\pi}{6}$ .

**5. Official Ans. by NTA (56)**

**Sol.** Given equation

$$\sin^4 \theta + \cos^4 \theta - \sin \theta \cos \theta = 0$$

$$\Rightarrow 1 - \sin^2 \theta \cos^2 \theta - \sin \theta \cos \theta = 0$$

$$\Rightarrow 2 - (\sin 2\theta)^2 - \sin 2\theta = 0$$

$$\Rightarrow (\sin 2\theta)^2 + (\sin 2\theta) - 2 = 0$$

$$\Rightarrow (\sin 2\theta + 2)(\sin 2\theta - 1) = 0$$

$$\Rightarrow \sin 2\theta = 1 \text{ or } \boxed{\sin 2\theta = -2}$$

(not possible)

$$\Rightarrow 2\theta = \frac{\pi}{2}, \frac{5\pi}{2}, \frac{9\pi}{2}, \frac{13\pi}{2}$$

$$\Rightarrow \theta = \frac{\pi}{4}, \frac{5\pi}{4}, \frac{9\pi}{4}, \frac{13\pi}{4}$$

$$\Rightarrow S = \frac{\pi}{4} + \frac{5\pi}{4} + \frac{9\pi}{4} + \frac{13\pi}{4} = 7\pi$$

$$\Rightarrow \frac{8S}{\pi} = \frac{8 \times 7\pi}{\pi} = 56.00$$

**6. Official Ans. by NTA (2)**

**Sol.**  $(32)^{\tan^2 x} + (32)^{\sec^2 x} = 81$

$$\Rightarrow (32)^{\tan^2 x} + (32)^{1+\tan^2 x} = 81$$

$$\Rightarrow (32)^{\tan^2 x} = \frac{81}{33}$$

In interval  $\left[0, \frac{\pi}{4}\right]$  only one solution

**7. Official Ans. by NTA (4)**

**Sol.**  $\sin 2\theta + \tan 2\theta > 0$

$$\Rightarrow \sin 2\theta + \frac{\sin 2\theta}{\cos 2\theta} > 0$$

$$\Rightarrow \sin 2\theta \frac{(\cos 2\theta + 1)}{\cos 2\theta} > 0 \Rightarrow \tan 2\theta (2 \cos^2 \theta) > 0$$

Note :  $\cos 2\theta \neq 0$

$$\Rightarrow 1 - 2 \sin^2 \theta \neq 0 \Rightarrow \sin \theta \neq \pm \frac{1}{\sqrt{2}}$$

Now,  $\tan 2\theta (1 + \cos 2\theta) > 0$

$$\Rightarrow \tan 2\theta > 0 \quad (\text{as } \cos 2\theta + 1 > 0)$$

$$\Rightarrow 2\theta \in \left(0, \frac{\pi}{2}\right) \cup \left(\pi, \frac{3\pi}{2}\right) \cup \left(2\pi, \frac{5\pi}{2}\right) \cup \left(3\pi, \frac{7\pi}{2}\right)$$

$$\Rightarrow \theta \in \left(0, \frac{\pi}{4}\right) \cup \left(\frac{\pi}{2}, \frac{3\pi}{4}\right) \cup \left(\pi, \frac{5\pi}{4}\right) \cup \left(\frac{3\pi}{2}, \frac{7\pi}{4}\right)$$

As  $\sin \theta \neq \pm \frac{1}{\sqrt{2}}$ ; which has been already considered

**8. Official Ans. by NTA (2)**

**Sol.**  $\cos x + \cos y - \cos(x+y) = \frac{3}{2}$

$$\cos^2 \left(\frac{x+y}{2}\right) - \cos \left(\frac{x+y}{2}\right) \cdot \cos \left(\frac{x-y}{2}\right)$$

$$+ \frac{1}{4} \cdot \cos^2 \left(\frac{x-y}{2}\right) + \frac{1}{4} \sin^2 \left(\frac{x-y}{2}\right) = 0$$

$$\Rightarrow \left(\cos \left(\frac{x+y}{2}\right) - \frac{1}{2} \cos \left(\frac{x-y}{2}\right)\right)^2 + \frac{1}{4} \sin^2 \left(\frac{x-y}{2}\right) = 0$$

$$\Rightarrow \sin \left(\frac{x-y}{2}\right) = 0 \text{ and } \cos \left(\frac{x+y}{2}\right) = \frac{1}{2} \cos \left(\frac{x-y}{2}\right)$$

$$\Rightarrow x = y \text{ and } \cos x = \frac{1}{2} = \cos y$$

$$\therefore \sin x = \frac{\sqrt{3}}{2}$$

$$\Rightarrow \sin x + \cos y = \frac{1 + \sqrt{3}}{2}$$

option (2)

**9. Official Ans. by NTA (11)**

**Sol.**  $3 \sin x + 4 \cos x = k + 1$

$$\Rightarrow k + 1 \in \left[-\sqrt{3^2 + 4^2}, \sqrt{3^2 + 4^2}\right]$$

$$\Rightarrow k + 1 \in [-5, 5]$$

$$\Rightarrow k \in [-6, 4]$$

No. of integral values of  $k = 11$

**10. Official Ans. by NTA (1)**

**Sol.**  $\sqrt{3} (\cos x)^2 - \sqrt{3} \cos x + \cos x - 1 = 0$

$$\Rightarrow (\sqrt{3} \cos x + 1)(\cos x - 1) = 0$$

$$\Rightarrow \cos x = 1 \text{ or } \cos x = -\frac{1}{\sqrt{3}} \text{ (reject)}$$

$$\Rightarrow x = 0 \text{ only}$$

## 11. Official Ans. by NTA (2)

$$\text{Sol. } x \in \left(0, \frac{\pi}{2}\right)$$

$$\log_{10} \sin x + \log_{10} \cos x = -1$$

$$\Rightarrow \log_{10} \sin x \cdot \cos x = -1$$

$$\Rightarrow \sin x \cdot \cos x = \frac{1}{10} \quad \dots\dots(1)$$

$$\log_{10}(\sin x + \cos x) = \frac{1}{2}(\log_{10} n - 1)$$

$$\Rightarrow \sin x + \cos x = 10^{\left(\log_{10} \sqrt{n-1}\right)} = \sqrt{\frac{n}{10}}$$

by squaring

$$1 + 2\sin x \cdot \cos x = \frac{n}{10}$$

$$\Rightarrow 1 + \frac{1}{5} = \frac{n}{10} \quad \Rightarrow \quad n = 12$$

## 12. Official Ans. by NTA (2)

$$\text{Sol. } (81)^{\sin^2 x} + (81)^{\cos^2 x} = 30$$

$$(81)^{\sin^2 x} + \frac{(81)^1}{(18)^{\sin^2 x}} = 30$$

$$(81)^{\sin^2 x} = t$$

$$t + \frac{81}{t} = 30$$

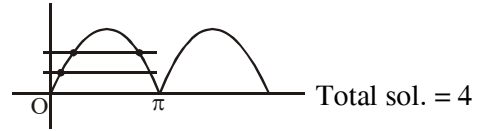
$$t^2 - 30t + 81 = 0$$

$$(t - 3)(t - 27) = 0$$

$$(81)^{\sin^2 x} = 3^1 \quad \text{or} \quad (81)^{\sin^2 x} = 3^3$$

$$3^{4\sin^2 x} = 3^1 \quad \text{or} \quad 3^{4\sin^2 x} = 3^3$$

$$\sin^2 x = \frac{1}{4} \quad \text{or} \quad \sin^2 x = \frac{3}{4}$$



## 13. Official Ans. by NTA (1)

$$\text{Sol. } \text{If } \cot x > 0 \Rightarrow \frac{1}{\sin x} = 0 \text{ (Not possible)}$$

$$\text{If } \cot x < 0 \Rightarrow 2\cot x + \frac{1}{\sin x} = 0$$

$$\Rightarrow 2\cos x = -1$$

$$\Rightarrow x = \frac{2\pi}{3} \text{ or } \frac{4\pi}{3} \text{ (reject)}$$